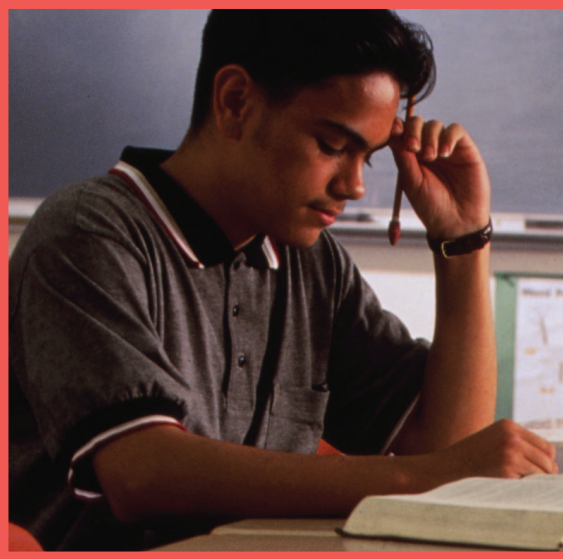
GRADE

8

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills



A Student and Family Guide to Grade 8
Reading • Mathematics • Social Studies

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills

Grade 8

Reading, Mathematics, and Social Studies

A Student and Family Guide

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A Letter from the Associate Commissioner of Education

Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS replaces the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on the TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this book is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Ann Smisko
Associate Commissioner
Curriculum, Assessment, and Technology
Texas Education Agency

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READING

INTRODUCTION

What Is This Book?

This is a study guide to help you strengthen the skills tested on the eighth-grade TAKS test. The guide has three sections—reading, mathematics, and social studies. This is the reading section.

How Is the Reading Section Organized?

The reading section has four parts.

- Part One: "Skills and Strategies" explains the skills and strategies tested on the eighth-grade TAKS test while guiding you through some practice questions.
- Part Two: "Guided Practice" leads you through sample reading selections with practice questions that will give you a chance to practice the skills and strategies you learned in the first part of this guide.
- Part Three: "Independent Practice" provides you with sample reading selections and questions like those on the TAKS test. These selections and questions give you a chance to see how well you understand the skills and strategies tested on TAKS.
- Part Four: "Reading Answer Key" gives you the answers to the practice questions in this guide and explains each of the answer choices.

What Is Tested on TAKS?

Careful readers use many different skills to get the most out of what they read. On TAKS, these reading skills and strategies are grouped under four objectives, or goals for learning. You show how well you understand an objective by answering questions related to some of the skills grouped under that objective. The reading selections and questions in this guide provide help and practice with all four reading objectives.

What Does a TAKS Reading Test Look Like?

The reading section of the eighth-grade TAKS test will include several reading selections. These might be short stories, newspaper or magazine articles, or informational essays. After reading a selection, you will be asked to answer some multiple-choice questions about the passage.

Note to Students

As you work through the sections of this study guide, practice your writing skills by writing your answers in the booklet. In the "Skills and Strategies" section, fill in the "Try It" boxes with your best answers. In the "Guided Practice" section, write your answers to the questions in the margins of the reading selections. Practicing your writing skills now will help you understand what you are reading and help prepare you for the ninth-grade TAKS reading test.

Steps to Success

_____

Student's Name

Directions: Use the following steps as you work through this study guide. When you complete each of the numbered steps below, place a check mark next to that section on the chart.

READING		
1	**Getting Started** • Read "What a Careful Reader Does" on page 11. • Now read "Always Together" on pages 12–14.	✓
2	**Help with Skills** • Read "Help with Skills" on pages 15–54. You should review the skills and strategies presented there.	✓
3	**"Always Together" Again** • Read "Always Together" again on pages 57–60. Answer the questions in the margins of the story as you read. • Read "Practice with Reading Skills" on pages 61–66.	✓
4	**"Ready If Needed: King Gill, the 12th Man"** • Read "Ready If Needed: King Gill, the 12th Man" on pages 67–69. Answer the questions in the margins of the story as you read. • Answer the practice questions on pages 70–73. Check your answers in the answer key on pages 87–89.	✓
5	**"Plaza Theater Ends 75-Year Run" and "Letter to the Editor"** • Read "Plaza Theater Ends 75-Year Run" and "Letter to the Editor" on pages 77–80. • Answer the practice questions on pages 81–83. Check your answers in the answer key on pages 89–91.	✓

Skills and Strategies

- What a Careful Reader Does
- "Always Together"
- Help with Skills

The purpose of pages 11–54 is to provide practice with the reading skills and strategies tested on the eighth-grade TAKS reading test. Read through each part and practice with the examples.

Throughout the skill and strategy explanations in this section, you will find "Try It" boxes with a number of blank lines following the questions. Practice your writing skills by writing your answers to these questions on the blank lines provided.

This section is not meant to be read all at once. You will benefit from working in short sessions that take place every day. If at any time you feel frustrated, take a break, ask for help, and try again later.

What a Careful Reader Does

BEFORE reading, a careful reader sets a purpose for reading. The reader asks

Why am I reading this selection?

Am I reading for entertainment or for information?

What does the selection seem to be about?

Is it about something or someone I already know?

Is it about something new I am learning?

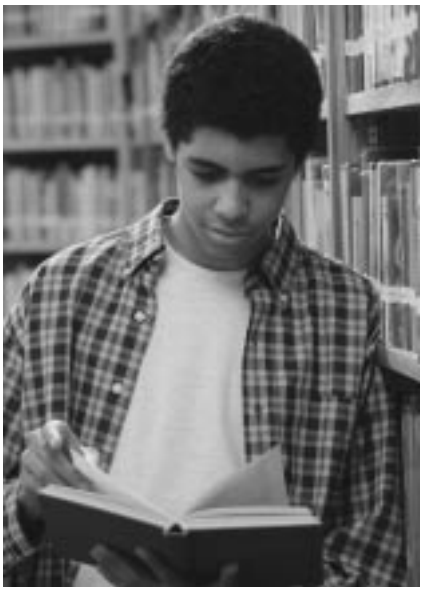
Is it about something I want to learn?

What kind of reading will I do?

Will I read about characters in a story?

Will I read to learn how to do something?

Will I read to discover interesting facts?



DURING reading, a careful reader asks

Do I understand what I'm reading?

Do I need to slow down?

Are there any words I don't know? Can I figure out what they mean?

Do I need to look for clues?

Do I need to read some parts over again?

How can I connect with what I'm reading?

Is it something I already know?

Is it something new I am learning?

Is it something I want to know more about?

AFTER reading, a careful reader asks

What do I remember about the selection?

Can I name the most important ideas in it?

Can I tell someone what the story was about?

Can I think of other ways to show that I understand it?

What do I think about the selection?

Did it add to something I already knew?

Did it tell me something new?

Did it make me want to learn more?

"Always Together"

Read the story below. It will be used throughout the skill and strategy explanations in the "Help with Skills" section that follows. The story is a sample of the type of selection you might find on a TAKS test. As you read, you may notice a number next to each paragraph. The paragraphs are numbered, just like they are on the TAKS test, to help you when answering questions.

Always Together

1 Desiree looked at her grandfather, who was half-asleep in the big recliner, and sighed. Why did Grandpa have to get sick today? She didn't blame Grandpa for feeling ill. She always enjoyed spending time with him, but today she had other plans. She was going to play basketball with her friend Marvin that afternoon. Because her mother had taken Desiree's brother to a birthday party, Desiree had to stay home in case her grandfather needed anything.

2 Desiree put the magazine down and studied Grandpa's weathered face. She couldn't believe it had been five years since he had come to live with her family. She loved having him around. He had more patience than her parents and was always ready to listen to her problems or help with her homework. Even when Desiree was in trouble, Grandpa could make her smile. He would tell her a story about a time when her mother had gotten into trouble, and they would laugh together. No matter how upset Desiree was, Grandpa could always make her laugh, but today he was just dozing in the living room. Desiree could barely <u>contain</u> her disappointment, though she tried to hide her feelings.

3 "Desiree, you can watch television if you like," Grandpa said without opening his eyes. "It won't bother me."

4 "That's okay, Grandpa," she responded, trying to sound happy. "There's nothing on anyway. Maybe I'll just practice the piano, if you don't mind."

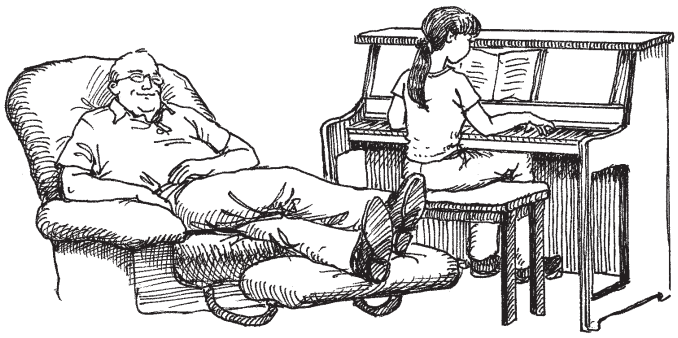
5 "Not a bit." Grandpa's face lit up as he <u>resettled</u> himself in the old chair. "That piano holds a lot of memories for me." Except for the recliner, the piano that had belonged to Desiree's grandmother was the only piece of furniture Desiree's grandfather had brought with him when he came to live with

Continued

them. Desiree sat down at the piano and ran her fingers over the smooth wood, gently sliding the cover back from the keys. She had been taking lessons for about a year.

6 After playing a couple of songs from her practice book, she sat back and frowned. Then, deciding to try something different, she took out her grandmother's old songbook. Desiree had tried playing some of the songs in it a few months before, but they had been too difficult. She thought that maybe she would be able to play some of the pieces now. She was thumbing through the pages, looking at the titles and music, when she found a piece of paper carefully folded between two pages. It was the music to a song called "Always Together." She looked over the notes and smiled. It seemed challenging, but she thought she could play it. Placing the sheet on the music stand, she started playing slowly and tentatively. As she played, she gained confidence and started playing a little faster. The beautiful melody rose and fell, settling over the room like a faint fragrance until, finally, she had reached the last note.

7 "Play it again, Desiree," Grandpa mumbled softly. "I haven't heard that in so long." Desiree looked over at the recliner. Grandpa still seemed to be sleeping. His eyes were closed, and he had a peaceful look on his face. Desiree began playing the song again, happy that Grandpa was enjoying it. She was playing it much better the second time.



8 "Do you remember when we first heard that?" Grandpa asked, startling Desiree. She started to tell him she had never heard it before, but before she could, he continued. "Ah, you were the prettiest girl at the dance. I can still see you in that red dress. I don't know what you ever saw in a farm boy like me," he murmured, chuckling. Desiree became concerned. For a moment

Continued

she almost stopped playing. Was Grandpa getting sicker? Then she noticed the name written in delicate handwriting on the corner of the music: Desiree Walker, her grandmother. Of course! She was named after her grandmother. Grandpa thought her grandmother was playing the piano!

9 "I almost didn't ask you to dance," Grandpa said. Desiree listened intently. She continued to play but remained spellbound, engrossed by his memories. "But I did, and you said yes. And you kept saying yes, even though I couldn't dance a lick." Grandpa laughed as Desiree came to the end of the song. "I was so nervous that I almost forgot to ask you your name." With this, his voice drifted off, but a soft smile remained on his lips. He seemed to be sleeping deeply, dreaming of times gone by. Desiree closed the piano softly and walked over to the couch. She sat there, studying the face she had grown up with. To her he had always been Grandpa, a kind old man who read stories to her, taught her games, and laughed at her corny jokes. Now she imagined him young and <u>vigorous</u>, dancing with his future wife for the first time, his whole life still ahead of him.

10 "We're home," Desiree's mother said as she opened the front door. "Were you two all right here by yourselves?"

11 "Oh, we were fine," Grandpa said, waking up. He seemed to be feeling better. "I had the most wonderful dream. I feel rejuvenated, as if I were 25 years old."

12 "Really? You'll have to tell me about it later," Desiree's mother said. "And what about you, Desiree?" she asked. "Did anything interesting happen to you today?"

13 "Yes," Desiree answered with a thoughtful expression on her face.

14 "Well, you can tell me about it later, too," her mother said.

15 "Maybe I won't," Desiree said quietly to herself. "This is between me and Grandpa."

You have finished Step 1 in your "Steps to Success." Be sure to place a check mark in your chart on page 8.

Help with Skills

Careful readers use many different skills to get the most out of what they read. These reading skills are grouped under four objectives, or goals for learning, that are listed throughout this section. Read through each objective and practice with the examples.

This section is not meant to be completed all at once. Take a break when you need one.

Objective 1: The student will demonstrate a basic understanding of culturally diverse written texts.

What is "a basic understanding"?

Having a basic understanding involves being able to do the following:

- Recognize words and their meanings while reading
- Learn the meanings of new words that you come across while reading
- Learn parts of words such as prefixes and suffixes
- Find the main idea of a paragraph, article, or story
- Find the supporting details in a paragraph, article, or story
- Paraphrase and summarize what you have read

Context Clues

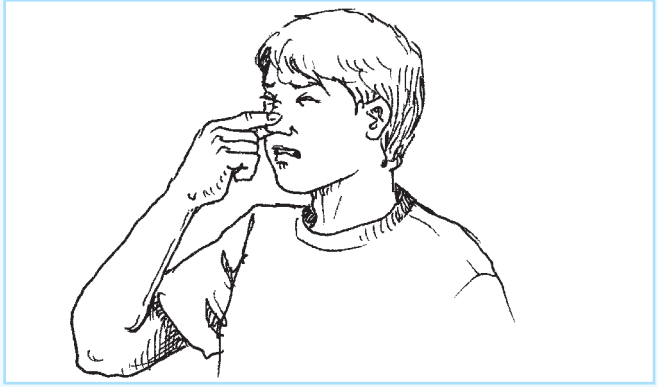
Sometimes when you read, you will see words or expressions that you do not know. Often the surrounding text will give you clues to help you figure out the meanings of the unknown words or expressions. These clues are called **context clues**.

What are some examples of context clues?

Synonyms—A synonym is a word that means the same or almost the same as another word. *Intricate* and *complicated* are synonyms, as are *effortless* and *simple*.

Read the sentences below. Can you find a synonym for the word *pungent*?

The pungent odor of vinegar filled the kitchen. "That vinegar sure smells strong, Dad," said Michael, holding his nose.



The word *strong* is a synonym for *pungent*.

TIP:

✔ Context clues may appear in the sentences near the word, or they may be spread throughout a paragraph or even be in another paragraph.

Antonyms—An antonym is a word that means the opposite of another word. *Ridiculous* and *sensible* are antonyms, as are *marvelous* and *terrible*.

Read the sentence below. Can you find an antonym for the word *tentative*?

Suzanne's first attempts at painting were tentative and awkward, but as she gained experience, she became confident of her artistic skills.

The word *confident* is an antonym for *tentative*.

Explanations, Definitions, and Descriptions—These explain, define, or describe the meanings of other words.

Read the sentences below. Can you find an explanation of the word *isthmus*?

> Chris studied the map carefully. He located the Isthmus of Panama and labled the narrow strip of land. Next he labeled the two landmasses that the strip connected.

The words "narrow strip of land" and "the two landmasses that the strip connected" explain the meaning of *isthmus*.

Examples—An example is an item that is similar to other items in a group. Spanish, Russian, and English are examples of languages. Peaches, mangoes, and apples are examples of fruits.

Read the sentences below. Can you find the word that helps you figure out the meaning of the word *schooner*?

> Kelly watched as several people waved from the schooner. "I'd love to own a boat like that someday," Kelly said to her mom.

A *schooner* is an example of a boat.

Prefixes and Suffixes

Sometimes you can find clues to help you understand the meaning of an unfamiliar word by breaking the word into its parts. A **prefix** is a word part that can be added to the beginning of a word to make a new word. Here are some prefixes that you might come across as you read:

pre- means "before"

de- means "the opposite of"

re- means "again"

Can you figure out the meaning of the underlined word in the sentences below from "Always Together"? Use what you know about prefixes.

"Not a bit." Grandpa's face lit up as he <u>resettled</u> himself in the old chair.

re- (again) + *settled* (to come to rest) = to come to rest again

A **suffix** is a word part that can be added to the end of a word to change the word's meaning. Here are some common suffixes that you might recognize as you read:

-ous means "possessing the qualities of"

-ize means "to cause to be"

-ful means "full of"

Can you figure out the meaning of the underlined word in the sentence below from "Always Together"? Use what you know about suffixes.

Now she imagined him young and <u>vigorous</u>, dancing with his future wife for the first time, his whole life still ahead of him.

vigor (strength) + *-ous* (possessing the qualities of) = possessing the qualities of strength

Try It

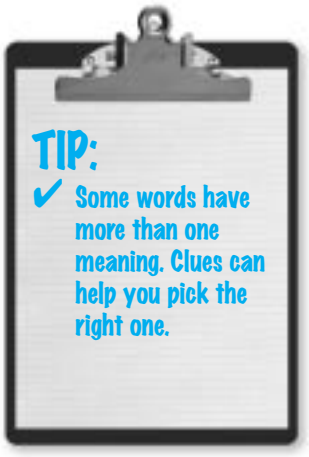
Can you figure out the meaning of the underlined word in the sentence below? Use what you know about prefixes and suffixes.

The coach knew that a loss would demoralize his team.

1. The prefix *de-* means _____.
2. The word *morale* means "confidence of a group."
3. The suffix *-ize* means _____.
4. So the word *demoralize* means _____.

1. The prefix *de-* means "the opposite of."
2. The word *morale* means "confidence of a group."
3. The suffix *-ize* means "to cause to be."
4. So the word *demoralize* means "to cause not to be confident."

Once you are familiar with these word parts, you can find the clues in a word by breaking it into its parts.



Multiple-Meaning Words

Since some words have more than one meaning, you have to consider which meaning the author intends. Keep in mind the context in which you read the word. The context will tell you which meaning to choose.

What does the word *contain* mean in the sentence below?

> The cowboy tried to contain the wild horses in the wooden corral, but it wasn't big enough to hold them all.

If you look up the word *contain* in a dictionary, you might see something like this:

> **contain** \kən-'tān\ *v* **1.** to include as part of **2.** to fit or accommodate **3.** to restrain oneself **4.** to encircle or enclose

In this sentence the words "to hold them" help you know that *contain* means "to encircle or enclose."

Try It

Now look at this sentence from paragraph 2 of "Always Together" on page 12.

> Desiree could barely <u>contain</u> her disappointment, though she tried to hide her feelings.

1. Look back at the dictionary entry for *contain*. What does the word *contain* mean in this sentence?

_____

2. What clues help you to know this?

_____

1. *Contain* means "to restrain oneself."

2. From the words "hide her feelings," you get the idea that Desiree is holding back her disappointment.

Remember that sometimes context clues aren't in the same sentence as the unfamiliar word or phrase. Clues may come before or after the sentence. They might even come before or after the paragraph.

TIP:

✔ Words, phrases, and sentences that surround a word can help you figure out its connotation.

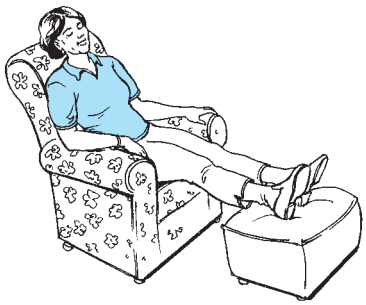
Denotative and Connotative Meanings

The **denotation** of a word is its dictionary definition, or meaning.

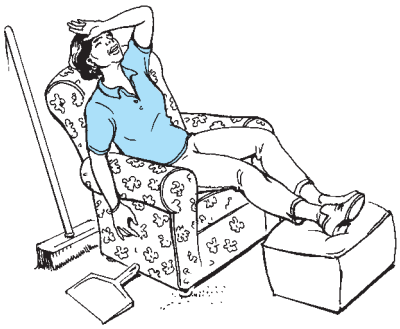
The **connotation** of a word is the feeling associated with it, in addtion to its dictionary meaning.

Words with the same denotation can have very different connotations—for example, *surprised* and *amazed*. Both words have similar meanings. But *amazed* has a stronger feeling connected with it than *surprised* does. Knowing word connotations can help you understand an author's message.

Look at the examples below:



Aunt June sat <u>lazily</u> in the chair.



Aunt June sat <u>wearily</u> in the chair.

Each sentence has a different feeling associated with it and presents Aunt June in a different way. The connotation of *lazily* makes Aunt June appear idle and unmotivated. The connotation of *wearily* makes Aunt June appear worn out or exhausted.

Help with Skills: Objective 1

Main Idea, Supporting Details, and Summary

Main idea—The main idea of a story or paragraph answers the question "What is the story or paragraph mainly about?"

Sometimes the main idea is clearly stated, making it easy to find.

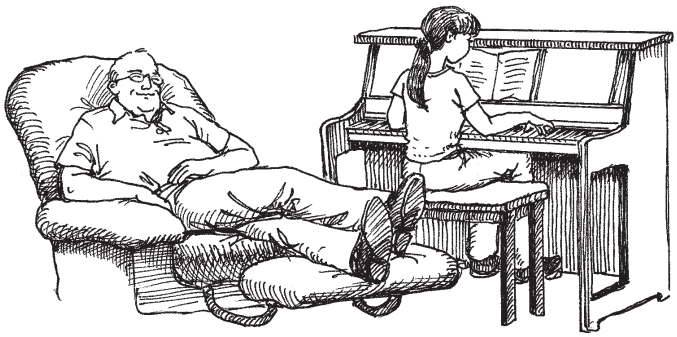
TIP:

✔ To find the main idea, imagine saying to a friend, "Well, basically this story is about _____."

Go back and reread paragraph 2 from "Always Together" on page 12.

What is the main idea of paragraph 2?
Desiree loves her grandfather and has a good relationship with him.

How do you know this is the main idea?
In the third sentence of paragraph 2, you see that Desiree loves having her grandfather around because Grandpa is patient with her and "could always make her laugh." From this information, the reader can determine that Desiree has a good relationship with her grandfather.



Other times the writer does not state the main idea directly in a story or paragraph. That means that you have to pay attention to the supporting details in the story or paragraph to figure out the main idea.

Now return to "Always Together" and reread paragraph 6 on page 13.

What is the main idea of paragraph 6?
Desiree plays songs on the piano and is pleased that she can play a challenging song, "Always Together," from her grandmother's songbook.

How do you know that this is the main idea?
In the first part of the paragraph, you learn that Desiree has decided to get out "her grandmother's old songbook." In the last two sentences of the paragraph, Desiree "gained confidence" as she played "the beautiful melody." These details support the main idea that Desiree is pleased about playing a challenging song.

TIP:
✔ Learning the supporting details of a story or paragraph—who, what, when, where, why, and how—can help you understand its main idea.

Supporting details—Supporting details explain the main ideas or make them clearer. Some details add depth and interest to a story by appealing to the senses and can bring a story to life for the reader. These details also answer the questions *who*, *what*, *when*, *where*, *why*, and *how*.

Another way to think about supporting details is to imagine a bicycle. A bicycle has a frame, a seat, brakes, and two wheels. Think of this bicycle as the main idea.

Now think of a few "extras" that make the bicycle more appealing—15 speeds, an adjustable seat, a shock absorber, and off-road tires. Think of these extras as supporting details. Just as extras can make a bicycle look and perform better, supporting details can make a story more interesting and fun to read.

Photo courtesy of Cannondale Corp.

TIP:
✔ Good readers often stop to summarize what they have read.

Summary—A summary briefly restates the most important events and shows how they are connected. When you write a summary, it is important to paraphrase, or restate, the author's ideas in your own words. A good summary usually tells what the whole story is about but focuses only on the important events.

Help with Skills: Objective 1

Read the following story. What is the main idea? What are the supporting details? How would you summarize the story?

The Barking Dog

"That crazy dog is barking again!" Melanie exclaimed. She leaped out of bed, wondering why she had begged her parents for a dog. Sam's constant barking was annoying, especially in the middle of the night. Melanie rubbed her eyes and with flashlight in hand went to find out what Sam was barking at this time. Sam was part German shepherd and very protective, but Melanie wanted to put an end to the barking.

Stepping out the back door, Melanie immediately heard a hissing noise behind the bushes. Sam's barking grew louder and louder as she ventured closer to the noise. Melanie expected to find a cat but was shocked to discover a small opossum paralyzed with fear, balancing on the fence. She struggled to corner Sam and then dragged him inside. The opossum jumped off the fence and scampered across the backyard to the safety of the neighbor's yard. As Melanie crawled back into bed, she decided that the barking must stop. Tomorrow she was going to take Sam to obedience school.

Main Idea of "The Barking Dog"

After Melanie has trouble stopping Sam's barking, she decides to take him to obedience school.

Supporting Details of "The Barking Dog"

Who is the story about?	Melanie and her dog Sam
What is Melanie trying to do?	She is trying to stop her dog from barking.
When does the story take place?	The story takes place at night.
Where does the story take place?	Melanie is in her bedroom and in her backyard.
Why does the dog continue to bark?	There is an opossum on the fence.
How does the story end?	After Melanie corners Sam and drags him inside, she decides to take him to obedience school.

Summary of "The Barking Dog"

Using the supporting details above, you can summarize the story as follows:

Melanie's dog Sam will not stop barking. When Melanie finds out that the dog is barking at an opossum, she corners Sam and drags him into the house. She realizes that she needs to take Sam to obedience school.

TIP:

✔ Take a text's main idea and the supporting details and put them in your own words to create a summary.

Try It

Return to "Always Together" on pages 12–14. As a review, answer the following questions about the supporting details of the story.

1. **Who** is the story about?

_____

2. **Where** does the story take place?

_____

3. **Why** does Desiree stay home with Grandpa? _____

_____

4. **What** makes Grandpa remember the past? _____

_____

5. **What** does Grandpa remember about his wife? _____

_____

6. **How** has Desiree's view of Grandpa changed by the end of the story?

_____

_____

_____

1. Desiree and Grandpa
2. The story takes place in Desiree's house.
3. She stays home with Grandpa to take care of him because he is sick.
4. Desiree plays the song "Always Together."
5. Grandpa and his wife, also named Desiree, met each other and danced together the first time they heard the song "Always Together."
6. Desiree not only views Grandpa as a kind old man, but she now also understands more about his earlier years as a young and vigorous man.

You can use the answers to the questions above as supporting details to write a summary for "Always Together."

Important Note

Readers can get confused when they talk about main ideas, supporting details, and summaries, so here is a comparison of the three. The **main idea** is the most important idea in a paragraph or reading selection. It can be stated in one sentence. The answers to questions such as *who*, *what*, *when*, *where*, *why*, and *how* are **supporting details**. You create a **summary** when you put together the main idea and the supporting details in your own words. It usually takes several sentences to write a good summary.

Objective 2: The student will apply knowledge of literary elements to understand culturally diverse written texts.

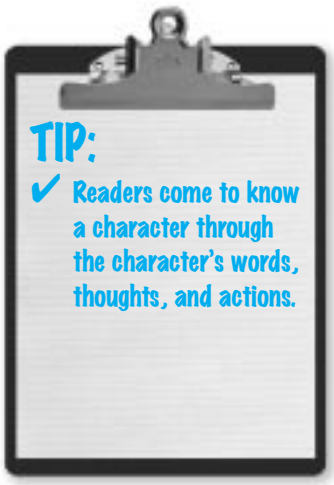
What are literary elements?

Literary elements are the basic parts that an author uses to create a story. These parts include the characters, the setting, the plot, and the conflict in a story.

An understanding of literary elements includes being able to do the following:

- Analyze characters, including their traits, motivations, conflicts, and points of view
- Analyze characters' relationships
- Analyze changes that characters go through, both personally and in their relationships with others or the world around them
- Recognize plot and identify events or conflicts that affect the plot
- Identify a story's setting and its effect on the story's meaning
- Identify how authors use literary devices such as flashback, foreshadowing, and symbolism to shape the reader's understanding of characters, events, and meaning of the story

Analyzing Characters



Characters are the people or animals in stories. You can tell a lot about a character by asking these questions as you read:

- How does the character feel?
- What does the character do?
- What does the character say?
- What do other characters say about the character?
- How do other characters act toward the character?
- How does the character change in the story?

When you **analyze a character**, you look for several kinds of clues in the story. An author often provides information about a character's traits, motivations, conflicts, points of view, relationships, and changes.

Traits—A character in a story has certain traits, or qualities, that make him or her unique. As you read, you'll discover a character's traits. Sometimes a character's traits remain the same; other times traits can change, especially as a character adopts new beliefs and attitudes during the course of the story.

To figure out which traits a character has, ask yourself, "Which words best describe this character?" Then use details from the story to support your choices.

Which of these words best describe Desiree?

caring, *jealous*, *talented*, *arrogant*

If you picked *caring* and *talented*, you are thinking about some important ideas expressed in the story. The story mentions that Desiree cares about Grandpa and what happens to him. You also find out that Desiree is a talented piano player. She is able to play some challenging music. These details from the story support the choice of both words to describe Desiree. There is no evidence from the story to support *jealous* or *arrogant* as character traits for Desiree.

Motivation—What makes characters behave the way they do in a story? A character's motivation is something that makes a character think, feel, or act in a particular way. When you know the reasons for a character's actions, you understand his or her motivation.

Think back to "Always Together." What motivates Desiree to play songs from her grandmother's songbook that were once too difficult for her to play?

Paragraph 6 on page 13 reveals that Desiree grows tired of the pieces she is practicing, so she looks for more challenging pieces in her grandmother's songbook.

You can see that a character's motivation influences his or her actions. The actions, in turn, reveal the character's traits.

Conflict—A conflict is a struggle between two opposing forces. Sometimes a character's traits will contribute to a conflict in a story. A conflict can be between two or more characters, between a character and an outside force, or within a character.

What conflict does Desiree face in "Always Together"?
Desiree faces a conflict within herself.

How do you know?
She loves Grandpa and enjoys spending time with him, but she is also disappointed because she wants to play basketball with her friend. Paragraphs 1 and 2 describe this conflict.

A character's conflict is usually resolved by the end of a story.

Point of View—A character's point of view is the way he or she views the events or circumstances in which he or she is involved. Characters often have differing points of view about a situation or another character.

As you read, be aware of each character's point of view toward other characters or situations in a story.

Relationships—One way to understand a character is to examine the character's relationships with other characters in the story. Does the character get along with others? Is the character in conflict with another character?

Read the following lines from "Always Together." What can you tell about Desiree's relationship with her grandfather in this part of the story?

> Desiree closed the piano softly and walked over to the couch. She sat there, studying the face she had grown up with. To her he had always been Grandpa, a kind old man who read stories to her, taught her games, and laughed at her corny jokes. Now she imagined him young and vigorous, dancing with his future wife for the first time, his whole life still ahead of him.

Desiree has a close, loving relationship with Grandpa. She fondly remembers the man she has grown up with. Desiree is also fascinated by Grandpa's earlier life.

Changes—Just as we are changed by our experiences in real life, characters also undergo changes. A character often changes as different events unfold in a story. The changes often affect the story's outcome.

Try It

Reread paragraph 9 of "Always Together" on page 14.

1. How does Desiree's view of Grandpa change toward the end of the story?

_____

_____

2. How do you know? What clues in the story tell you that her point of view has changed?

_____

_____

3. How does this change affect how the story ends?

_____

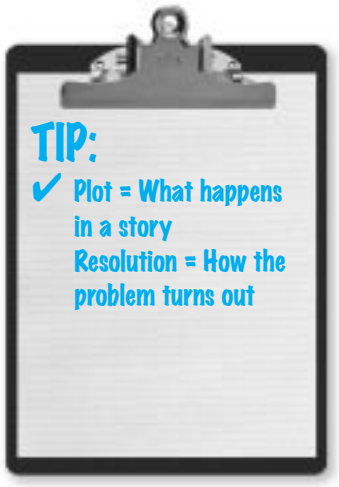
_____

1. Desiree views Grandpa with fascination. She no longer thinks of him as just "a kind old man."

2. At the end of paragraph 9, Desiree studies Grandpa's familiar face and realizes that he was once young and energetic, with "his whole life still ahead of him."

3. Desiree feels closer to Grandpa now that she has seen a different aspect of him.

Help with Skills: Objective 2



Story Plot and Problem Resolution

The **plot** is the sequence of events that make up a story. The story is usually centered around a problem, or **conflict**.

In "Always Together," Desiree must stay home with Grandpa even though she has already made plans. Desiree's problem causes her to feel that her day is ruined.

As one or more characters try to solve a problem, events build to a **climax**, or turning point. Then comes the **resolution**, in which the reader learns how the problem turns out.

To figure out the resolution to the problem, ask yourself, "How does the main character solve his or her problem?"

Use the sequence chart below to lead you through the story plot and problem resolution for "Always Together."

Problem: Desiree feels that her day is ruined.

↓

First event: A disappointed Desiree stays home with Grandpa.

↓

Second event: Desiree decides to play the piano.

↓

Third event: Desiree discovers "Always Together" and begins to play it.

↓

Fourth event: The song reminds Grandpa of the first time he met Desiree's grandmother.

↓

Fifth event: Grandpa describes his memories as Desiree continues to play.

↓

Resolution: A fascinated Desiree no longer feels that her day has been ruined.

How is the problem resolved?
Desiree learns about Grandpa's life as a young man. This new information fascinates her, and she no longer feels that her day has been ruined.

Setting and Its Importance

Setting is the time and place in which the events of a story happen. The setting can be either real or imaginary and can take place in the past, present, or future. In some stories the author is very specific about when and where the events take place. In others the author may tell you either the time or the place that the events occur, but not both.

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TIP:

✔ Setting is more than just a story's location. Settings can create a mood and help you better understand character motivation.

Setting can be an important part of a story. It can create a particular mood or help you understand why characters act as they do or play an important part of the plot. Sometimes the setting will change. When you read, look for clues that tell you about the setting.

> At nine o'clock in the morning on a cool September day, Samantha entered the high school for the first time.

This sentence tells <u>where</u> and <u>when</u> something happens.

> Emily felt the cold hand of fear grip her heart just before the coach blew the whistle.

This sentence tells <u>when</u> something happens.

> A flock of birds landed in the mesquite tree outside Dylan's bedroom window.

This sentence tells <u>where</u> something happens.

Try It

Think back to the story "Always Together" on pages 12–14.

1. What is the setting of this story?

_____

_____

2. Why is the setting important to the story?

_____

_____

_____

1. The story takes place in Desiree's home in the afternoon.

2. The setting is important to the story because it is where Grandpa, Desiree, and the piano all come together. If Grandpa hadn't come to live with Desiree and her family, Desiree might not have heard Grandpa reminiscing about the first time he met his wife. If the setting had been different, the events of the story wouldn't have happened as they did.

You learn from Grandpa's memories about a time and a place that are very different from when and where the story is actually taking place.

Literary Devices

Writers bring their stories to life by using special tools called **literary devices**. These tools include flashback, foreshadowing, and symbolism. Literary devices help you understand characters, events, and the meanings of stories.

Flashback—Have you ever watched a movie in which the image on the screen becomes wavy or hazy and the action shifts from the present to the past? This is a technique often used to indicate a flashback. A flashback interrupts the sequence of events in a story to show something that happened before the story's beginning. The flashback helps the reader understand a character's present situation.

Not all stories are told in chronological order. As you read, pay attention to when each event in a story actually takes place.

Read the story below. Can you tell where the flashback occurs?

Moving Day

Victor stacked the last box on top of the others in the hallway and went back into his room for a last look. As he walked through the familiar doorway, he noticed the pencil marks on the doorjamb. Five marks, one for each year he and his family had lived in the house they were now leaving. The marks showed how much he had grown over the years since his family had first moved into the house. Victor could still remember that first day as clearly as if it had been yesterday.

"And this is your room, Victor," his mother had said, her eyes shining with happiness.

Victor had stood in the open doorway and stared in wonder, not daring to breathe for fear that the spell would be broken. "Do I have to share it with Tony?" he had finally asked in a whisper. For as long as he could remember, he and his little brother Tony had shared a room.

"No, Victor. You and Tony will both have your own rooms," his mother had replied, gently pushing him into the room.

"No way!" he had shouted, running around the empty room, imagining all the things he could do with his new room.

Victor sighed, the memory fading as quickly as it had come. He liked living here, but now that his father had gotten a new job in another state, he and his family were moving again. Victor quietly closed the door to his room, now empty again, and hoped that his new room in his new home would be as nice.

At what point in the story does the forward movement in time stop? The flashback begins when Victor's mother says, "And this is your room, Victor. . . ." The author includes the flashback to help you understand how important the room is to Victor and how sad he is that he has to move.

Help with Skills: Objective 2

Foreshadowing—In foreshadowing, an author gives hints about what might happen later in a story. These clues can be in the form of events in the story. Clues can also come from information from the narrator or a character. Foreshadowing is used to build suspense or tension in a story.

Return to paragraph 5 of "Always Together" on pages 12–13. Reread the first three sentences of the paragraph.

What is foreshadowed by this description?
The description shows that Grandpa cares deeply about the piano. The author is foreshadowing the connection between the piano and Grandpa's memories. When Grandpa sits in the recliner and hears Desiree play "Always Together" on the piano, he remembers the first time he met Desiree's grandmother. This is an important event in the story.

As you read, look for the use of foreshadowing and think about what future event this clue might be suggesting.

Symbolism—Another device writers use is symbolism. A symbol is something that stands for, or represents, something else. For example, the U.S. flag is a symbol of freedom to Americans. What do these symbols represent to you: a flashy car? a red rose? a fox? To many people a flashy car symbolizes vanity; a rose, love; a fox, cunning.

Sometimes an author will use a symbol in a story to create a certain feeling in the reader or make an important point. The writer also uses a symbol to tell the reader something significant about a character or a situation in the story. In a story a symbol can be a person, an object, or even a situation.

Can you find the symbol in the story "Always Together"? Reread paragraphs 7 and 8 on pages 13–14. The author uses a symbol to tell the reader something significant about Grandpa.

What does the song "Always Together" represent, or symbolize, for Grandpa?
The song symbolizes Grandpa's memories of the first time he met Desiree's grandmother.

How do you know?
When Desiree plays the song, Grandpa reminisces about the first time he met his wife. When he hears the song, he is reminded that they will always be together in his memories.

Symbols can often give the reader clues about the theme of a story. As you read, look for symbols the author uses to help you understand the theme of the story.

Important Note

Objectives 3 and 4 both require students to analyze culturally diverse written texts.

What does "analyzing a text" mean?

Analyzing a text means recognizing the way an author organizes information in order to understand the author's purpose. By analyzing a text, careful readers move beyond reading the words on the page and begin to think critically about the information presented.

Objective 3: The student will use a variety of strategies to analyze culturally diverse written texts.

What are "strategies," and how do they help students analyze text?

Strategies help readers think about what they are reading by providing a structure, or framework, for looking at what they are reading in different ways. Reading strategies allow readers to understand the different ways authors present ideas and how these ideas are connected across a text.

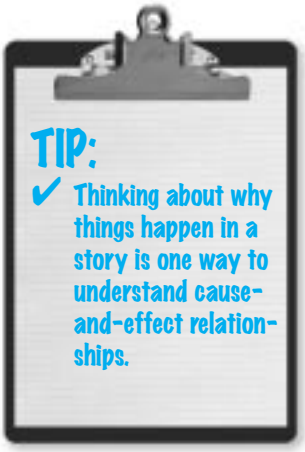
Using reading strategies includes being able to do the following:

- Locate and recall information using organizational clues such as cause and effect and chronological order
- Identify similarities and differences among texts
- Use graphic organizers to analyze texts
- Identify an author's purpose for writing
- Examine ways an author's perspective affects his or her writing

Help with Skills: Objective 3

Using a Text's Structure to Locate and Recall Information

Authors put the events of a story in a certain order so that the events will make sense. When you read, look for patterns in the way events have been organized and how those events relate to one another. You can use patterns of organization, such as cause and effect or chronology, to help you locate information in a selection.



Cause and effect—Authors often write stories as a series of causes and effects. Knowing how to recognize these causes and effects helps you better understand a writer's message. A cause makes something happen. An effect is what happens as a result.

As you read stories and articles, you will notice that authors don't always state every cause-and-effect relationship. Instead, you must use clues from the story to figure out these relationships.

Try It

Find the causes and effects in the paragraphs below:

"Can you help me stake this tent before it rains?" Kamil asked. Seth turned the fish in the frying pan one last time and then walked over to where his friend was standing.

"I'll start on this side," said Seth as he drove a tent stake into the ground. In no time they had almost finished. "This last one is stubborn," Seth said as he wiped his brow.

"I know," Kamil replied. "No matter where I try to move it, it won't sink into the ground. I just keep hitting solid rock." The two boys took turns pounding the stake until it snapped in half.

"Oh, great!" Seth exclaimed, somewhat exasperated. "Now what are we going to do?" They sat down to rest and waited for Kamil's dad to come back from the car.

After a while Kamil sniffed the air. "What's that smell?" he asked.

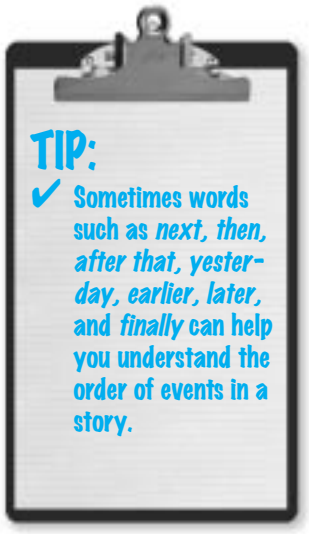
"Oh no!" shouted Seth as he raced over to the campfire.

Cause		Effect
Kamil needs help staking the tent.	→	Seth leaves the fish to fry on the campfire.
Seth forgets the fish.	→	

What was the effect of Seth's forgetting about the fish? Write your answer in the blank box.

Although you are not told directly, you can use the information in the story to conclude that because Seth forgets about the fish, it has burned in the frying pan.

Chronological order — Another way of presenting events is in time (chronological) order, or the sequence in which events occur in the story. Authors will often use clue words to help you see the order clearly.



Read the paragraphs below from "Always Together." What clues help you understand the order in which things happen in the story?

"We're home," Desiree's mother said as she opened the front door. "Were you two all right here by yourselves?"

"Oh, we were fine," Grandpa said, waking up. He seemed to be feeling better. "I had the most wonderful dream. I feel rejuvenated, as if I were 25 years old."

"Really? You'll have to tell me about it later," Desiree's mother said. "And what about you, Desiree?" she asked. "Did anything interesting happen to you today?"

"Yes," Desiree answered with a thoughtful expression on her face.

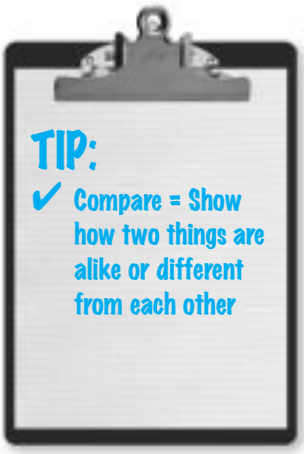
"Well, you can tell me about it later, too," her mother said.

"Maybe I won't," Desiree said quietly to herself. "This is between me and Grandpa."

In this part of the story, what happens first?
Desiree's mother comes home.

What happens after Grandpa wakes up?
He tells Desiree's mother that he feels rejuvenated.

What happens last?
Desiree decides to keep her special moment with Grandpa to herself.



Comparing Two Selections

Have you ever watched a movie or a television show and thought, "Hey, that was kind of like another show I saw"? If you have, you already know how to compare two stories.

No two stories are exactly alike. However, sometimes two pieces of writing can have a lot in common. Here are some ways in which two or more pieces of writing might be similar and different:

- **Main Idea:** Two stories might be about different characters in different settings but still have the importance of courage or some other theme as the main idea.
- **Subject:** You might read a narrative story about a stand-up comic, an autobiography written by a stand-up comic, an article about how stand-up comics make a living, or a poem about the thrill of performing in front of a live audience. Each of these would have the same subject (stand-up comedy), but each would be presented in a different way.
- **Setting:** Stories might be set in the same place and at the same time but be about different characters.
- **Characters:** You might read two stories about the same characters. Or you might read the same story told by two different characters.
- **Plot:** Two stories might have the same plot. Even if the setting and the characters are different, you might recognize that the action is similar in each story.
- **Conflict:** Two stories might have the same basic conflict, such as people struggling against the forces of nature.
- **Organization:** Two authors might choose to organize their writing in similar ways, such as by using chronological order or cause and effect.

At times the eighth-grade TAKS reading test will have two stories that are meant to be read together. These are called paired selections. When reading paired selections, you must be able to recognize how the two stories are similar and how they are different.

Compare the two stories that follow. First, take a look at this story:

A Reminder

Cecilio followed Max slowly as they climbed the rocky cliff and walked along the narrow path. "I know it's got to be here somewhere," he heard Max say. "It seems like just yesterday we were up here, huh, Cee?"

It had been several years since Max had come to visit, and Cecilio had forgotten all about the tree fort. A lot had happened since the summer after fourth grade. Mama had had the baby. Uncle Julio and Aunt Delia had moved in. Papa had lost his job. Max's voice interrupted Cecilio's thoughts.

"Look what I found!" Max called out. He held up a tattered wooden sign that read "Keep Out." "I guess the tree fort is gone, but at least we still have the sign," Max added. Cecilio turned away. He didn't care about the sign. In fact, he hated looking at it. Back then everything had seemed easier, but now nobody ever had time for him. That sign reminded Cecilio only about how different things were now.

Now look at this story:

Just Yesterday

Max and Cecilio climbed the rocky cliff and made their way along the narrow path. "I know it's got to be here somewhere," said Max. "It seems like just yesterday we were up here, huh, Cee?"

Cecilio did not answer. His shoulders slumped, and he turned away from Max. Max tried to study his friend's face. Something was different. He wished Cecilio would confide in him. To Max, it felt like they had just built the tree fort, but it had been several years since that summer after the fourth grade. Things were different now. If only Cecilio would talk to him.

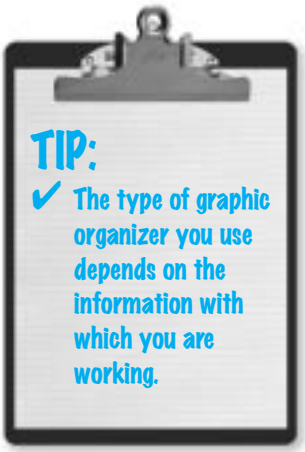
Suddenly Max tripped over an old board. He picked it up, and a smile broke out across his face. "Look what I found!" Max called out. He held up a tattered wooden sign that read "Keep Out." "I guess the tree fort is gone, but at least we still have the sign," Max added. Max hoped that the sign would make Cecilio smile. Maybe remembering that summer would make whatever was bothering Cecilio go away.

How are these two stories alike?
They both have the same characters, setting, and plot. Both stories are centered around a tree fort and a sign, which symbolize the past. Both stories contain a reference to past events that helps the reader understand how each character views the tree fort.

How are these stories different?
Each story is told from a different character's point of view. Each story focuses on a different character's thoughts and feelings, revealing how he feels about the sign.

In "A Reminder," Cecilio is unhappy because nobody seems to have time for him. The sign reminds him of happier times, which in turn makes him feel sad about his present situation. In "Just Yesterday," Max knows that something is bothering Cecilio. He hopes that the sign will bring back good memories and make Cecilio feel happier.

Representing Information in Different Ways



One way to keep track of the information you read is to make notes. Another way is to use graphic organizers such as outlines, time lines, or cluster diagrams. Seeing information in a graphic organizer can help you answer questions about what you read.

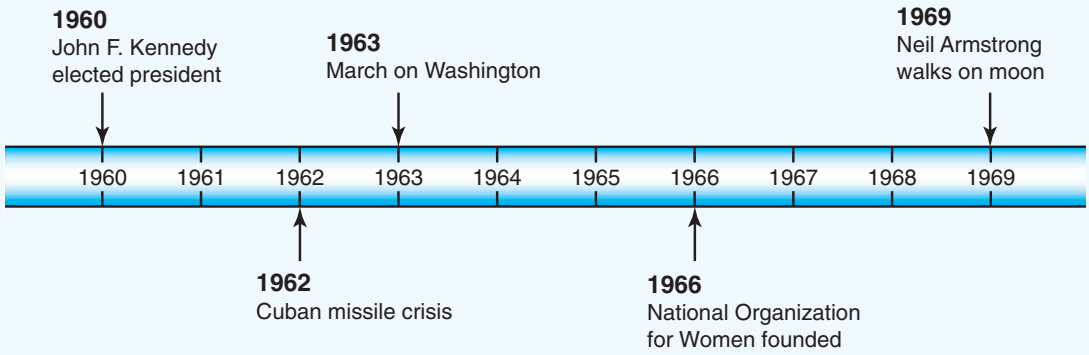
Outlines—An outline is an organized list of main ideas and important details. You can use an outline to show the relationship between ideas in informational text. Outlines usually follow the form shown below.

Skateboarding

I. Equipment
  A. Skateboard
  B. Helmet
  C. Pads
    1. Knee
    2. Elbow
II. Places to skate
  A. Empty swimming pool
  B. Sidewalk
  C. Skatepark
    1. Half-pipe
    2. Funbox

Time lines—This type of graph shows an order of events and the amount of time that passes between each event. Marks on a time line represent units of time.

Suppose you read an informational text about major events that occurred in the 1960s. You might show the information on a time line to help keep track of the order of events.



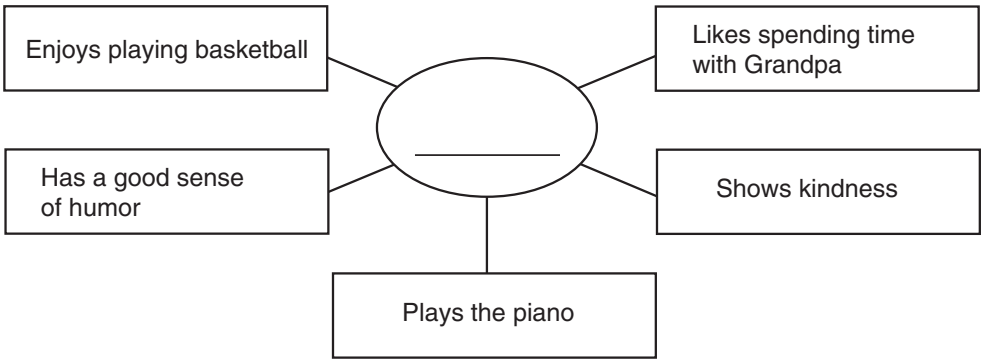
When did Neil Armstrong walk on the moon?
Armstrong walked on the moon in 1969.

How many years after John F. Kennedy's election was the National Organization for Women founded?
Six years after Kennedy was elected, the National Organization for Women was founded.

Cluster diagrams—This type of graphic organizer can help you keep track of information, such as the traits of a character in a story. It can also help you keep track of ideas or possible solutions to a story's problem.

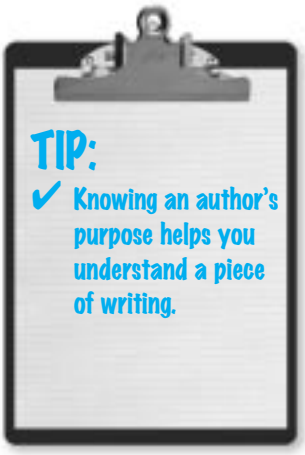
Try It

Here's a cluster diagram you might make to describe a character in the story "Always Together." Can you figure out who it is?



Which character do you think it is? Add your answer to the blank line.

You know from the traits and qualities listed in the diagram that the character being described must be Desiree.



Purposes of a Text

As you read, ask yourself, "Why did the author write this?" Authors write stories for different reasons, or purposes. Some of these reasons are:

To entertain—By telling a good story, an author entertains the reader. An adventure story about a trip in space is a good example.

To inform—Readers learn factual information through articles in newspapers, encyclopedias, or textbooks. A good example might be an article about the Hubble telescope.

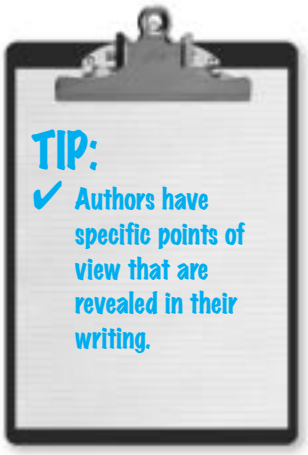
To explain—Some pieces of writing give the reader specific instructions or directions. A pamphlet about constellations might give directions telling how to locate the Big Dipper in the night sky.

To persuade—An author of a persuasive article uses powerful words to influence the reader to believe a certain thing or feel a certain way. For example, a reporter could write a strong editorial in favor of spending more money on the space program.

Of course, some writing has more than one purpose. A story about an encounter with a javelina might entertain you and teach you about javelinas at the same time. An advertisement for a chemistry set might include suggestions for experiments you can do at home.

Why do you think the author of "Always Together" wrote the story? The author's purpose is to entertain as well as to offer insight. The author has written an interesting and heartwarming story that contains a message. The reader is entertained by Desiree's situation and by the way she handles her perceived problem. By the story's end, Desiree, as well as the reader, gains valuable insight about her grandfather. She understands that her grandfather is a more complex person than she believed. The author's message may be that elderly people have much to offer.

Author's Perspective



An author's perspective, or point of view, is the way the author looks at the world. This perspective is a combination of ideas, beliefs, attitudes, and feelings.

Understanding an author's **point of view** is sometimes easier in a nonfiction selection. The author will often state his or her opinion directly, as in "I have always loved the sight of morning dew on a freshly mowed lawn" or "Using the Internet is the fastest way to access important information."

Authors of fictional, or made-up, stories don't usually state their opinions directly in their works. Still, you can often figure out an author's point of view by looking closely at story elements such as plot, character, and theme.

Read the following student letter about the cafeteria in a school newspaper.

> Dear Editor:
>
> This is my second year at this school. Lunch used to be my favorite time of day. I liked hanging out with my buddies. Even some of the food was good. But this year we can't enjoy lunchtime. The food is lousy. What happened to the mashed potatoes, gravy, and chicken-fried steak? The meat tastes like cardboard, and the gravy is lumpy. Who can eat that? The problems with this stuff they call food are too many to count.
>
> An eighth-grade student

What is the point of view of the author of this letter?
The author's point of view is that the cafeteria's food is bad.
A letter written by the cafeteria manager would probably express a different point of view. The cafeteria manager would probably think the food is acceptable, given how much the food costs and the work it takes to prepare school lunches.

How authors think, feel, and believe affects their view of the world and how they write about it.

Objective 4: The student will apply critical-thinking skills to analyze culturally diverse written texts.

What are critical-thinking skills, and how do they help students analyze texts?

Careful readers understand that reading requires them to make connections between what they have read and what they already know. Critical-thinking skills help careful readers use the information they are reading in order to develop an understanding that goes beyond the lines of a text. Reading in this way is an important tool for thinking and learning in daily life.

Thinking critically about a text includes being able to do the following:

- Understand when deeper meanings are implied rather than directly stated in a selection
- Form conclusions based on the information within a selection
- Make reasonable predictions about what might happen next
- Distinguish between facts and opinions, especially in persuasive texts such as articles and advertisements, whose purpose is to persuade the reader about a specific topic
- Support conclusions, generalizations, and interpretations with ideas and sentences from the selection
- Make connections between the themes and issues presented in texts
- Analyze the ways in which authors organize and present ideas
- Assess the ways that style, tone, and mood affect a text

Inferences

Authors don't always directly state every idea in a story or article. When you recognize these unstated ideas, you are making an **inference**. You make inferences by drawing conclusions, forming a generalization, or making a prediction.

TIP:
✔ When you make an inference, look for evidence in the text that will support your "educated guess."

Conclusions—One type of inference, or "educated guess," is a conclusion. When you draw a conclusion as you read, you ask questions such as:

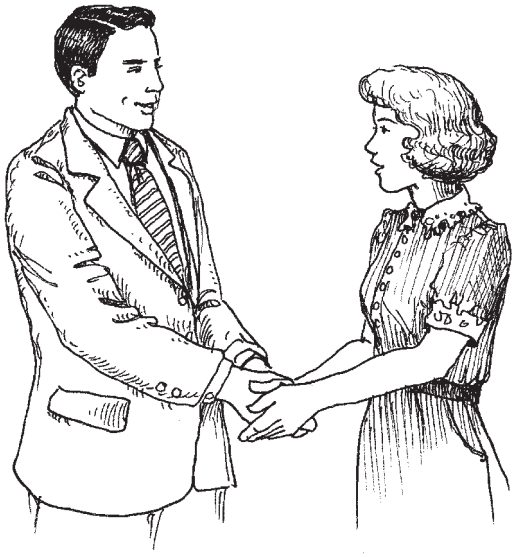
What is the author suggesting in this selection?

What does my own experience tell me about this?

What "educated guess" makes sense based on what I'm reading?

Drawing conclusions helps you better understand characters and situations in a story.

Reread paragraph 9 from "Always Together" on page 14. What conclusion can you draw from this paragraph?



What conclusion can you draw about Desiree's grandmother?
She was very patient.

How do you know?
Even though Grandpa couldn't dance, Desiree's grandmother still agreed to dance with him several times. She must not have minded that he couldn't dance very well.

Help with Skills: Objective 4

Generalizations—A generalization is a judgment. Like drawing conclusions, making generalizations depends on putting together what you know and what you have read.

Suppose you read a magazine article that tells about new softball leagues that are being formed in many cities across the United States. Based on this information, you might make this generalization:

> Softball is becoming a popular sport in the United States.

©Charles Gupton/CORBIS

For a generalization to be valid, or truthful, it must be based on evidence. Valid generalizations are not guesses. Valid generalizations often use words such as *many*, *often*, and *some*:

Many people own cell phones.

Generalizations that are not valid make statements that are too broad. They may use words such as *all*, *always*, *every*, and *never*:

Every student has read the *The Outsiders*.

Making generalizations will help you connect information that you read with your own experiences.

Try It

Go back to paragraphs 5 and 6 of "Always Together" on pages 12–13. Reread the information carefully; pay close attention to the details.

1. What generalization can you make about piano playing based on Desiree's experience?

_____

_____

_____

2. What evidence from the text supports this generalization?

_____

_____

_____

1. Generalization: A person's ability to play the piano improves with practice.

2. Support for this generalization: In paragraph 5, you learn that Desiree has been taking piano lessons for about a year. In paragraph 6, Desiree decides to try to play some of the songs in her grandmother's songbook that were too difficult a few months earlier. She chooses the song "Always Together," and the more she plays it, the better she gets. Since Desiree's piano playing has improved with practice and since you know that skills improve with practice in general, you can say that a person's ability to play the piano improves with practice.

Help with Skills: Objective 4

Predictions—When you make a prediction about a story, you make a guess about what might happen. You use information in the story and your own experience to make the prediction.

To make a prediction, notice the following as you read:

- Details about character, plot, and setting
- What a character says and does
- Foreshadowing, or hints about what might happen in the future

What prediction can you make based on the story below?

Closed

Emilio rode his bike to the public library. The dozen books he had checked out two weeks ago were due today. Emilio parked his bike in the bike rack in front of the library. He unzipped his backpack and pulled out an armful of books. As he walked toward the library entrance, Emilio noticed a sign in the window that read "Closed" in big red letters. "Oh, great," Emilio muttered to himself. "If I hadn't had to pump up my flat tire, I would have been here on time." Emilio stood there and added up the late charges in his head. Suddenly a small door with a shiny silver handle caught his eye. "Looks like I won't have any late charges after all," Emilio said brightly.

What is your prediction?
Emilio will pull the handle and place his books in the book-return slot.

What clue or clues helped you make this prediction?
When Emilio spots a small door with a handle, he says that he won't have any late charges after all.

As you read, you may find that you have to change a prediction based on new information in the story.

Supporting Your Inference with Text Evidence

A key ingredient in making an inference is the information an author gives you. The author's thoughts and ideas, combined with the reader's prior knowledge, allow a reader to provide powerful support, or evidence, for a particular interpretation of a text.

When interpreting a text, you may use the actual words an author has written to support your interpretation. Other times you will paraphrase, or restate in your own words, what the author has written. When you paraphrase, you often think again about your own knowledge and experiences that are connected to what you just read.

Fact and Opinion

A **fact** is a statement that can be proved true.

An **opinion** is a statement that cannot be proved true or false. An opinion tells what someone thinks, feels, or believes.

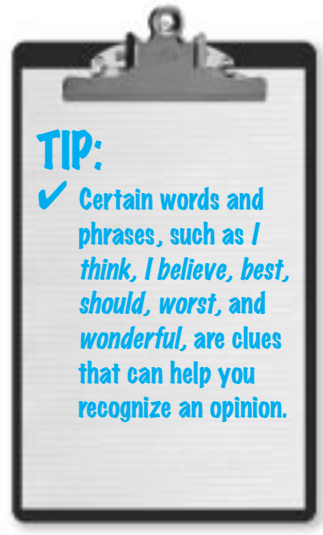
Read the sentences below. Which sentence states a fact? Which states an opinion?

> Sentence 1: Steven Spielberg directed the movie *E.T.*
>
> Sentence 2: *E.T.* was the best movie made in the 1980s.

Sentence 1 is a fact. You can prove it by using a reference source about motion pictures.

Sentence 2 is an opinion. It tells what a person believes.

Certain words help you recognize an opinion. Some of these are *I think* and *I believe*. Other clues to opinions are words such as *best*, *should*, *worst*, and *wonderful*.



Try It

Read this beginning of a report written about pianos.

> The piano is one of the most difficult instruments to play. I should know—I've been playing for over six years. There are 88 keys on the average piano. The keys control hammers inside the piano. The hammers cause the vibrations that create the sound. It takes lots of practice to learn how to play all 88 keys just right.

1. Which underlined sentence is a fact?

_____

2. Which underlined sentence is an opinion?

_____

1. The second underlined sentence is a fact. It can be proved by using a reference source, such as an encyclopedia.

2. The first underlined sentence is an opinion. While some people may think that the piano is the most difficult instrument to play, others may disagree. There is no way to prove that it is the most difficult instrument to play.

TIP:

✔ You get the most out of reading when you try to connect and compare ideas as you read.

Connecting and Comparing Ideas

An important way to make sense of what you read is to think about the ideas in a story or article. This is especially helpful when you read two selections based on the same topic or theme. You might ask yourself these questions as you read:

What can I learn when I connect the main ideas?

How are the ideas alike? How are they different?

Suppose you read a selection about emus and another selection about ostriches. You might compare the ideas in both selections using a Venn diagram like the one shown below:

Emus

Found mainly in Australia, emus are the world's second-largest birds. Emus are covered with long, thick, shaggy feathers that look and feel like hair. They stand about six feet tall and can weigh up to 120 pounds. While emus can run up to 30 miles per hour, they are not able to fly. Males and females look similar, with brown feathers, pointy beaks, long legs, and three-toed feet. Emus are usually calm, docile creatures. However, when a male emu feels threatened, he can become aggressive, kicking and scratching with his powerful legs.

Emu

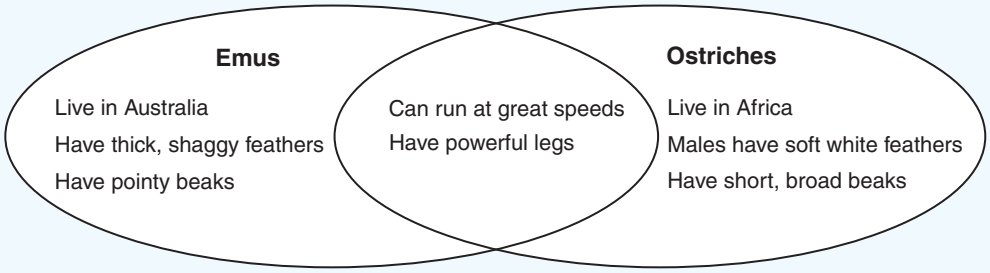
©Galen Rowell/CORBIS

Ostriches

Ostriches are the world's largest birds. These natives of Africa are flightless. They have long necks, small heads, and short, broad beaks. Ostriches can weigh up to 300 pounds. Males are distinguished by their soft white feathers, and the female's plumage is usually a dull grayish brown. Ostriches grow to be over eight feet tall and run at speeds greater than 40 miles per hour. An ostrich's powerful legs can inflict serious injury to any animal that it kicks.

Ostrich

©Paul A. Souders/CORBIS



Analyzing the Ways Authors Present Ideas

Authors organize information in different ways. Knowing how a text is organized will help you find the main ideas. For example, if you notice that an author has used a cause-and-effect pattern, you will know to look for more causes and effects as you read.

Here are some of the patterns authors use to arrange and link ideas:

Cause and effect: The author focuses on something that has happened (effect) and tells you why it has happened (cause).

Compare and contrast: The author focuses on how two or more things are alike or different.

Specific-to-general reasoning (inductive): The author begins with facts and examples and leads you to a general conclusion based on the facts and examples.

Generalization-to-conclusion reasoning (deductive): The author begins with a general statement and provides facts and examples that support the general statement.

Chronological order: The author tells events in the order in which they occurred in time.

Read the following story excerpt. Which organizational pattern has the author used?

> Jasmine threw her backpack on the floor. She stepped over scattered papers, carelessly flinging her sweater across them. She picked her way through discarded socks, jeans, and books. Her empty trash can was surrounded by wads of crumpled-up paper. Jasmine didn't care, though. She liked the cozy feel of her room—even though she was the messiest girl on the planet!

How has the author organized the excerpt?
The author has used specific-to-general reasoning.

What clues tell you this?
The specific details about Jasmine's room lead the reader to the general conclusion that Jasmine is messy.

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Paying attention to how an author organizes a selection can help you understand the information. Often authors will use more than one organizational pattern in their writing.

Style, Tone, and Mood

Suppose you read a story about someone who is late for a doctor's appointment. Then you read a story about the same subject by a different author. What sets these stories apart? The main difference might be that the authors do not use language in the same way. Another difference might be that one author may tell the story in a humorous way, while the other may tell the story in a serious way. The overall feeling of each story might be different, too.

No two authors write in the same way. The writing of different authors is likely to differ in style, tone, and mood.

- **Style** is the way an author uses words, phrases, and sentences. Two authors' styles can be as different as their speaking voices or signatures.
- **Tone** is an author's attitude toward the subject he or she is writing about. The tone of a story can be lighthearted, curious, angry, and so on. The way an author feels about a subject determines the language he or she uses.
- **Mood** is the overall feeling of a story. A story's mood, or atmosphere, can be peaceful, mysterious, suspenseful, and so on. An author carefully chooses the words he or she uses to create a certain atmosphere.

Suppose you read this sentence in a story.

> Michael's heart pounded wildly as he desperately felt his way along the dark hallway.

How would you describe the mood in this sentence?
The words *pounded, wildly, desperately,* and *dark* create a suspenseful mood.

Suppose the sentence was written differently.

> Michael's heart thumped rapidly as he raced to the hallway and flipped on the lights.

How would you describe the mood in this sentence?
In this sentence the writer creates a different mood. A lively, fast-paced mood is created by the words *thumped, rapidly, raced,* and *flipped.*

Great work! You've finished Step 2 in your "Steps to Success."
Be sure to place a check mark in your chart on page 8.

GUIDED PRACTICE

- "Always Together" Again
- "Ready If Needed: King Gill, the 12th Man"

The purpose of pages 57–73 is to provide you with reading selections so that you can apply the skills and strategies you learned in the last section. Read each selection carefully, look at the questions in the margins of both stories, and try to answer them as you read.

Remember that taking notes will help you better understand what you are reading. Writing the answers to the questions in the margins of both stories will help guide your thinking and will help you practice a very important life skill.

This section is not meant to be read all at once. Remember to take a break and try again later if you get tired or frustrated. Work a little bit every day to get the most benefit out of these practice sessions.

"Always Together" Again

Now you will read "Always Together" for a second time. This time you will see questions written next to the story. Although the selections on the actual TAKS test do not include questions in the margins, questions are included here to help guide your thinking. They are examples of the types of questions you should ask yourself when you are reading a narrative. Answer the questions, and they will help you understand the story better.

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"Always Together" is a **narrative**. It is something you might read for fun, but you might also read it to learn something new. When you read a narrative, look for these things:

- **Characters**, such as people or animals
- A **place** where the story happens
- A **time** when the story happens
- A **conflict**, or problem that a character has
- A **resolution**, or solution to the problem

Reading Selection

Always Together

(Question 1) Why does Desiree think that her day has been ruined?

1 Desiree looked at her grandfather, who was half-asleep in the big recliner, and sighed. Why did Grandpa have to get sick today? She didn't blame Grandpa for feeling ill. She always enjoyed spending time with him, but today she had other plans. She was going to play basketball with her friend Marvin that afternoon. Because her mother had taken Desiree's brother to a birthday party, Desiree had to stay home in case her grandfather needed anything.

(Question 2) Why does Desiree enjoy having Grandpa live with her family?

2 Desiree put the magazine down and studied Grandpa's weathered face. She couldn't believe it had been five years since he had come to live with her family. She loved having him around. He had more patience than her parents and was always ready to listen to her problems or help with her homework. Even when Desiree was in trouble, Grandpa could make her smile. He would tell her a story about a time when her mother had gotten into trouble, and they would laugh together. No matter how upset Desiree was, Grandpa could always make her laugh, but today he was just dozing in the living room. Desiree could barely contain her disappointment, though she tried to hide her feelings.

3 "Desiree, you can watch television if you like," Grandpa said without opening his eyes. "It won't bother me."

4 "That's okay, Grandpa," she responded, trying to sound happy. "There's nothing on anyway. Maybe I'll just practice the piano, if you don't mind."

(Question 3) What symbol in paragraph 5 is a link among Desiree, her grandfather, and her grandmother?

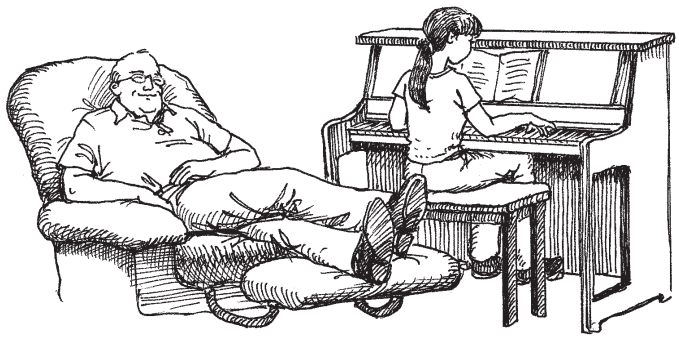
5 "Not a bit." Grandpa's face lit up as he resettled himself in the old chair. "That piano holds a lot of memories for me." Except for the recliner, the piano that had belonged to Desiree's grandmother was the only piece of furniture Desiree's grandfather had brought with him when he came to live with them. Desiree sat down at the piano and ran her fingers over the smooth wood, gently sliding the cover back from the keys. She had been taking lessons for about a year.

Continued

(Question 4)
In paragraphs 5–9, what information helps you conclude that Desiree's grandmother is no longer alive?

6 After playing a couple of songs from her practice book, she sat back and frowned. Then, deciding to try something different, she took out her grandmother's old songbook. Desiree had tried playing some of the songs in it a few months before, but they had been too difficult. She thought that maybe she would be able to play some of the pieces now. She was thumbing through the pages, looking at the titles and music, when she found a piece of paper carefully folded between two pages. It was the music to a song called "Always Together." She looked over the notes and smiled. It seemed challenging, but she thought she could play it. Placing the sheet on the music stand, she started playing slowly and tentatively. As she played, she gained confidence and started playing a little faster. The beautiful melody rose and fell, settling over the room like a faint fragrance until, finally, she had reached the last note.

7 "Play it again, Desiree," Grandpa mumbled softly. "I haven't heard that in so long." Desiree looked over at the recliner. Grandpa still seemed to be sleeping. His eyes were closed, and he had a peaceful look on his face. Desiree began playing the song again, happy that Grandpa was enjoying it. She was playing it much better the second time.



(Question 5)
What are paragraphs 8 and 9 mainly about?

(Question 6)
Why doesn't Desiree realize at first that Grandpa thinks he is talking to her grandmother?

8 "Do you remember when we first heard that?" Grandpa asked, startling Desiree. She started to tell him she had never heard it before, but before she could, he continued. "Ah, you were the prettiest girl at the dance. I can still see you in that red dress. I don't know what you ever saw in a farm boy like me," he murmured, chuckling. Desiree became concerned. For a moment she almost stopped playing. Was Grandpa getting sicker? Then she noticed the name written in delicate handwriting on the

Continued

corner of the music: Desiree Walker, her grandmother. Of course! She was named after her grandmother. Grandpa thought her grandmother was playing the piano!

(Question 7) What mood does the author create in paragraph 9?

9 "I almost didn't ask you to dance," Grandpa said. Desiree listened intently. She continued to play but remained spellbound, engrossed by his memories. "But I did, and you said yes. And you kept saying yes, even though I couldn't dance a lick." Grandpa laughed as Desiree came to the end of the song. "I was so nervous that I almost forgot to ask you your name." With this, his voice drifted off, but a soft smile remained on his lips. He seemed to be sleeping deeply, dreaming of times gone by. Desiree closed the piano softly and walked over to the couch. She sat there, studying the face she had grown up with. To her he had always been Grandpa, a kind old man who read stories to her, taught her games, and laughed at her corny jokes. Now she imagined him young and vigorous, dancing with his future wife for the first time, his whole life still ahead of him.

(Question 8) How would you describe the character of Grandpa?

10 "We're home," Desiree's mother said as she opened the front door. "Were you two all right here by yourselves?"

(Question 9) What does the word rejuvenated mean?

11 "Oh, we were fine," Grandpa said, waking up. He seemed to be feeling better. "I had the most wonderful dream. I feel rejuvenated, as if I were 25 years old."

12 "Really? You'll have to tell me about it later," Desiree's mother said. "And what about you, Desiree?" she asked. "Did anything interesting happen to you today?"

13 "Yes," Desiree answered with a thoughtful expression on her face.

14 "Well, you can tell me about it later, too," her mother said.

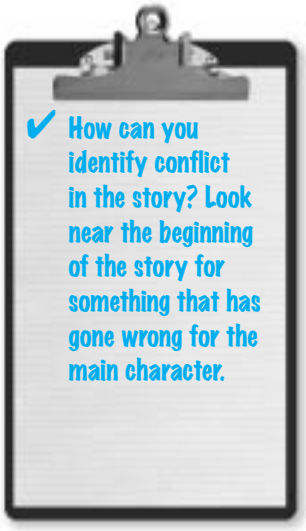
(Question 10) How would you summarize "Always Together" in one paragraph?

15 "Maybe I won't," Desiree said quietly to herself. "This is between me and Grandpa."

Practice with Reading Skills

Below are the questions you answered as you read "Always Together." Read the questions again and the answer explanations that follow.

If you answered the questions incorrectly, the explanations will show you how to get the correct answers.



Question 1: Why does Desiree think that her day has been ruined?

In the last two sentences of paragraph 1, you find out that Desiree had planned to play basketball with her friend Marvin. Instead, she has to stay home in case Grandpa needs anything. The last sentence in paragraph 2 mentions Desiree's disappointment. Desiree's conflict is that she isn't able to spend time with her friend. Therefore, she believes that her day has been ruined.

Turn to page 30 for more help with analyzing characters' conflicts.

Question 2: Why does Desiree enjoy having Grandpa live with her family?

Let's look at the second paragraph. This paragraph tells you that Desiree thinks that Grandpa has "more patience than her parents." Grandpa's patience is the foundation for his caring, loving relationship with Desiree. This is the reason Desiree enjoys having Grandpa live with her family.

Turn to page 30 for more help with analyzing character relationships.

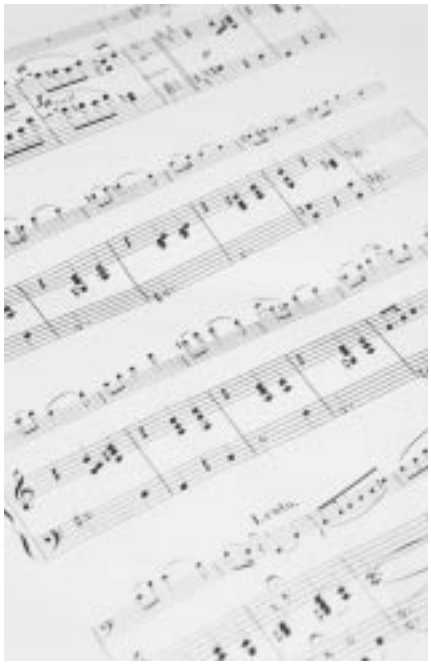
TIP:

✔ Remember that a symbol is something that represents something else.

Question 3: What symbol in paragraph 5 is a link among Desiree, her grandfather, and her grandmother?

Look in paragraph 5 for an object that Desiree and her grandparents care about. The fourth sentence says "Except for the recliner, the piano that had belonged to Desiree's grandmother was the only piece of furniture Desiree's grandfather had brought with him when he came to live with them." The next sentence shows that Desiree treats the piano with great care.

You can tell from this information that both Desiree and her grandfather value the piano. The piano links the three family members and is a symbol of family ties. This is also reflected in the title, "Always Together."



Turn to page 36 for more help with recognizing symbolism.

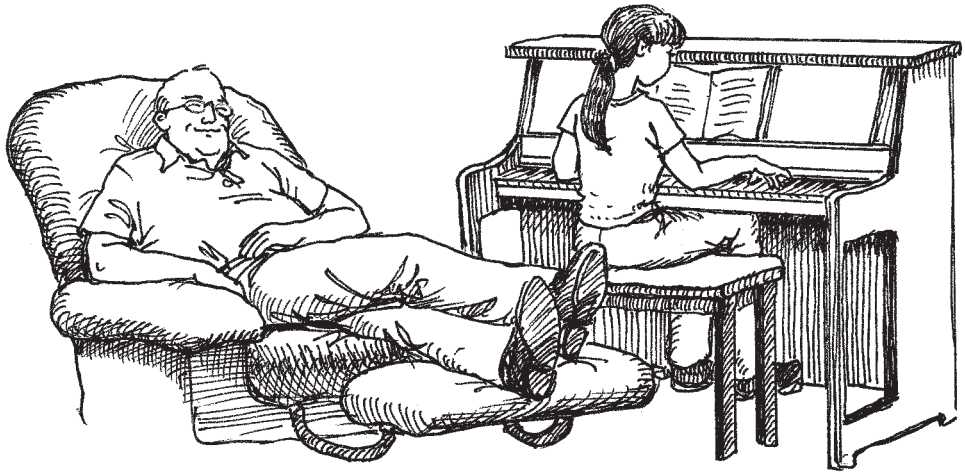
Question 4: In paragraphs 5–9, what information helps you conclude that Desiree's grandmother is no longer alive?

Let's look at the facts and details the author provides in paragraphs 5–9 to support this conclusion. The fourth sentence in paragraph 5 says "Except for the recliner, the piano that had belonged to Desiree's grandmother was the only piece of furniture Desiree's grandfather had brought with him when he came to live with them." The story mentions only the piano that had belonged to Desiree's grandmother; it never mentions Desiree's grandmother coming to live with the family. This is a clue that Desiree's grandmother is no longer alive.

Turn to page 47 for more help with drawing conclusions.

TIP:

✔ Look for clues in a story to help you draw a conclusion and support it.

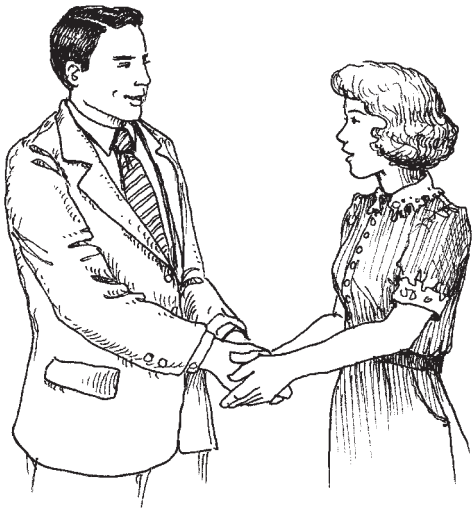


TIP:

✔ Remember that the answers to the questions *who, what, when, where, why,* and *how* can help you figure out the main idea.

Question 5: What are paragraphs 8 and 9 mainly about?

Let's go back to paragraphs 8 and 9 of the story and look at the details to help you determine the main idea. Answer this question: Who is Grandpa talking to? At first it seems that Grandpa is talking to his granddaughter Desiree. By the end of paragraph 8, though, it is clear that Grandpa is talking to his wife, who was also named Desiree.



Now let's use other details from paragraphs 8 and 9 to answer these questions: What are Grandma and Grandpa doing? When did this event happen? Where did this event happen? Sentences 1, 3, 4, and 5 of paragraph 8 tell you that Grandpa and his future wife met for the first time at a dance a long time ago. Information in paragraph 9 also answers these questions. The supporting details help you determine that paragraphs 8 and 9 are mainly about the first time Desiree's grandfather met her grandmother.

Turn to page 23 for more help with finding the main idea.

Question 6: Why doesn't Desiree realize at first that Grandpa thinks he is talking to her grandmother?

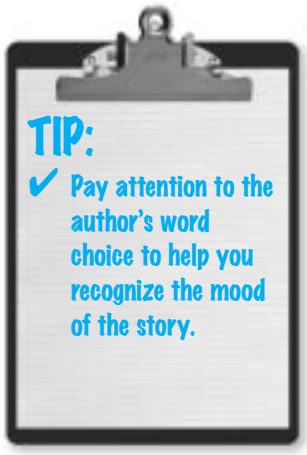
The question can be phrased another way: "Desiree doesn't realize at first that Grandpa thinks he is talking to her grandmother because—" The word *because* signals that you should look for a cause. The effect is that "Desiree doesn't realize at first that Grandpa thinks he is talking to her grandmother." In paragraph 8, Desiree notices her grandmother's name written on the corner of the sheet music: Desiree Walker.

Now read the following information from the end of paragraph 8: "Of course! She was named after her grandmother. Grandpa thought her grandmother was playing the piano!" Desiree and her grandmother shared a name.

This is the reason for, or cause of, Desiree's confusion. She doesn't realize at first that Grandpa thinks he is talking to her grandmother, because she and her grandmother are both named Desiree.

Turn to page 38 for more help with identifying cause and effect.

Question 7: What mood does the author create in paragraph 9?



Reread paragraph 9. Notice the author's word choice when describing Desiree's reaction to Grandpa's story. Ask yourself, "What is the author trying to tell me by choosing these words?"

The words *intently*, *spellbound*, and *engrossed* describe someone who is fascinated. At the end of paragraph 9, Desiree studies Grandpa's face and imagines him young and vigorous. All this information suggests that the mood of paragraph 9 is one of fascination.

Turn to page 54 for more help with recognizing mood.

Question 8: How would you describe the character of Grandpa?

Let's look back at paragraph 2. Sentences 4 and 5 say, "He had more patience than her parents and was always ready to listen to her problems or help with her homework. Even when Desiree was in trouble, Grandpa could make her smile." And sentence 7 says "No matter how upset Desiree was, Grandpa could always make her laugh." Therefore, Grandpa is patient and has a good sense of humor.

Turn to page 28 for more help with analyzing characters.

Question 9: What does the word rejuvenated mean?

To figure out the meaning of *rejuvenated*, look for a word that you recognize. You probably know that the word *juvenile* means "young." *Rejuvenated* and *juvenile* share the same root, *juvenis*, meaning "young person." The prefix *re-* means "again." You can put together the prefix and root to figure out that *rejuvenated* means "made young again."

Now look at how this word is used in the story to determine whether "made young again" makes sense. The sentence in the story says "I feel rejuvenated, as if I were 25 years old." You can tell from the phrase "as if I were 25 years old" that to be *rejuvenated* does in fact mean "to feel young again."

Turn to page 18 for more help with prefixes, suffixes, and other vocabulary words.

Question 10: How would you summarize "Always Together" in one paragraph?

There are several ways in which this story could be summarized so that all the main events and ideas are covered. The summary that follows is one way:

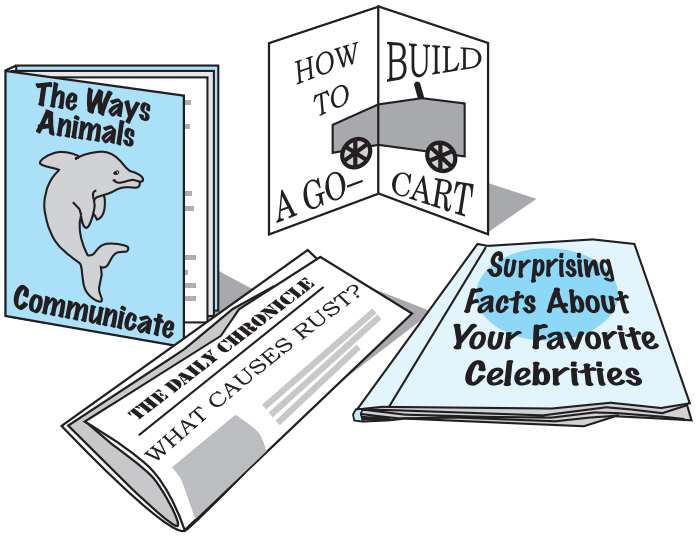
> Desiree stays home to take care of her ill grandfather. As Desiree plays "Always Together," Grandpa reminisces about the day he met Desiree's grandmother. Listening to him causes Desiree to view her grandfather with greater appreciation.

Turn to page 24 for more help with summaries.

Excellent! You've finished Step 3 in your "Steps to Success." Be sure to place a check mark in your chart on page 8.

"Ready If Needed: King Gill, the 12th Man"

Next you will read "Ready If Needed: King Gill, the 12th Man." This is an **informational article**. An informational article is about real people, places, or events. It might explain or describe something. It might also tell how to do something.



When you read an informational article, look for these things:

- Facts about the real world
- Information about . . .
  - what something is like
  - how to do something
  - what happened long ago
  - an interesting person, place, or thing
- Examples to help readers understand the topic
- New words to learn

You may want to read an informational article more slowly than a narrative. An informational article can give you a lot to think about.

Here are some helpful hints for reading an informational article:

- Take notes while you read.
- Try to connect the information with what you already know.
- When you have finished the article, try to retell the important ideas using your own words.

Just as with the story "Always Together," you will see questions written next to the article. Although the selections on the actual TAKS test do not include questions in the margin, they are provided here as examples of the types of questions you should ask yourself as you read an informational article. They will help guide your thinking and help you understand the information in the article better.

Reading Selection

Ready If Needed: King Gill, the 12th Man

What pattern does the author use to organize the information in this article?

In the first paragraph, why does the author use details that seem to contradict one another?

1 Some consider it the biggest football game in the history of Texas A&M University. It was certainly one of the most unusual. A small but scrappy group of Texas A&M players surprised undefeated national powerhouse Centre College, 22 to 14, in the 1922 Dixie Classic, the forerunner to the Cotton Bowl. The game featured some of the best players in the country, but it was an unlikely sophomore who became a legend and inspired a tradition that lives to this day. King Gill didn't make any tackles, nor did he score any points. In fact, he wasn't even officially a part of the team that day, yet he achieved immortality without ever getting into the game.

As you read about the 1922 Dixie Classic in the next few paragraphs, what conclusions can you draw?

What information in the second paragraph supports the idea that Centre College was considered the better team?

2 Gill was watching the game from the press box that day in early January. He had been a reserve halfback for the team for most of the season. As a substitute Gill hadn't expected to play much in the Dixie Classic, so after the team's final regular-season game, he had decided to give up football for the rest of the year to concentrate on basketball. He spent most of the first half of the game spotting plays for a local newspaper reporter who was covering the game. As a spotter Gill helped the reporter identify players in the game. Gill was, no doubt, as surprised as anyone at how well his former teammates were playing. Hardly anyone had expected the Texas A&M Aggies to beat the Centre College Colonels, and yet, as the game neared the halfway point, the Aggie boys clung to a slim lead.

What does the word dilemma mean in paragraph 3?

Why did Coach Bible ask Gill to help the reporter spot plays?

3 Although they were leading, not everything was going well for the Aggies. The game was brutal, and the team had suffered several serious injuries. When Bugs Morris, one of the team's quarterbacks, was hurt, it left only one healthy substitute available to play. At the rate his players were being carried off the field, Dana Bible, the Aggies' head coach, figured he would run out of players before the game ended. This was a troubling dilemma for Bible, who feared his team would be forced to forfeit if more players had to leave the game. Then Bible remembered Gill. He and Gill had shared a taxi ride to the stadium that morning. In fact, it had been Coach Bible who had asked Gill to spot plays for the newspaper reporter.

Continued

Why did Coach Bible ask Gill to put on a uniform?

4 Bible sent word to the press box for Gill to come down to the field and speak with him. A few minutes later Gill arrived and was shocked to find out that Coach Bible wanted him to put on a uniform and be ready to play if needed. Having given up football for basketball, Gill had already turned in his uniform and equipment. He would have to borrow the uniform of Heinie Weir, who had injured his leg on the third play of the game. With no dressing rooms at the stadium, Gill and Weir sneaked beneath the stands and changed outfits. Still somewhat in disbelief, Gill stood on the sidelines throughout the remainder of the game, ready to play if called on.

What does the word negligible mean in paragraph 5?

Why would Gill's willingness to enter the game not have been as inspiring earlier in the season?

5 As it turned out, the team did not need the faithful volunteer after all. The Aggies went on to beat the Colonels without Gill ever playing in the game. It was one of the greatest victories in Aggie football history. Gill's negligible part in the game might even have been forgotten had it not been for a student yell leader named Harry Thompson. It was Thompson whom Coach Bible had sent to the press box to ask Gill to come down to the field. When the victorious team arrived back at the Texas A&M campus, Thompson organized a rally and yell practice on the steps of a nearby YMCA. Thompson was so inspired by the team's victory and Gill's willingness to answer the call of duty that he dubbed Gill the Aggies' 12th man. He challenged the entire student body to demonstrate the same kind of spirit. From that night on Aggie students have considered themselves the team's 12th man—ready to support the 11 players on the field in any way they can. To honor this spirit, the Aggie Corps of Cadets, a student group studying military science, adopted the custom of standing throughout every Aggie football game, win or lose.

How would you summarize this article?

Why did the author write this article?

6 Though he is best known for a game he never played in, King Gill was one of the finest athletes ever to play at Texas A&M. He went on to star in three sports for the Aggies. But it was his willingness to help his comrades on the field that has come to mean something special to football fans everywhere. Today this 12th-man spirit extends far beyond the bounds of the football field. It has come to symbolize the spirit of all those who offer emotional support to others facing difficult challenges. It serves as a reminder that friends are ready and willing to help if needed.

Practice Questions

Here are questions similar to the ones you thought about as you read "Ready If Needed: King Gill, the 12th Man." Each question now has four answer choices. See whether one of the choices matches the answer you thought of as you were reading. If not, think about the four choices and choose the best one. Mark your answer in the book.

Note: These questions are not in the same order in which they appear next to the selection.

Question 1

In paragraph 3, the word dilemma means —

A an inability to continue

B a problem with no apparent solution

C an important consideration

D a lack of ideas

TIP:
✔ Remember to use context clues to determine a word's meaning.

Answer Key: page 87

Question 2

Which of these best summarizes the selection?

A Afraid that his team would be forced to forfeit because of injuries, the Texas A&M coach asked a former player to suit up and be ready to play. Although the player did not get to play in the game, his willingness to help the team inspired a tradition that continues to this day.

B No one expected it, but a small yet determined Texas A&M football team defeated Centre College in the 1922 Dixie Classic. Although the Aggies won the football game, several of their best players suffered serious injuries and had to be carried off the field.

C After an exciting game in which the Texas A&M football team defeated a team that was considered unbeatable, a Texas A&M yell leader organized a rally at a local YMCA. He inspired students to create a new tradition to support the football team.

D A backup football player gave up the sport to concentrate on basketball but later volunteered as a spotter for a local reporter who was covering the Dixie Classic. The player rejoined the football team one last time when the coach invited him back.

TIP:
✔ Remember that a good summary briefly expresses the key ideas and events, and identifies the connection between them.

Answer Key: page 87

Question 3

Gill's willingness to enter the game would not have been as inspiring earlier in the season because —

A he was playing basketball at the time

B Texas A&M had won all its other games easily

C he was still a member of the football team then

D there were no injuries during any other games

Answer Key: page 87

Question 4

In paragraph 1, why does the author use details that seem to contradict one another?

A To inform readers of Gill's athletic ability

B To explain why Gill is considered a legend

C To make readers curious about Gill

D To describe how Gill and the Aggies won a game

Answer Key: page 87

Question 5

What can the reader conclude from the author's description of the 1922 Dixie Classic?

A The Aggies did not suffer any serious injuries after Gill arrived on the field.

B Gill decided to quit basketball and return to the football team.

C Injured players were allowed to return to the game.

D Coach Bible was angry at Gill for giving up football for basketball.

Answer Key: page 87

Question 6

Which sentence in this selection supports the idea that Centre College was considered the better team prior to the game?

A *The game featured some of the best players in the country, but it was an unlikely sophomore who became a legend and inspired a tradition that lives to this day.*

B *Hardly anyone had expected the Texas A&M Aggies to beat the Centre College Colonels, and yet, as the game neared the halfway point, the Aggie boys clung to a slim lead.*

C *But it was his willingness to help his comrades on the field that has come to mean something special to football fans everywhere.*

D *At the rate his players were being carried off the field, Dana Bible, the Aggies' head coach, figured he would run out of players before the game ended.*

TIP:
✔ This question is asking how you know a particular idea is true. You must choose the sentence that captures that idea best.

Answer Key: page 88

Question 7

Which of the following best describes the overall organization of this selection?

A The author gives examples to inspire people to help others facing difficult situations.

B The author explains the plan the Aggies used to defeat the Centre College Colonels.

C The author compares what people expected to happen in the Dixie Classic to what actually did.

D The author relates the events that led to the 12th-man tradition at Texas A&M.

TIP:
✔ Look for patterns in the text to help you determine how the author organized the information.

Answer Key: page 88

Question 8

Coach Bible asked Gill to put on a uniform because —

A some of the other players were getting tired

B the Aggies were losing the football game

C he knew that Gill wanted to play in the game

D he was worried that he would run out of players

Answer Key: page 88

Question 9

Why did Coach Bible ask Gill to help the reporter spot plays?

A The reporter did not know the names of the injured players.

B Bible felt sorry that Gill was no longer part of the team.

C As a former player Gill knew the team members well.

D Bible wanted Gill to be nearby in case he needed him to play.

Answer Key: page 88

Question 10

In paragraph 5, the word negligible means —

A unreliable

B surprising

C unimportant

D famous

Answer Key: page 88

Question 11

The author wrote this selection to —

A give a complete history of Texas A&M football

B tell the life story of one of the Aggies' finest athletes

C persuade athletes to continue playing football

D relate the origin of a well-known college-football tradition

Answer Key: page 88

You have finished Step 4 in your "Steps to Success." Be sure to place a check mark in your chart on page 8.

Independent Practice

- "Plaza Theater Ends 75-Year Run" and "Letter to the Editor"

The purpose of pages 77–83 is to provide you with an opportunity to practice what you have learned by reading stories and answering questions similar to those that are on the TAKS test. The two selections "Plaza Theater Ends 75-Year Run" and "Letter to the Editor" on pages 78–80, are paired selections and should be read together before you answer the questions that follow. Like the selections on the TAKS test, selections in this part do not have questions in the margins to guide your thinking. You may want to use "What a Careful Reader Does" on page 11 to help you remember what types of questions to ask yourself as you read.

"Plaza Theater Ends 75-Year Run" and "Letter to the Editor"

Now you will read two selections, "Plaza Theater Ends 75-Year Run" and "Letter to the Editor." These are paired selections and should be read together before you answer the questions that follow.

The first selection, "Plaza Theater Ends 75-Year Run," is an **informational article**. You know from reading "Ready If Needed: King Gill, the 12th Man" that an informational article contains facts. It explains or describes something.

The second selection is "Letter to the Editor." In a **letter to the editor**, a person explains how he or she feels about a topic. Usually the writer wants to persuade the readers to agree with him or her. To do so, the writer may give reasons or examples that support his or her opinion. The writer might also use emotional language to win the reader's agreement.

Both selections you will now read are about the closing of an historic movie theater. You will find that some information is present in both the article and the letter, yet they express different points of view concerning the closing of the theater. Together, they provide you with a more complete understanding of what the closing of the theater means to the town.

As you read both selections, think about how they are alike and how they are different. Try to understand the purpose of each selection. Compare the main ideas. If the same idea is mentioned in both the article and the letter to the editor, it is probably important.

Plaza Theater Ends 75-Year Run

1 Minutes after the last movie ended yesterday at the Plaza Theater, employees were busy sweeping up spilled popcorn and gathering candy wrappers and soda cups. It was a scene that had been repeated many times in the theater's 75-year history. This time, however, the cleanup was a little different. As one group of workers carried out the trash and wiped down the concession counters, another group began removing seats, light fixtures, and other theater equipment in preparation for the building's demolition. Wrecking crews were to begin tearing down the historic landmark at 10 A.M. today.

2 The film classic *The Last Picture Show*, starring Timothy Bottoms, Jeff Bridges, and Cybill Shepherd, was the last movie shown in the old theater. Though the movie is 30 years old, most of the 250 seats were filled with teary-eyed theater patrons wanting to say good-bye to the old building. Theater owner Ed Bradford said he chose the movie because it seemed appropriate. The movie is set in a small town where the only movie theater is preparing to close down.

3 Bradford said that large, new multiscreen theaters in the city made it impossible for the Plaza to compete. "Film companies want to put their movies on several screens with many different show times," he said. "That was something we just couldn't offer." Bradford added that the theater's location was also a factor. "This used to be the center of town," he said. "Now the area is mostly office buildings and warehouses."

Photo courtesy of El Paso Community Foundation.

***In 1961 the Plaza featured the hit movie* Dr. No.**

4 Bradford said the theater's demise has been coming for some time. Late last year the theater closed briefly when it was unable to meet operating costs. Donations

Continued

Continued from page 1

from local residents helped Bradford reopen the theater, but it was only a short-term solution. Last week Mayor Kathy Sullivan and other city leaders suggested the city might be interested in turning the old theater into a museum and public meeting place. However, these plans were abandoned because of financial problems. City leaders wanted Bradford to donate the building and property to the city. In exchange, Mayor Sullivan had pledged that the city would pay to renovate the building. The cost of remodeling and repairing the building was estimated at over a million dollars. However, Bradford declined. "I've got to earn a living, too," he said.

5 Instead, Bradford sold the building and land to a local development firm, Parkens and Cliff. The firm plans to build a downtown shopping complex on the land where the theater is located. Parkens and Cliff has already purchased many of the small businesses surrounding the Plaza Theater building. The firm is considering making a multiscreen theater part of the shopping center.

6 "I'm sad to see it go," Jim Phelps said as he strolled out of the theater late last night. "This is where my wife and I went on our first date." Phelps was one of several people who traveled hundreds of miles to see the show. "It was worth the drive," he said.

7 The theater dates back to the time of silent movies and until recently still featured piano music during the intermissions between shows. The movie screen could be pulled up to reveal a large stage where plays and other live entertainment events were sometimes performed.

8 Mayor Sullivan had mixed emotions about the closing of the Plaza Theater. "While the closing of this historic site is sad to us all," she said, "a new shopping center would bring much-needed jobs to the downtown area."

9 The crowd bid its final farewell as Bradford locked the doors for the last time. After 75 years the Plaza Theater had shown its last movie. The theater will be missed.

The Morning Journal

August 16, 2001 Letters to the Editor Section D, Page 1

Editor:

1 What a shame that yet another local treasure has been lost in the name of progress. To many of us, the Plaza Theater was more than just a place to go watch a play or a movie. It was a big part of our lives, a place where we shared not only entertainment, but also tragedy and triumph.

2 I remember going to the Plaza for Saturday matinees as a young boy. A dime got you in to see two movies. If you were lucky, one of them might feature Roy Rogers, who was one of the biggest stars in the movies at that time. My friends and I went almost every weekend, even if we had seen the movie before.

3 Several years later my first job was at the Plaza. I made only a dollar an hour, but I thought I was rich. The Plaza is also where I met my future wife. She worked the ticket booth, and I worked the refreshment counter.

4 Unfortunately, this is also where we saw the first pictures of Pearl Harbor and other horrible images from World War II. The newsreels that were played before the movies were the only way we could see our servicemen in action. The Plaza was also where we later saw the images of victory parades across the country when the war ended.

5 I know a shopping mall will bring jobs and improve our economy, but I can't help but feel we have lost more than just a movie theater and an old building; we have lost a part of ourselves. There are a lot of places in our city to shop, but the Plaza Theater was one of the few places where people could experience history.

6 It is too bad Mayor Sullivan and our city leaders did not appreciate what the Plaza Theater meant to our city. The mayor has squandered yet another opportunity to preserve a local landmark. What a senseless loss.

Robert McCourty

Chairman, City Historical Society

Practice Questions

Answer these questions about the selections you just read, "Plaza Theater Ends 75-Year Run" and "Letter to the Editor." Use the skills you have learned already while working on "Always Together" and "Ready If Needed: King Gill, the 12th Man."

Use "Plaza Theater Ends 75-Year Run" (pp. 78–79) to answer questions 12–14.

Question 12

What can the reader conclude about the other businesses bought by the development company?

A They are in bad condition.

B They are also movie theaters.

C They are also being torn down to make room for the shopping center.

D They are doing well and making a lot of money.


Answer Key: page 89

Question 13

Why did the owner choose to show *The Last Picture Show* on the final night?

A It was the only movie that is still available.

B It was a movie about the history of the theater.

C It had been the town's favorite movie for the last 75 years.

D It has a plot that seemed to fit the occasion.


Answer Key: page 89

Question 14

Which sentence from the article suggests that many people wanted the theater to stay open?

A *Donations from local residents helped Bradford reopen the theater, but it was only a short-term solution.*

B *The movie is set in a small town where the only movie theater is preparing to close down.*

C *Bradford said the theater's demise has been coming for some time.*

D *The firm plans to build a downtown shopping complex on the land where the theater is located.*


Answer Key: page 89

Use "Letter to the Editor" (p. 80) to answer questions 15–18.

Question 15

How does McCourty's position with the City Historical Society affect his letter to the editor?

A He includes many historical facts about the theater.

B He focuses on the theater's place in history.

C He remembers learning how to play the piano in the theater.

D He explains the history of World War II.

Answer Key: page 89

Question 16

Which of these is an opinion in McCourty's letter to the editor?

A As a child McCourty attended matinees on Saturdays.

B The theater showed newsreels of Pearl Harbor.

C When he was younger, McCourty worked at the theater.

D Mayor Sullivan does not appreciate the Plaza Theater.

Answer Key: page 90

Question 17

Paragraphs 2 through 4 of the letter to the editor are mainly about —

A Robert McCourty's memories of the Plaza Theater

B when Robert McCourty worked at the Plaza Theater

C how Robert McCourty met his wife

D the victory parades Robert McCourty watched after World War II

Answer Key: page 90

Question 18

What does the word squandered mean in paragraph 6 of the letter to the editor?

A Believed

B Found

C Thrown away

D Remembered

Answer Key: page 90

Use "Plaza Theater Ends 75-Year Run" and "Letter to the Editor" to answer questions 19–21.

Question 19

In what way is the information about Mayor Sullivan's efforts to save the theater presented differently in the article and in the letter to the editor?

A The article expresses regret that the mayor did not do more to save the building, but the letter commends the mayor for trying her best.

B The article presents factual information about the mayor's efforts, but the letter criticizes the mayor for not doing more to preserve the landmark.

C The article does not discuss the mayor's efforts, but the letter provides many details about the solutions she proposed.

D The article shows that the mayor did not want to save the theater, but the letter explains why the mayor thought a new shopping center would help the town's economy.

Answer Key: page 90

Question 20

From information in both selections, the reader can tell that the theater was —

A poorly maintained and in danger of collapsing

B more important to city officials than to local citizens

C a place where many people had experienced important events

D as popular at its closing as it was when it opened

Answer Key: page 90

Question 21

One similarity between these selections is that both discuss —

A the new jobs the shopping center is expected to create

B the reasons Ed Bradford had for closing the theater

C the newsreels watched by moviegoers during World War II

D the mayor's offer to turn the theater into a museum

Answer Key: page 91

Congratulations! You've finished the last step in your "Steps to Success." Be sure to place a check mark in your chart on page 8.

Answer Key

- "Ready If Needed: King Gill, the 12th Man"
- "Plaza Theater Ends 75-Year Run" and "Letter to the Editor"

The answer key on pages 87–91 provides the answers and explanations for the practice questions that follow "Ready If Needed: King Gill, the 12th Man" and the paired selections "Plaza Theater Ends 75-Year Run" and "Letter to the Editor." Check each of your answers to see whether you chose the right one. Don't worry if you miss some questions; the explanations included with each question will help you know how to answer the question correctly.

Reading Answer Key

"Ready If Needed: King Gill, the 12th Man"

Question 1 (page 70)

A Incorrect. The word *dilemma* cannot mean "an inability to continue," because the rest of the sentence says that the coach "feared his team would be forced to forfeit if more players had to leave the game." This shows that the team was continuing to play.

B Correct. The context clues "figured he would run out of players" and "feared his team would be forced to forfeit" show that *dilemma* means "problem."

C Incorrect. The context clues "troubling" and "feared" shows that a *dilemma* cannot mean just an "important consideration"—it is something problematic.

D Incorrect. There is no context in this paragraph or the ones that follow to support the idea that Coach Bible had "a lack of ideas." Instead, the information shows that the coach tried to think through his problem for a solution.

If you missed this question, turn to page 16 to read more about context clues.

Question 2 (page 70)

A Correct. This choice identifies and connects the major points of the article.

B Incorrect. This choice is not a good summary of the article because it focuses on minor details and doesn't mention King Gill.

C Incorrect. This choice is not a good summary of the article because it focuses on minor details and doesn't mention King Gill.

D Incorrect. This choice is not correct because it does not accurately describe what happened in the article.

If you missed this question, turn to page 24 to read more about summarizing texts.

Question 3 (page 71)

A Incorrect. Although paragraph 2 shows that Gill quit football to concentrate on basketball, this doesn't explain why his actions were so inspiring.

B Incorrect. The article does not describe any other games.

C Correct. If Gill had still been a member of the football team, there would have been nothing inspiring about his willingness to play. But as paragraph 4 shows, he was willing to leave the press box, change into a borrowed uniform beneath the stands, and stand by for the rest of the game.

D Incorrect. The article does not describe any other games.

If you missed this question, turn to page 39 to read more about understanding a text's chronology.

Question 4 (page 71)

A Incorrect. The paragraph does not focus on Gill's athletic ability. In fact, it suggests the opposite by pointing out that "Gill didn't make any tackles, nor did he score any points."

B Incorrect. The paragraph does say that Gill is considered a legend, but it does not explain why this is so.

C Correct. The paragraph states "The game featured some of the best players in the country, but it was an unlikely sophomore who became a legend." It also states that Gill did not make any tackles or score any points and that he wasn't even officially part of the team, yet he "achieved immortality." The point of these contradictory details is to make the reader curious about how Gill helped the team.

D Incorrect. The paragraph does state that the Aggies won a game and that Gill was somehow involved, but the paragraph does not describe how this happened.

If you missed this question, turn to page 54 to read more about an author's style.

Question 5 (page 71)

A Correct. Paragraph 5 says that "the team did not need the faithful volunteer after all." The Aggies did not lose any other players to injuries after Gill suited up to play. Therefore, Gill did not have to play.

B Incorrect. Paragraph 2 says Gill had given up football in order to concentrate on basketball.

C Incorrect. Paragraph 3 indicates that once injured players were carried off the field, they did not return to the game.

D Incorrect. Paragraph 3 shows that Gill and the coach had a friendly relationship. They had shared a taxi ride that morning, and the coach had asked Gill to spot plays for the newspaper reporter.

If you missed this question, turn to page 47 to read more about drawing conclusions.

Question 6 (page 72)

A Incorrect. This sentence contains no information about Centre College's team.

B Correct. The first part of the sentence shows that most people expected Centre College to win.

C Incorrect. This sentence contains no information about Centre College's team.

D Incorrect. This sentence contains no information about Centre College's team.

If you missed this question, turn to page 50 to read more about using texts to support responses.

Question 7 (page 72)

A Incorrect. The selection describes a specific set of events. It does not give examples of people helping others facing difficult situations.

B Incorrect. The selection is about the game the Aggies played against the Centre College Colonels but does not describe any particular plan.

C Incorrect. The selection does say that it was a surprise that the Aggies won the game, but it does not focus on what people expected to happen.

D Correct. The selection describes a series of events—the Aggies were in the lead, the team suffered injuries, the coach asked Gill to put on a uniform—that led up to the 12th-man tradition.

If you missed this question, turn to page 53 to read more about how an author organizes information.

Question 8 (page 72)

A Incorrect. The article says nothing about whether the players were tired.

B Incorrect. Paragraph 2 says the Aggies were in the lead.

C Incorrect. The article never says Gill wanted to play in the game. In fact, paragraph 4 says "Gill had already turned in his uniform and equipment."

D Correct. Paragraph 3 describes Coach Bible's fear that he would "run out of players before the game ended." Paragraph 4 says Coach Bible asked Gill to "be ready to play if needed."

If you missed this question, turn to page 24 to read more about finding supporting details.

Question 9 (page 73)

A Incorrect. Paragraph 2 does say that Gill helped to identify players but this choice doesn't explain why Coach Bible asked Gill to help the reporter.

B Incorrect. The selection never says whether Coach Bible was sorry that Gill was no longer part of the team.

C Correct. Paragraph 2 explains that "spotting plays" involves identifying players. It also mentions that the Aggies were Gill's former teammates.

D Incorrect. Paragraph 3 shows that Coach Bible did not think of using Gill as a substitute player until he was in danger of having to forfeit the game.

If you missed this question, turn to page 38 to read more about recognizing cause-and-effect structures.

Question 10 (page 73)

A Incorrect. Gill was reliable. Paragraph 4 states that Gill "stood on the sidelines throughout the remainder of the game."

B Incorrect. The words "might even have been forgotten" show that "surprising" cannot be the correct choice.

C Correct. The words "might even have been forgotten" are a context clue that tells you that *negligible* means "unimportant."

D Incorrect. The words "might even have been forgotten" show that Gill was not yet *famous* for what he did.

If you missed this question, turn to page 16 to read more about using context clues to determine the meanings of words.

Question 11 (page 73)

A Incorrect. The article does not give the complete history of Texas A&M football.

B Incorrect. Paragraph 6 does call Gill "one of the finest athletes ever to play at Texas A&M," but the article does not tell his life story.

C Incorrect. The article is not trying to persuade anyone to do anything.

D Correct. The article describes the Texas A&M football tradition of the 12th man.

If you missed this question, turn to page 44 to read more about author's purpose.

"Plaza Theater Ends 75-Year Run" and "Letter to the Editor"

Question 12 (page 81)

A Incorrect. Nothing in the article suggests the other businesses are in bad condition.

B Incorrect. Nothing in the article suggests the other businesses are movie theaters.

C Correct. Paragraph 5 says that the development firm "plans to build a downtown shopping complex on the land where the theater is located" and that it "has already purchased many of the small businesses surrounding the Plaza Theater building." Paragraph 1 says that the theater is being torn down to make room for the shopping complex. It makes sense that the developers would also tear down the surrounding businesses to make room for the shopping complex.

D Incorrect. The article does not say the other businesses are doing well. In fact, paragraph 8 suggests they are not. The mayor said that the shopping center would bring "much-needed jobs to the downtown area."

If you missed this question, turn to page 47 to read more about drawing conclusions.

Question 13 (page 81)

A Incorrect. Nothing in the article suggests that *The Last Picture Show* was the only movie available.

B Incorrect. The movie is about a theater that closes down, but it is not about the Plaza Theater. Paragraph 2 says the movie is "set in a small town" with only one movie theater. Paragraph 3 of the article mentions the "large, new multiscreen theaters in the city" where the Plaza is located. Therefore, the movie theater in the movie and the Plaza Theater are not the same theater.

C Incorrect. Paragraph 1 says the Plaza Theater is 75 years old, but the article does not say *The Last Picture Show* is the town's favorite movie.

D Correct. Paragraph 1 states that the Plaza Theater has closed down. Paragraph 2 says that the plot of the movie involves a theater closing down. Also, in paragraph 2, the theater owner said he chose the movie because "it seemed appropriate."

If you missed this question, turn to page 29 to read more about characters' motivations.

Question 14 (page 81)

A Correct. The fact that residents gave donations shows that they cared about the theater and wanted to help it stay open.

B Incorrect. This sentence does not show that many people wanted the theater to stay open.

C Incorrect. This sentence does not show that many people wanted the theater to stay open.

D Incorrect. This sentence does not show that many people wanted the theater to stay open.

If you missed this question, turn to page 50 to read more about using texts to support responses.

Question 15 (page 82)

A Incorrect. McCourty does not focus on historical facts about the theater itself. He does not say, for example, how old the theater is, who built it, or what movie it first showed.

B Correct. McCourty's position with the City Historical Society affects the focus of his letter. The point of McCourty's letter is to show the theater's place in history. The theater was there when movies cost only a dime and when Roy Rogers was a big star. It was there when young people thought a dollar an hour was a lot of money. And it was there during and after World War II. These details help to establish the theater's place in history.

C Incorrect. McCourty's letter does not mention playing the piano.

D Incorrect. McCourty mentions World War II in paragraph 4 to make his point about how important the Plaza was in his life. He does not give the history of World War II.

If you missed this question, turn to page 45 to read more about how an author's perspective affects a text.

Question 16 (page 82)

A Incorrect. In paragraph 2, McCourty says that he went to Saturday matinees. This is not an opinion; it is a fact that could be verified by other people, such as McCourty's parents or childhood friends.

B Incorrect. In paragraph 4, McCourty says that the theater showed newsreels of Pearl Harbor. This is not an opinion; it is a fact that could be verified by other people, such as the other audience members.

C Incorrect. In paragraph 3, McCourty says that he worked at the theater. This is not an opinion; it is a fact that could be verified by other people, such as his former boss or his wife.

D Correct. This is the only choice that cannot be verified by other people. Some people might agree with McCourty that Mayor Sullivan does not care about the Plaza, but other people might disagree.

If you missed this question, turn to page 51 to read more about fact and opinion.

Question 17 (page 82)

A Correct. All three paragraphs are about McCourty's memories of the Plaza. They tell about his going to see movies, working there, meeting his wife, and seeing images of World War II at the theater.

B Incorrect. Paragraph 3 says McCourty worked at the theater, but it does not say exactly when, and paragraphs 2 and 4 are not about his working there.

C Incorrect. Paragraph 3 says McCourty met his wife at the theater, but paragraphs 2 and 4 are not about this.

D Incorrect. Paragraph 4 tells of these victory parades, but paragraphs 2 and 3 do not.

If you missed this question, turn to page 23 to read more about identifying main ideas.

Question 18 (page 82)

A Incorrect. This choice does not make sense in the context of the sentence. How could the mayor "believe" an opportunity?

B Incorrect. This choice does not make sense in the context of paragraph 6. The rest of the paragraph says that the mayor has failed to preserve the theater, not that she has found an opportunity to preserve it.

C Correct. Paragraph 6 contains the context clues "did not appreciate" and "senseless loss." These clues tell you that *squandered* means "thrown away."

D Incorrect. This choice does not make sense in the context of paragraph 6. The rest of the paragraph says that the mayor has failed to preserve the theater, not that she has remembered to preserve it.

If you missed this question, turn to page 16 to read more about context clues.

Question 19 (page 83)

A Incorrect. The article does not express any regret over the mayor's actions. It says that people loved the old theater, but it also says that the shopping center will probably be good for downtown. And the letter does not commend the mayor; it criticizes her.

B Correct. The article gives facts about the mayor's efforts. Paragraphs 4 and 8 describe what she said and did, but they do not say whether the author of the article agrees with her. In contrast, McCourty's letter does not contain information about the mayor's efforts. The letter just gives his negative opinion of the mayor when he says she "did not appreciate what the Plaza Theater meant" and "has squandered yet another opportunity."

C Incorrect. The article discusses the mayor's efforts, while the letter does not.

D Incorrect. The article shows that the mayor did want to save the theater, but it also shows that she thinks the shopping center will benefit the town. It is the letter that assumes that the mayor did not want to save the theater.

If you missed this question, turn to page 53 to read more about how an author organizes information.

Question 20 (page 83)

A Incorrect. Neither the letter nor the article suggests the theater was about to collapse. If it had been, people could not have attended the showing of *The Last Picture Show*.

B Incorrect. Both the letter and the article suggest the opposite—that the theater was more important to local citizens than to city officials. Paragraph 4 of the article says that local residents donated money to help the theater stay

open. McCourty's entire letter shows how important the theater was to him, a private citizen.

C Correct. Paragraphs 2–4 of McCourty's letter describe important experiences that he and his generation had at the Plaza. Paragraph 6 of the newspaper article tells about another man and his wife having their first date at the theater.

D Incorrect. Neither the letter nor the article actually says how popular the theater was when it opened. Also, paragraph 3 of the article says it was "impossible for the Plaza to compete" with the city's large, new multiscreen theaters, suggesting the Plaza is less popular now than it used to be.

If you missed this question, turn to page 52 to read more about connecting ideas across texts.

Question 21 (page 83)

A Correct. Paragraph 8 of the article mentions that the shopping center will bring jobs to the downtown area. Paragraph 5 of McCourty's letter also mentions these jobs.

B Incorrect. McCourty's letter says nothing about Bradford's reasons for closing the theater.

C Incorrect. The article says nothing about newsreels.

D Incorrect. McCourty's letter says nothing about the mayor's offer to turn the theater into a museum.

If you missed this question, turn to page 40 to read more about finding similarities across texts.

MATHEMATICS

INTRODUCTION

What Is This Book?

This is a study guide to help you strengthen the skills tested on the Grade 8 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in eighth grade, you will be better prepared to succeed on the Grade 8 TAKS and during the next school year. This study guide is organized into three sections. This section is about mathematics.

What Are Objectives?

Objectives are goals for the knowledge and skills that students should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is the Mathematics Section Organized?

The mathematics section of this study guide is divided into the six objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

How Can I Use This Book?

First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.

What Are the Helpful Features of the Mathematics Section?

- There are several words in the mathematics section that are important for you to understand. These words are bold-faced in the text and are defined when they are introduced. Locate the bold-faced words and review the definitions.
- Examples are contained inside shaded boxes.
- Each objective has "Try It" problems based on the examples in the review sections.
- A Grade 8 Mathematics Chart is included on pages 96–97 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.
- Look for the following features in the margin:

  Ms. Mathematics provides important instructional information for a topic.

Detective Data offers a question that will help remind you of the appropriate approach to a problem.

Do you see that . . . points to a significant sentence in the instruction.

How Should the "Try It" Problems Be Used?

"Try It" problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each "Try It" problem features lines for your responses. The answers to the "Try It" problems are found immediately following each problem.

While completing a "Try It" problem, cover up the answer portion with a sheet of paper. Then check the answer.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 8 TAKS test. There are two types of questions in the mathematics section.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.
- Griddable Questions: Some practice questions use a seven-column answer grid like those used on the Grade 8 TAKS test.

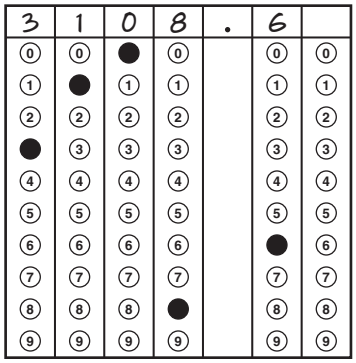
How Do You Use an Answer Grid?

The answer grid contains seven columns, including columns for two decimal places: tenths and hundredths.

Suppose 3,108.6 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 3 is in the thousands place, 1 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, and 6 is in the tenths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 3,108.6 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the hundredths place, because this zero will not affect the value of the correct answer.



Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of the mathematics section on pages 238–248. Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.

Grade 8
Mathematics Chart

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grade 8 Mathematics Chart

Perimeter	square	$P = 4s$
	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	square	$A = s^2$
	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism	$V = Bh^*$
	cylinder	$V = Bh^*$
	pyramid	$V = \frac{1}{3}Bh^*$
	cone	$V = \frac{1}{3}Bh^*$
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Simple Interest Formula		$I = prt$

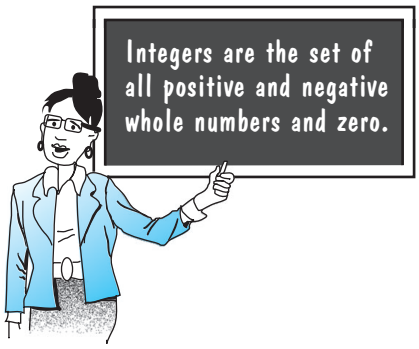
Objective 1

The student will demonstrate an understanding of numbers, operations, and quantitative reasoning.

For this objective you should be able to

- understand that different forms of numbers are appropriate for different situations; and
- select and use appropriate operations to solve problems and justify solutions.

What Are Rational Numbers?



Rational numbers are numbers that can be written as the ratio of two integers where zero is not the denominator. A ratio can be expressed as a fraction.

The fraction $\frac{-2}{3}$ is an example of a rational number; it is the ratio of two integers. This rational number could also be written as $-\frac{2}{3}$.

Rational numbers include any real number that can be written as a fraction. Integers, percents, and some decimals are rational numbers.

Type of Number	Example	As a Ratio of Two Integers
An integer	-9	$\frac{-9}{1}$, or $-\frac{9}{1}$
A decimal number that terminates or forms a repeating pattern	0.5 $0.\overline{3}$	$\frac{5}{10}$, or $\frac{1}{2}$ $\frac{1}{3}$
A percent	25%	$\frac{25}{100}$, or $\frac{1}{4}$

How Do You Select the Appropriate Form of a Rational Number to Solve Problems?

When you solve problems that involve rational numbers, you may need to convert the numbers from one form to another. For example, to find a percent of change, convert the percent to a decimal before multiplying. In a problem that includes both fractions and decimals, it may be helpful to convert all the numbers to either fractions or decimals.

At a restaurant Agnes ordered food that cost $14.89. The tax on her bill was 7%. She gave the waiter a 15% tip on the total bill, including tax. Write an expression that can be used to represent the tip Agnes left.

- First rewrite each of the percents as a decimal.

$$7\% = 0.07$$

$$15\% = 0.15$$

- Represent the 7% tax on Agnes's bill. Multiply 0.07 by the cost of the food.

$$0.07 \cdot \$14.89$$

- Represent the total bill, including tax. Add the tax to the cost of the food.

$$\$14.89 + (0.07 \cdot \$14.89)$$

- Represent the 15% tip Agnes left. Multiply 0.15 by the total bill, including tax.

$$0.15[\$14.89 + (0.07 \cdot \$14.89)]$$

The expression $0.15[\$14.89 + (0.07 \cdot \$14.89)]$ can be used to represent the tip Agnes left.

What Are Irrational Numbers?

Irrational numbers are numbers that cannot be written as the ratio of two integers. An irrational number cannot be expressed precisely in decimal form because the decimal does not terminate or form a repeating pattern.

- The number $\sqrt{2}$ is an irrational number. No matter how many decimal places $\sqrt{2}$ is written to, it is still only an approximation because $\sqrt{2}$ cannot be expressed precisely in decimal form.

  The decimal 1.41 is an approximation of $\sqrt{2}$.

  $$1.41 \cdot 1.41 = 1.9881$$

  The number 1.9881 is close to 2.

  The decimal 1.414213562 is a better approximation of $\sqrt{2}$.

  $$1.414213562 \cdot 1.414213562 = 1.999999999$$

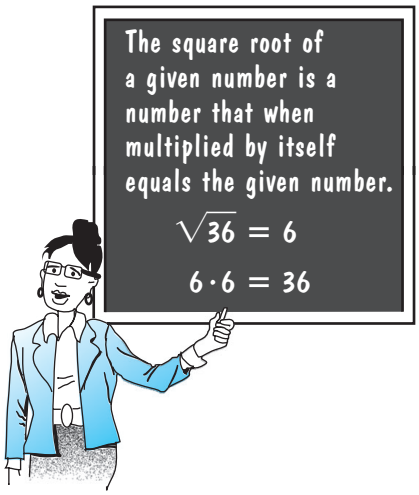
  The number 1.999999999 is closer to 2.

- The number pi, π, is also an irrational number. Here is an estimate of π expressed to 20 decimal places.

  $$3.14159265358979323846$$

  The value of π has been calculated to millions of digits by computers. It is not a repeating decimal. There is no pattern to its digits.

  Since π cannot be expressed precisely in decimal form, π is an irrational number.



How Do You Estimate the Value of an Irrational Number?

One way to estimate the value of an irrational number is to find two consecutive rational numbers with the value of the irrational number between them.

Approximate the value of $\sqrt{32}$.

Find a pair of consecutive integers. The first integer squared should be less than 32. The second integer squared should be greater than 32. The value of $\sqrt{32}$ will be between these two consecutive integers.

Since $5^2 = 25$ and $25 < 32$, the value of $\sqrt{32}$ is greater than 5.

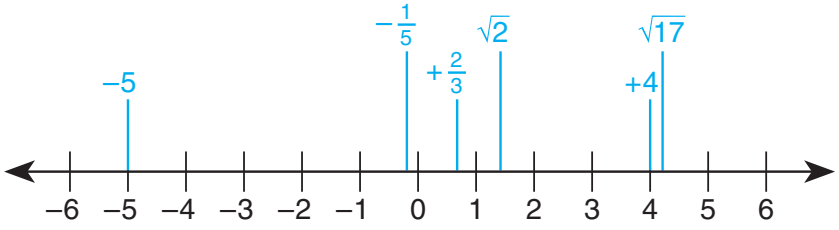
Since $6^2 = 36$ and $36 > 32$, the value of $\sqrt{32}$ is less than 6.

The value of $\sqrt{32}$ is between 5 and 6.

Any decimal number between 5 and 6 is an approximate value of $\sqrt{32}$.

What Is a Real Number?

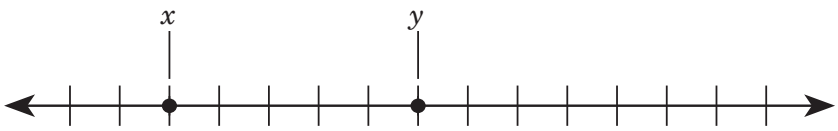
A **real number** is any rational or irrational number. The set of real numbers can be graphed on a number line. Every point on the number line has a real number associated with it.

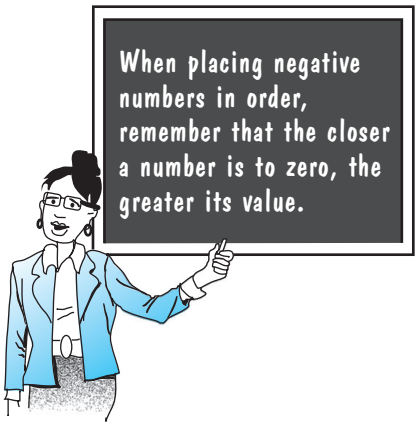


How Do You Compare and Order Rational Numbers?

A number line can help you compare and order rational numbers. On a number line, positive numbers are to the right of 0, and negative numbers are to the left of 0.

Look at this number line with the rational numbers x and y represented.

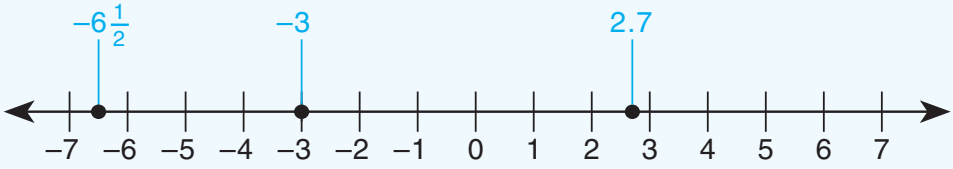




Use these guidelines to compare and order rational numbers on a number line.

- If x is to the left of y on the number line, then $x < y$.
- If y is to the right of x on the number line, then $y > x$.

The rational numbers $-6\frac{1}{2}$, -3, and 2.7 are graphed on the number line below.



You can use the number line to see that $-6\frac{1}{2} < -3$ and $-3 < 2.7$.

$$-6\frac{1}{2} < -3 < 2.7$$

You can also compare and order rational numbers without a number line. If the rational numbers you are comparing are in different forms, use these guidelines to convert them into the same form.

Conversion	Guideline	Example
Fraction to a decimal	• Divide the numerator by the denominator.	$\frac{3}{4} = 4\overline{)3.00}^{0.75}$
Decimal less than 1 to a fraction	• Use the smallest place value, the one farthest to the right, to determine the denominator of the fraction. • Use the digits to the right of the decimal point to determine the numerator of the fraction.	$0.35 = \frac{35}{100}$
Decimal greater than 1 to a mixed number	• Use the digits to the left of the decimal point as the whole-number part of the mixed number. • Convert the digits to the right of the decimal point to a fraction.	$3.28 = 3\frac{28}{100}$
Decimal to a percent	• Move the decimal point two places to the right. • Put a percent sign after the number.	$0.45 = 45\%$
Percent to a decimal	• Move the decimal point two places to the left. • Drop the percent sign.	$3.5\% = 0.035$
Fraction to a percent	• First convert the fraction to a decimal. • Then convert the decimal to a percent.	$\frac{1}{5} = 0.20 = 20\%$
Percent to a fraction	• Express the percent as a fraction with a denominator of 100. • If the percent is greater than 100%, it may be expressed as a mixed number.	$35\% = \frac{35}{100}$ $125\% = \frac{125}{100} = 1\frac{25}{100}$

If the rational numbers you wish to compare are in the same form, use these rules to compare and order them.

- Order decimal numbers by comparing the digits in each place value from left to right.

  If the decimal numbers do not have the same number of decimal places, write zeros behind the last digit to the right of the decimal point. This does not change the value of a decimal. For example, 3.51 is equal to 3.5100.

- If two fractions have the same denominator, compare their numerators.

  For example, $\frac{5}{23} < \frac{8}{23}$ because $5 < 8$.

- If two fractions do not have the same denominator, find a common denominator and then compare the numerators.

  Compare $\frac{2}{3}$ and $\frac{5}{11}$. Since $\frac{2}{3}$ is equal to $\frac{22}{33}$ and $\frac{5}{11}$ is equal to $\frac{15}{33}$, compare $\frac{22}{33}$ and $\frac{15}{33}$. Since $22 > 15$, then $\frac{22}{33} > \frac{15}{33}$ and $\frac{2}{3} > \frac{5}{11}$.

Place this list of numbers in order from least to greatest.

$$0.35, \quad -4\tfrac{1}{2}, \quad -1, \quad \tfrac{3}{8}, \quad 30\%$$

- One way to help you order the numbers is to write them all in decimal form.

$$-4\tfrac{1}{2} = -4.5$$

$$\tfrac{3}{8} = 0.375$$

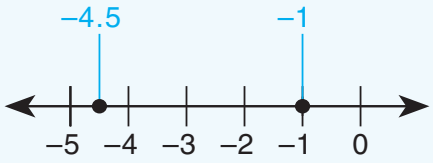
$$30\% = 0.30$$

- To compare the decimals, write each number to three decimal places.

$$0.350, \quad -4.500, \quad -1.000, \quad 0.375, \quad 0.300$$

- Order the negative numbers first.

  When graphed on a number line, -4.5 is to the left of -1.



$-4.500 < -1.000$

The number -4.500 is the smallest, so write it first.

The number -1.000 is the next smallest. Write it after -4.500.

- Order the three remaining numbers: 0.350, 0.375, and 0.300. They all have the same value, 3, in the tenths place. Look at the hundredths place: 0.3$\underline{5}$0, 0.3$\underline{7}$5, 0.3$\underline{0}$0.

  Since $0 < 5$, then $0.300 < 0.350$.

  Since $5 < 7$, then $0.350 < 0.375$.

- List the numbers in order from least to greatest.

$$-4.500, \quad -1.000, \quad 0.300, \quad 0.350, \quad 0.375$$

Written in their original form, the list of numbers in order from least to greatest is: $-4\frac{1}{2}$, -1, 30%, 0.35, $\frac{3}{8}$.

Try It

For five days last winter, Clara's science class recorded the lowest outdoor temperatures in degrees Celsius. The results are shown below.

Lowest Outdoor Temperatures

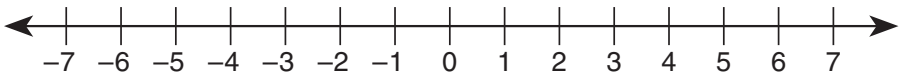
Day	Lowest Temperature (°C)
Monday	3
Tuesday	−2
Wednesday	−1
Thursday	5
Friday	−4

List these temperatures in order from warmest to coldest. List the days in order from warmest to coldest.

One way to solve this problem is to graph the temperatures on a

_____.

Place a mark on the number line below for each temperature.



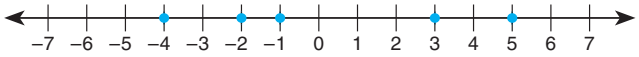
List the temperatures in order from warmest to coldest:

_____°C, _____°C, _____°C, _____°C, and _____°C.

Use the table to match the temperatures to their corresponding days.

The days listed in order from warmest to coldest are _____, _____, _____, _____, and _____.

One way to solve this problem is to graph the temperatures on a number line.



The temperatures listed in order from warmest to coldest are 5°C, 3°C, −1°C, −2°C, and −4°C. The days listed in order from warmest to coldest are Thursday, Monday, Wednesday, Tuesday, and Friday.

How Do You Solve Problems Involving Rational and Irrational Numbers?

You solve problems involving rational and irrational numbers in the same way you solve any other problem. First understand the problem. Identify the quantities involved and the relationships between them. Write an equation that can be used to find the answer. Solve the equation and then check your answer to see whether it is reasonable.

At the grocery store bananas cost $0.55 for 2 pounds, and melons cost $0.19 per pound. James bought 3 pounds of bananas and a melon that weighed 4.77 pounds. What was the total cost of the fruit he bought?

- Find the amount that James spent on bananas.

  Write a proportion, which is a statement that shows two ratios are equal.

  Find two ratios that compare the number of pounds of bananas purchased to the cost of the bananas. The price of 2 pounds of bananas is $0.55, and James bought 3 pounds of bananas for x.

$$\frac{\text{pounds}}{\text{cost}} \quad \frac{2}{0.55} = \frac{3}{x}$$

  To solve the proportion for x, find the cross products and divide by 2.

$$2x = 3 \cdot 0.55$$
$$2x = 1.65$$
$$x = 0.825$$

  Round 0.825 to the nearest cent. James paid $0.83 for the bananas.

- Find the cost of a melon.

  Multiply $0.19, the cost per pound for melons, by 4.77, the weight of the melon James bought.

$$0.19 \cdot 4.77 = 0.9063$$

  Round 0.9063 to the nearest cent. James paid $0.91 for the melon.

- Find the total cost of the fruit James bought.

$$0.83 + 0.91 = 1.74$$

James paid a total of $1.74 for the bananas and the melon.

The area of a square is 12 square centimeters. Find a reasonable estimate of the length of a side of the square.

- Use the formula for the area of a square.

$$A = s^2$$

- Substitute 12 into the formula for A, the area of the square.

$$12 = s^2$$

- Solve for s. Take the square root of both sides of the equation.

$$\sqrt{12} = \sqrt{s^2}$$

$$\sqrt{12} = s$$

- To approximate the value of $\sqrt{12}$, find two consecutive integers with the value of $\sqrt{12}$ between them. The first integer squared should be less than 12, and the second integer squared should be greater than 12.

$$3^2 = 9 \text{ and } 4^2 = 16$$

$$\sqrt{9} < \sqrt{12} < \sqrt{16}$$

$$3 < \sqrt{12} < 4$$

Since $\sqrt{12}$ is between 3 and 4, any number between 3 and 4 is a reasonable estimate of the length of a side of the square.

Try It

Darren has a circular piece of paper that covers an area of 157 square inches. What is the approximate radius in inches of the piece of paper?

The formula for the area of a circle is _____.

Substitute _____ for A, the area of the circle.

$$\_\_\_\_\_ = \pi r^2$$

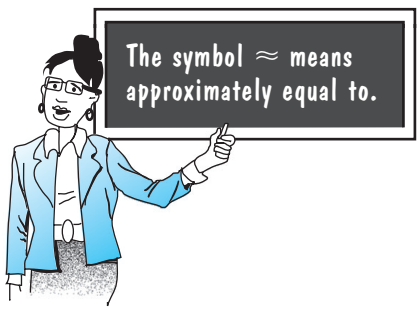
To solve the equation, divide both sides of the equation by

_____. Use 3.14 as an estimate of the value of π.

$$\frac{\square}{3.14} \approx \frac{3.14\, r^2}{3.14}$$

$$\_\_\_\_\_ \approx r^2$$

$$r \approx \sqrt{\_\_\_\_\_}$$



To approximate the value of $\sqrt{\_\_\_\_\_}$, find two consecutive integers with the value of $\sqrt{\_\_\_\_\_}$ between them.

The first integer squared should be less than _____, and the second integer squared should be greater than _____.

Since _____ = 49 and _____ = 64, the value of $\sqrt{\_\_\_\_\_}$ is between _____ and _____.

An answer slightly larger than _____ would be reasonable.

The radius of the paper would be slightly larger than _____ inches.

The formula for the area of a circle is $A = \pi r^2$. Substitute 157 for A, the area of the circle: $157 = \pi r^2$. To solve the equation, divide by π.

$$\frac{157}{3.14} \approx \frac{3.14r^2}{3.14}$$

$$50 \approx r^2$$

$$r \approx \sqrt{50}$$

To approximate the value of $\sqrt{50}$, find two consecutive integers with the value of $\sqrt{50}$ between them. The first integer squared should be less than 50, and the second integer squared should be greater than 50. Since $7^2 = 49$ and $8^2 = 64$, the value of $\sqrt{50}$ is between 7 and 8. An answer slightly larger than 7 would be reasonable. The radius of the paper would be slightly larger than 7 inches.

How Do You Determine Whether the Answer to a Problem Is Reasonable?

One way to determine whether the answer to a problem is reasonable is to estimate the solution and see how big or small the answer should be. Then compare your estimate to the answer you calculated. The estimate and your calculation should be close to each other.

You can estimate an answer by rounding all the numbers in a problem before doing any calculations. Then perform the operations with the rounded numbers. Think about how rounding the numbers before calculating will affect the answer. Determine whether the exact answer should be greater or less than your estimate.

Tom measured a rectangular box. The dimensions were 5.8 inches, 8.1 inches, and 3.9 inches. He then calculated the box's volume to be about 19,000 cubic inches. Is 19,000 $in.^3$ a reasonable value for the volume of the box?

To decide whether Tom's answer is reasonable, estimate the volume.

- The volume of a rectangular prism (Tom's box) is $V = lwh$.
- One way to estimate the volume is by rounding each dimension to the nearest whole number.

  5.8 rounds to 6

  8.1 rounds to 8

  3.9 rounds to 4

- The volume of the box is approximately $6 \cdot 8 \cdot 4$, or about $48 \cdot 4$. This is close to $50 \cdot 4$, which is 200 $in.^3$. The answer should be close to 200 $in.^3$.

Tom's calculation of 19,000 $in.^3$ is not close to the estimate of 200 $in.^3$, so it is not a reasonable value for the volume of the box.

Try It

Gloria knows that the radius of a circular table is between 9 and 10 inches. Is it reasonable to say that the circumference of the table is about 60 inches?

The formula for the circumference of a circle is

$C =$ _____ .

Find the smallest and largest possible values for the circumference by substituting first _____ and then _____ for the radius of the table in the circumference formula.

$C \approx 2($_____$)($_____$); C \approx$ _____

$C \approx 2($_____$)($_____$); C \approx$ _____

The circumference of the table is between _____ and _____ inches, so 60 inches _____ a reasonable estimate of the circumference of the table.

The formula for the circumference of a circle is $C = 2\pi r$. Find the smallest and largest possible values for the circumference by substituting first 9 and then 10 for the radius of the table in the circumference formula.

$$C \approx 2(3.14)(9);\ C \approx 56.52$$

$$C \approx 2(3.14)(10);\ C \approx 62.8$$

The circumference of the table is between 56.52 and 62.8 inches, so 60 inches is a reasonable estimate of the circumference of the table.

What Is Scientific Notation?

Scientific notation is a way of expressing numbers using powers of 10. Using scientific notation helps keep track of decimal places in very large or very small numbers and makes it easier to do arithmetic with them.

When a number is expressed in scientific notation, it is written as the product of a factor and a power of 10. The factor must be a number that is equal to or greater than 1 but less than 10. Look at these two examples.

Do you see that . . .

Written in scientific notation, 2,300,000,000 is 2.3×10^9.

Written in scientific notation, 0.00000000034 is 3.4×10^{-10}.

Powers of 10

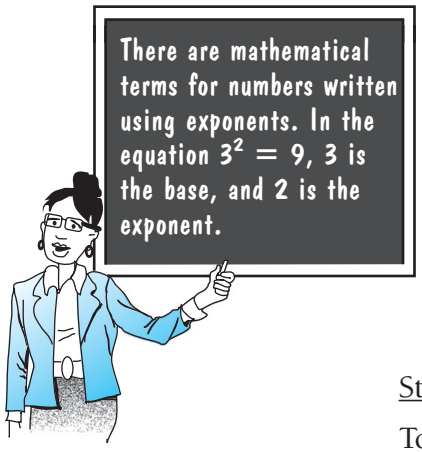
$10^1 = 10$	$10^{-1} = \frac{1}{10} = 0.1$
$10^2 = 10 \cdot 10 = 100$	$10^{-2} = \frac{1}{100} = 0.01$
$10^3 = 10 \cdot 10 \cdot 10 = 1{,}000$	$10^{-3} = \frac{1}{1{,}000} = 0.001$
$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10{,}000$	$10^{-4} = \frac{1}{10{,}000} = 0.0001$

How Do You Convert Between Scientific and Standard Notation?

Numbers written in regular place-value form are in **standard notation**. For example, the numbers 34,285 and 5.7 are both in standard notation.

Scientific to Standard Notation

To change a number from scientific to standard notation, move the decimal point the number of places shown in the exponent of 10.



- If the exponent of 10 is positive, the number in standard notation will be greater than or equal to 10. Move the decimal point to the right.

$$2.35 \times 10^5 = 235{,}000.$$
1 2 3 4 5

- If the exponent of 10 is negative, the number in standard notation will be less than 1. Move the decimal point to the left.

$$7.05 \times 10^{-5} = 0.0000705$$
5 4 3 2 1

Standard to Scientific Notation

To change a number from standard to scientific notation, move the decimal point until the number is greater than or equal to 1 and less than 10. The exponent of 10 is the number of places you moved the decimal point.

- If the number is greater than or equal to 10, move the decimal point to the left. Make the exponent positive.

$$5{,}200{,}000. = 5.2 \times 10^6$$
6 5 4 3 2 1

- If the number is less than 1, move the decimal point to the right. Make the exponent negative.

$$0.000000164 = 1.64 \times 10^{-7}$$
1 2 3 4 5 6 7

Write 17,800 in scientific notation.

- The number 17,800 is greater than 10. Move the decimal point to the left until you have a factor that is greater than or equal to 1 but less than 10.
- Move the decimal point four places to the left so the factor is 1.78.
- Since the decimal point was moved four places to the left, the exponent of 10 is 4, so write 10^4.

The number 17,800 written in scientific notation is 1.78×10^4.

Write 0.000063 in scientific notation.

- The number 0.000063 is less than 1. Move the decimal point to the right until you have a factor that is equal to or greater than 1 but less than 10.
- Move the decimal point five places to the right so the factor is 6.3.
- Since the decimal point was moved five places to the right, the exponent of 10 is -5, so write 10^{-5}.

The number 0.000063 written in scientific notation is 6.3×10^{-5}.

Write 1.41×10^4 in standard notation.

- The exponent of 10 is 4. Move the decimal point four places to the right.
- The factor 1.41 does not have four digits to the right of the decimal point. Write two zeros to the right of the hundredths place in order to have four places.

$$1.41 = 1.4100$$

- Move the decimal point four places to the right to get the number 14,100.

The expression 1.41×10^4 written in standard notation is 14,100.

Write 3.9×10^{-3} in standard notation.

- The exponent of 10 is -3. Move the decimal point three places to the left.
- The factor 3.9 does not have three digits to the left of the decimal point. Write two zeros to the left of the ones place in order to have three places.

$$3.9 = 003.9$$

- Move the decimal point three places to the left to get the number 0.0039.

The expression 3.9×10^{-3} written in standard notation is 0.0039.

Try It

The speed of light written in scientific notation is 2.99793×10^8 meters per second. Express the speed of light in standard notation.

The exponent of 10 is _____.

Move the decimal point _____ places to the _____.

In standard notation the speed of light is _____ meters per second.

The exponent of 10 is 8. Move the decimal point eight places to the right. In standard notation the speed of light is 299,793,000 meters per second.

Try It

A microscopic cell measures 0.0038 centimeter in diameter. Express the diameter of the cell in scientific notation.

To obtain a number equal to or greater than _____ but less than _____, move the decimal point _____ places to the _____.

The exponent of 10 is _____.

In scientific notation the diameter of the cell is _____ cm.

To obtain a number equal to or greater than 1 but less than 10, move the decimal point three places to the right. The exponent of 10 is -3. In scientific notation the diameter of the cell is 3.8×10^{-3} cm.

Now practice what you've learned.

Question 1

Brian's science teacher told him that he needed a score of at least 82% on his next quiz to earn a passing grade in the class. Which fraction represents a quiz score high enough for Brian to earn a passing grade?

A $\frac{15}{20}$

B $\frac{22}{25}$

C $\frac{80}{100}$

D $\frac{39}{50}$

Answer Key: page 238

Question 2

Five friends went out to lunch together. The bill for all their meals was $42.25 before tax. The tax was 4% of the bill. The friends split the total bill, including tax, evenly. Which expression can be used to find the amount of tax that each person paid?

A $(42.25 + 0.04) \div 5$

B $(42.25 \cdot 0.04) \cdot 5$

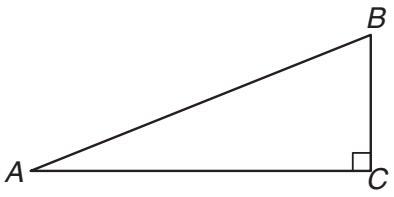
C $(42.25 \div 0.04) \cdot 5$

D $(42.25 \cdot 0.04) \div 5$

Answer Key: page 238

Question 3

In the triangle below, the length of side AB is $\sqrt{29}$ inches. What is the approximate value of $\sqrt{29}$?



A Between 4 and 5

B Between 3 and 4

C Between 5 and 6

D Between 6 and 7

Answer Key: page 238

Question 4

The distance from Earth to the moon is approximately 384,000 miles. Which of these numbers shows the approximate distance from Earth to the moon in scientific notation?

A 3.84×10^{6} mi

B 3.84×10^{5} mi

C 3.84×10^{-6} mi

D 3.84×10^{-5} mi

Answer Key: page 238

Question 5

At the local grocery store, beans cost $0.88 per pound, and bread costs $1.48 for 2 loaves. Which equation can be used to find t, the total cost if Don buys 1.19 pounds of beans and 1 loaf of bread?

A $t = (1.19 + 0.88) + (1.48 + 2)$

B $t = (1.19 \cdot 0.88) + (2 \cdot 1.48)$

C $t = (1.19 \cdot 0.88) + (1.48 \div 2)$

D $t = (1.19 \cdot 0.88) + (2 \div 1.48)$

Answer Key: page 238

Question 6

A car-rental company advertises a price of $230.93 to rent a car for 7 days. At that rate, what would it cost in dollars and cents to rent a car for 10 days?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.		
⓪	⓪	⓪	⓪		⓪	⓪
①	①	①	①		①	①
②	②	②	②		②	②
③	③	③	③		③	③
④	④	④	④		④	④
⑤	⑤	⑤	⑤		⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨

Answer Key: page 238

Question 7

A phone company charges $29.50 per month for the first 1,000 minutes of local calls and $0.04 a minute for any local calls beyond 1,000 minutes. Last month Carol made 1,215 minutes of local calls. What was her total cost for local calls last month?

A $29.50

B $48.60

C $38.10

D $78.10

Answer Key: page 238

Question 8

Mario wrote four checks in the following amounts: $8.39, $12.22, $11.48, and $19.02. He estimated the total of the checks he had written by adding 8, 12, 11, and 19. Which of these statements best describes the actual total of the checks?

A Less than the estimate, because the values Mario added were all less than the actual amounts

B Greater than the estimate, because the values Mario added were all less than the actual amounts

C Less than the estimate, because the values Mario added were all greater than the actual amounts

D Greater than the estimate, because the values Mario added were all greater than the actual amounts

Answer Key: page 238

Question 9

Barb cut out a circular piece of stained glass with a diameter of 8.1 centimeters. She then calculated the area of the piece of glass. Which is a reasonable value for Barb's calculation of the area of the piece of glass?

A 206 cm^2

B 824 cm^2

C 50 cm^2

D 25 cm^2

Answer Key: page 239

Question 10

Albert earns $78 for working 6 hours. Which equation could be used to find the number of dollars, d, that Albert earns in 9 hours?

A $\frac{78}{d} = \frac{9}{6}$

B $78d = 6 \cdot 9$

C $6 + 9 = 78 + d$

D $6d = 78 \cdot 9$

Answer Key: page 239

Objective 2

The student will demonstrate an understanding of patterns, relationships, and algebraic reasoning.

For this objective you should be able to

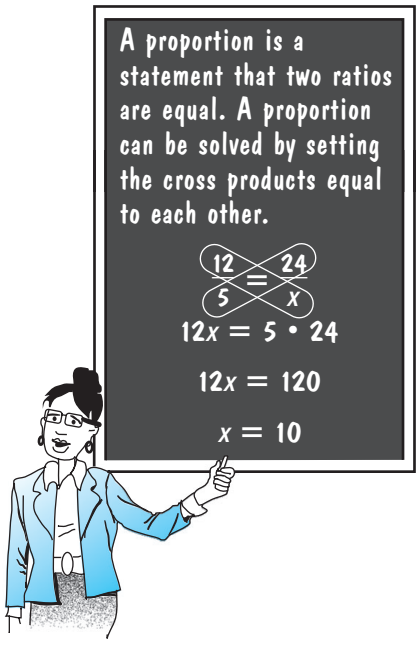
- identify proportional relationships in problem situations and solve problems;
- make connections among various representations of a numerical relationship; and
- use graphs, tables, and algebraic representations to make predictions and solve problems.

What Is a Proportional Relationship?

A **ratio** is a comparison of two quantities. A **proportion** is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you can use proportions when converting units of measurement. You also can use proportions to solve problems involving percent and rates.

To solve problems that involve proportional relationships, follow these guidelines.

- Identify the ratios to be compared. Be certain to compare the corresponding quantities in the same order.
- Write a proportion using the two ratios.
- Solve the proportion.



How many inches are in 7.5 feet?

- The number of inches in 7.5 feet is proportional to the number of inches in 1 foot. Let x represent the number of inches in 7.5 feet. Write a proportion.

$$\frac{\text{inches}}{\text{feet}} \quad \frac{x}{7.5} = \frac{12}{1}$$

- Solve the proportion.

$$\begin{array}{c} x \quad 12 \\ \overline{7.5} = \overline{1} \end{array}$$

$$1 \cdot x = 7.5 \cdot 12$$

$$x = 90$$

There are 90 inches in 7.5 feet.

In some problems you may be asked whether two ratios form a proportion.

A survey of 100 drivers in Dallas found that 52 drove cars, and the remaining people surveyed drove other types of vehicles. A second survey of 250 drivers found that 127 drove cars. Are these two survey results proportional?

To determine whether the two survey results are proportional, compare the ratios of the number of people who drove cars to the total number of people surveyed. If the two ratios are equal, the results are proportional.

- In the first survey the ratio of car drivers to the number of people surveyed is $\frac{52}{100}$. In the second survey the ratio of car drivers to the number of people surveyed is $\frac{127}{250}$.
- The relationship is proportional if the two ratios are equal.

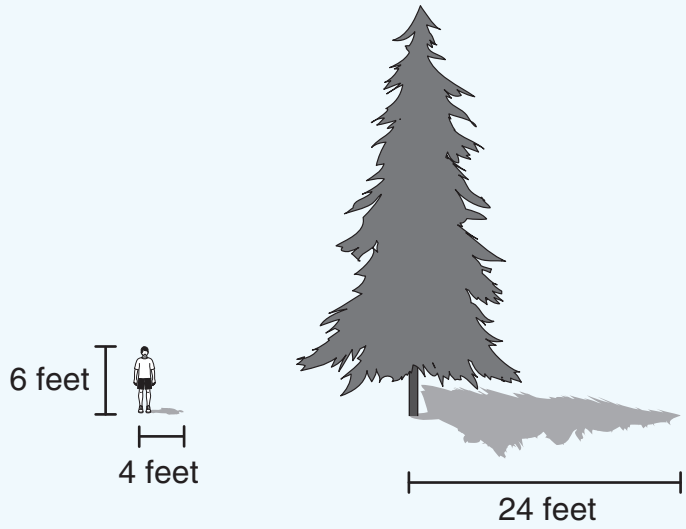
$$\frac{52}{100} \stackrel{?}{=} \frac{127}{250}$$

- Compare the cross products. Are they equal?

$$52 \cdot 250 \stackrel{?}{=} 100 \cdot 127$$
$$13{,}000 \neq 12{,}700$$

Since the cross products are not equal, the ratios are not equal. This means that the ratios do not form a proportion. Therefore, the survey results are not proportional.

Julio is 6 feet tall. While standing in the sun, he measured his shadow. It was 4 feet long. At the same time the tree near him cast a shadow 24 feet long. What is the height of the tree in feet?



- The problem involves a proportional relationship. The ratio of Julio's height to his shadow's length is equal to the ratio of the tree's height to its shadow's length.
- Write a proportion. Set the two corresponding ratios of height to shadow length equal to each other. Let x equal the height of the tree in feet.

$$\frac{\text{height}}{\text{shadow}} \quad \frac{6}{4} = \frac{x}{24}$$

- Solve the proportion using cross products.

$$6 \cdot 24 = 4x$$

$$144 = 4x$$

$$\frac{144}{4} = \frac{4x}{4}$$

$$36 = x$$

The tree is 36 feet tall.

Try It

Gina knows that 2 out of every 3 of her relatives live in Texas. If Gina has 48 relatives, how many live in Texas?

This problem involves a _____ relationship.

Let n represent the total number of Gina's relatives who _____.

One ratio comparing the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{\square}{\square}$.

Another ratio that compares the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{n}{\square}$.

Write a _____, a statement that two ratios are _____.

$$\frac{\square}{\square} = \frac{n}{\square}$$

Solve the proportion by setting the _____ equal to each other.

$$2 \cdot \_\_\_\_\_ = \_\_\_\_\_ n$$

$$\_\_\_\_\_ = 3n$$

$$\frac{\square}{\square} = \frac{3n}{\square}$$

$$\_\_\_\_\_ = n$$

Gina has _____ relatives who live in Texas.

This problem involves a proportional relationship. Let n represent the total number of Gina's relatives who live in Texas. One ratio comparing the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{2}{3}$. Another ratio that compares the number of Gina's relatives who live in Texas to the total number of her relatives is $\frac{n}{48}$. Write a proportion, a statement that two ratios are equal.

$$\frac{2}{3} = \frac{n}{48}$$

Solve the proportion by setting the cross products equal to each other.

$$2 \cdot 48 = 3n$$

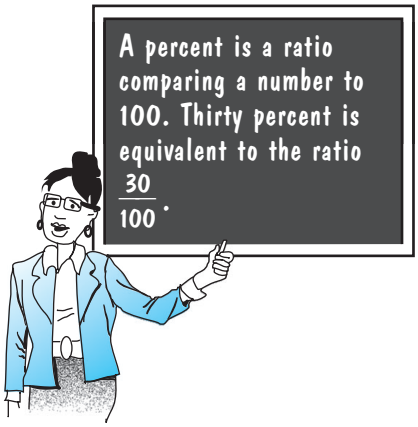
$$96 = 3n$$

$$\frac{96}{3} = \frac{3n}{3}$$

$$32 = n$$

Gina has 32 relatives who live in Texas.

Objective 2



Problems that involve percent can also be solved using proportions.

Mr. Cruz asked the seniors at a high school where they would like to go on a class trip. Of those surveyed, 65 percent voted for Washington, D.C. If there are 320 seniors, how many voted for Washington, D.C.?

First write a proportion that can be used to find n, the number of seniors who voted for Washington, D.C.

- Of those surveyed, 65 percent voted for Washington, D.C. A percent is a ratio that compares a number to 100. Write a ratio that compares 65 to 100.

$$\frac{65}{100}$$

- There are 320 seniors, and n of those seniors voted for Washington, D.C. Write a ratio that compares n to 320.

$$\frac{n}{320}$$

- Write a proportion, a statement that two ratios are equal.

$$\frac{65}{100} = \frac{n}{320}$$

Solve the proportion. Set the cross products equal to each other.

$$65 \cdot 320 = 100n$$

$$20{,}800 = 100n$$

$$\frac{20{,}800}{100} = \frac{100n}{100}$$

$$208 = n$$

So 208 seniors voted to go to Washington, D.C., on a class trip.

Try It

A can of mixed nuts contains 4 ounces of walnuts, 2 ounces of pecans, and 10 ounces of peanuts. What percent of the total weight of nuts in the can is made up of peanuts?

Let x represent the percent of the nuts' weight that is made up of peanuts.

The total weight of nuts in the can is

_____ + _____ + _____ = _____ ounces.

Write a proportion.

$$\frac{\text{weight of peanuts}}{\text{total weight}} \qquad \frac{\square}{\square} = \frac{x}{100}$$

Use cross products to solve the proportion.

_____ • 100 = _____ x

_____ = _____ x

_____ = x

Peanuts make up _____% of the total weight of nuts in the can.

The total weight of nuts in the can is $4 + 2 + 10 = 16$ ounces. Write a proportion.

$$\frac{10}{16} = \frac{x}{100}$$

$$10 \cdot 100 = 16x$$

$$1{,}000 = 16x$$

$$62.5 = x$$

Peanuts make up 62.5% of the total weight of nuts in the can.

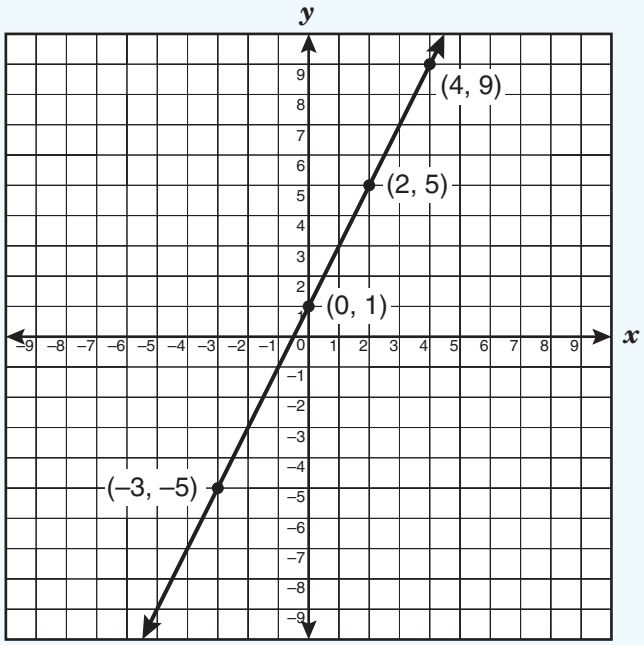
How Do You Compare Different Representations of a Relationship?

Sometimes you are asked to compare one representation of a relationship to another representation that is written in a different form. For example, you might be asked to compare a table to a graph or to find an equation to describe a relationship given in words.

The following table and graph are different representations of the same relationship.

x	y
−3	−5
0	1
2	5
4	9

Every ordered pair in the table is also a point on the line graphed below. Every point on the line fits the pattern shown in the table.



To see whether two different representations of a relationship are equivalent, follow these guidelines.

- See whether the pairs of related data from one representation fit the pattern for the other representation.
- Look for exceptions. Finding just one pair of values that works in both representations is not sufficient. Check as many pairs as possible.

When customers use a calling card, a phone company charges a 55-cent connection fee plus 3 cents for each minute the phone conversation lasts. The table below shows the costs of calls lasting 3, 5, 8, and 12 minutes.

Calling-Card Charges

Length of Call (minutes) (x)	Cost of Call (cents) (y)
3	64
5	70
8	79
12	91

Could you use the equation $y = 3x + 55$ to find y, the cost in cents of a calling-card telephone call lasting x minutes?

You are being asked whether two different representations of a relationship, a table and an equation, are equivalent.

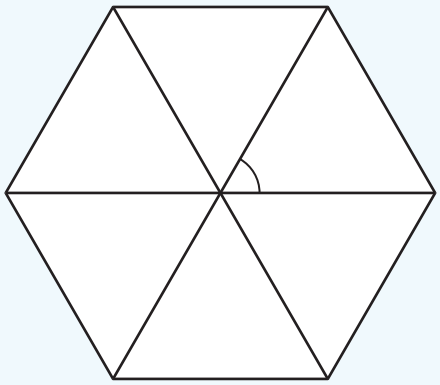
Substitute the values from the table for x and y and see whether they satisfy the equation $y = 3x + 55$.

x	$y = 3x + 55$	y	Yes/No
3	$64 \stackrel{?}{=} 3(3) + 55$ $64 \stackrel{?}{=} 9 + 55$ $64 = 64$	64	Yes
5	$70 \stackrel{?}{=} 3(5) + 55$ $70 \stackrel{?}{=} 15 + 55$ $70 = 70$	70	Yes
8	$79 \stackrel{?}{=} 3(8) + 55$ $79 \stackrel{?}{=} 24 + 55$ $79 = 79$	79	Yes
12	$91 \stackrel{?}{=} 3(12) + 55$ $91 \stackrel{?}{=} 36 + 55$ $91 = 91$	91	Yes

All the pairs of related numbers from the table fit the pattern for the equation.

If this pattern continues, the equation $y = 3x + 55$ could be used to find y, the cost in cents of a calling-card telephone call lasting x minutes.

To find the measures of the angles at the center of regular polygons, divide 360° by the number of sides in the polygon.



Does the table below show this relationship?

Number of Sides (n)	Angle Measure
4	90°
5	72°
6	60°
7	50°
8	45°

You are being asked whether two different representations of a relationship, a verbal description and a table, are equivalent.

Pick values from the table for the number of sides and the measure of the angle and see whether they satisfy the verbal description of this relationship.

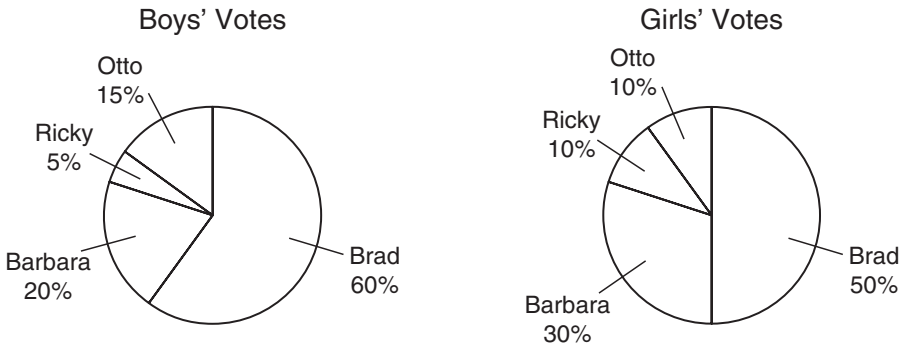
n	$\frac{360°}{n}$	Angle Measure	Yes/No
4	$\frac{360°}{4} = 90°$	90°	Yes
5	$\frac{360°}{5} = 72°$	72°	Yes
6	$\frac{360°}{6} = 60°$	60°	Yes
7	$\frac{360°}{7} = 51.42°$	50°	No
8	$\frac{360°}{8} = 45°$	45°	Yes

Not all the pairs of related numbers from the table fit the verbal description; there is one exception. The table's values do not match the verbal description of the relationship when the polygon has 7 sides.

The table does not match the verbal description of this relationship.

Try It

The circle graphs below show the percent of boys and girls in the junior class at Donley High School who voted for various candidates for class president.



Does the table below accurately represent the information in the graphs?

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	180	100
Barbara	60	60
Ricky	15	20
Otto	45	20

The two circle graphs give the voting data in the form of

_____.

To compare the graphs to the table, convert the data in the table to

_____.

To find what percent of the boys voted for Brad, _____ the number of boys who voted for Brad by the total number of boys. Then convert the quotient to a percent.

_____ ÷ 300 = _____

_____ = _____%

According to the table, _____% of the boys voted for Brad.

Compare this value to the circle graph. The graph shows that _____% of the boys voted for Brad.

These numbers agree, but _____ of the pairs of numbers must agree if these two representations are equivalent.

Complete the table below by converting the numbers of votes listed in the original table to percents.

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	60%	50%
Barbara		
Ricky		
Otto		

All the data in the table _____ with the data in the graphs. Therefore, the table accurately represents the information in the graphs.

The two circle graphs give the voting data in the form of percents. To compare the graphs to the table, convert the data in the table to percents. To find what percent of the boys voted for Brad, divide the number of boys who voted for Brad by the total number of boys.

$$180 \div 300 = 0.60$$

$$0.60 = 60\%$$

According to the table, 60% of the boys voted for Brad. The graph shows that 60% of the boys voted for Brad. These numbers agree, but all of the pairs of numbers must agree if these two representations are equivalent.

Junior Class President Election

Candidate	Boys' Votes	Girls' Votes
Brad	60%	50%
Barbara	20%	30%
Ricky	5%	10%
Otto	15%	10%

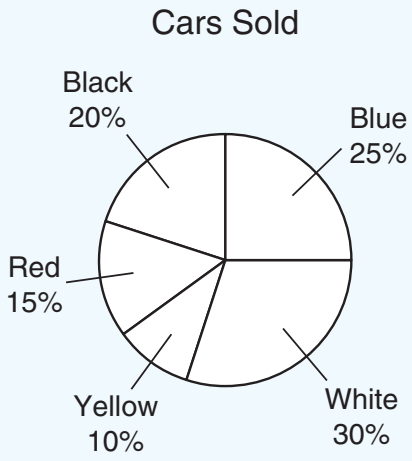
All the data in the table agree with the data in the graphs.

How Can Problems Be Solved Using Tables, Graphs, or Equations?

Problems that involve tables, graphs, and equations can be solved in the same way as other problems.

- Understand the problem. Identify the quantities that are involved and the relationships between those quantities.
- Write an equation you can use to solve the problem.
- Solve the equation. Answer the problem.
- See whether the answer you obtained to the problem is reasonable.

The graph shows the colors of cars a dealership sold in January.



The dealership expects to sell a total of 140 cars in February. If the number of red cars the dealership expects to sell in February is proportional to the number of red cars sold in January, how many red cars should the dealership expect to sell in February?

- Identify the percent of cars sold in January that were red. According to the graph, 15 percent of the cars sold in January were red.
- Use this percent to write a proportion that can be used to find the number of red cars that the dealership should expect to sell in February.

$$\begin{aligned} 15\% &= \frac{15}{100} \\ \frac{15}{100} &= \frac{x}{140} \\ 15 \cdot 140 &= 100x \\ 2{,}100 &= 100x \\ x &= 21 \end{aligned}$$

The dealership should expect to sell 21 red cars in February.

The table shows the total number of people who swam at a new neighborhood pool the first four days that it was open.

Pool Attendance

Day	Number of Swimmers
Monday	55
Tuesday	70
Wednesday	85
Thursday	100

The pool management schedules one lifeguard for each group of 40 or fewer swimmers. If the attendance pattern at the pool continues, how many lifeguards should be scheduled for Friday?

- First use the pattern of data in the table to predict Friday's attendance.

  From Monday to Tuesday, the attendance increased by 15 swimmers ($70 - 55 = 15$).

  From Tuesday to Wednesday, the attendance increased by 15 swimmers ($85 - 70 = 15$).

  From Wednesday to Thursday, the attendance increased by 15 swimmers ($100 - 85 = 15$).

  You can use this pattern to predict that Friday's attendance will be 15 swimmers greater than Thursday's attendance.

  $$100 + 15 = 115$$

  If the pattern continues, Friday's attendance should be 115 swimmers.

- For each group of 40 or fewer swimmers, the management schedules one lifeguard. Calculate the number of groups of 40 in 115 swimmers.

  $$115 \div 40 = 2.88$$

  There are 2 groups of 40 swimmers and 1 group of fewer than 40 swimmers. A lifeguard is needed for each group of 40 or fewer swimmers.

If the pattern continues, 3 lifeguards should be scheduled for Friday.

Try It

The table below describes an electric company's charges (c) in terms of the total number of kilowatt-hours of electricity (n) used during the month.

Residential Electric Billing Rates

Kilowatt-Hours Used per Month (n)	0–1,000	1,001–2,000	> 2,000
Amount Charged in Dollars (c)	$c = 30 + 0.05n$	$c = 35 + 0.04n$	$c = 50 + 0.03n$

The Wilson family used 1,650 kilowatt-hours of electricity last month. What was their electric bill for the month?

Since 1,650 is between 1,001 and 2,000, use the second formula in the table, _____, to determine the Wilsons' electric bill for the month.

Substitute _____ for n in the formula.

$c = 35 + 0.04n$

$c = 35 + 0.04($_____$)$

$c = 35 +$ _____

$c =$ _____

The Wilsons' electric bill for the month was $_____.

Since 1,650 is between 1,001 and 2,000, use the second formula in the table, $c = 35 + 0.04n$, to determine the Wilsons' electric bill for the month. Substitute 1,650 for n in the formula.

$c = 35 + 0.04n$

$c = 35 + 0.04(1{,}650)$

$c = 35 + 66$

$c = 101$

The Wilsons' electric bill for the month was $101.

How Can You Use an Algebraic Expression to Represent Any Term in a Sequence?

A **sequence** is a set of numbers written in a particular order. For example, 1, 9, 25, 49 is a sequence of four numbers. The number 1 is the first term in the sequence, 9 is the second term, 25 is the third term, and 49 is the fourth term.

What rule can be used to find the *n*th term in this sequence?

4, 8, 12, 16, . . .

Look at the relationship between the terms in the sequence and their position in the sequence.

Position	1	2	3	4	. . .	*n*
Value of Term	4	8	12	16	. . .	?

The 1st term is $4 \cdot 1 = 4$.

The 2nd term is $4 \cdot 2 = 8$.

The 3rd term is $4 \cdot 3 = 12$.

The 4th term is $4 \cdot 4 = 16$.

Each term in this sequence is equal to 4 times its position number in the sequence.

Represent the relationship algebraically. The value of the *n*th term is $4 \cdot n$, or $4n$.

What is the 15th term in this sequence?

4, 7, 10, 13, . . .

- First compare the position of a term to its value.

Position	Value of Term
1	4
2	7
3	10
4	13
n	?

- Look for a pattern that shows the relationship between a term's value and its position number.

  The 1st term is 4. Maybe the pattern is $4n$.

  The 2nd term is 7. Does the pattern $4n$ work for the second term? No. If n is 2, then $4n = 4(2) = 8$. The pattern does not work.

  Try a different multiple. Then add or subtract from the product to get the correct value. Try $3n + 1$.

  The rule works for the 1st term: $3(1) + 1 = 4$.

  The rule works for the 2nd term: $3(2) + 1 = 7$.

- Check to see whether the rule works for the next two terms in the sequence.

  The 3rd term in this sequence is 10 because $3(3) + 1 = 10$.

  The 4th term in this sequence is 13 because $3(4) + 1 = 13$.

  The nth term in the sequence is $3n + 1$.

- Find the value of the 15th term in the sequence. For the 15th term in the sequence, n is 15. Substitute 15 for n in the rule $3n + 1$.

$$3(15) + 1 = 45 + 1 = 46$$

The 15th term in the sequence is 46.

Look at this sequence of numbers.

$$-4, -6, -8, -10, -12, \ldots$$

Does the expression $-2(n + 1)$ show the relationship between any term and n, its position in the sequence?

Use the table to see whether this rule works for all the terms in the sequence.

Position	$-2(n + 1)$	Value of Term	Correct?
1	$-2(1 + 1) = -4$	-4	Yes
2	$-2(2 + 1) = -6$	-6	Yes
3	$-2(3 + 1) = -8$	-8	Yes
4	$-2(4 + 1) = -10$	-10	Yes
5	$-2(5 + 1) = -12$	-12	Yes

For the sequence $-4, -6, -8, -10, -12, \ldots$, the expression $-2(n + 1)$ shows the relationship between any term and n, its position in the sequence.

Try It

Which algebraic expression best describes the *n*th term in the sequence 2, 6, 12, 20, . . . , where *n* represents a term's position in the sequence?

$2n$ $\quad$ $n^2 + 2$ $\quad$ $n + n^2$

Check each expression for the values of *n* given in the table. Identify the rule that produces the correct term in each case.

Position	Rule #1 $2n$	Rule #2 $n^2 + 2$	Rule #3 $n + n^2$	Value of Term	Which Rule(s) Give the Correct Value?
1	$2(1) = 2$	$1^2 + 2 = 1 + 2 = 3$	$1 + 1^2 = 1 + 1 = 2$	2	Rule #1 Rule #3
2				6	
3				12	
4				20	

The expression _____ describes the *n*th term in the sequence because it is the only rule that works for all four numbers given.

Position	Rule #1 $2n$	Rule #2 $n^2 + 2$	Rule #3 $n + n^2$	Value of Term	Which Rule(s) Give the Correct Value?
1	$2(1) = 2$	$1^2 + 2 = 1 + 2 = 3$	$1 + 1^2 = 1 + 1 = 2$	2	Rule #1 Rule #3
2	$2(2) = 4$	$2^2 + 2 = 4 + 2 = 6$	$2 + 2^2 = 2 + 4 = 6$	6	Rule #2 Rule #3
3	$2(3) = 6$	$3^2 + 2 = 9 + 2 = 11$	$3 + 3^2 = 3 + 9 = 12$	12	Rule #3
4	$2(4) = 8$	$4^2 + 2 = 16 + 2 = 18$	$4 + 4^2 = 4 + 16 = 20$	20	Rule #3

The expression $n + n^2$ describes the *n*th term in the sequence because it is the only rule that works for all four numbers given.

Now practice what you've learned.

Question 11

Sam can type 280 words in 8 minutes. If Sam continues to type at the same rate, which equation can be used to find n, the number of words he can type in half an hour?

A $\frac{280}{8} = \frac{n}{2}$

B $\frac{280}{8} = \frac{n}{30}$

C $\frac{280}{8} = \frac{30}{n}$

D Not Here

Answer Key: page 239

Question 12

Briana delivers newspapers. She can deliver 60 papers in 45 minutes. Which of these represents an equivalent rate of delivering newspapers?

A 30 papers in $\frac{1}{2}$ hour

B 75 papers in 1 hour

C 120 papers in $1\frac{1}{2}$ hours

D 100 papers in 1 hour

Answer Key: page 239

Question 13

At Austin Shoe Factory 5 pairs of shoes, on average, can be placed in shoe boxes every 3 minutes. At this rate, how many pairs of shoes can be placed in shoe boxes during 8 hours of work?

A 48

B 2,880

C 13.3

D 800

Answer Key: page 240

Question 14

At Cantor Middle School 78% of the students ride the bus to school. If 975 students ride the bus, how many students attend the school?

A 760

B 975

C 1,250

D 1,053

Answer Key: page 240

Question 15

The Williamson Lumber Company charges a fee of $25 for a lumber delivery plus an additional fee based on the number of pieces of lumber being delivered.

Williamson Lumber Company

Number of Pieces of Lumber	Additional Fee
20	$4.80
30	$7.20
40	$9.60
50	$12.00
100	$24.00

Which equation can be used to find the total cost in dollars, c, to deliver n pieces of lumber?

A $c = 0.48n + 25$

B $c = 0.24n + 25$

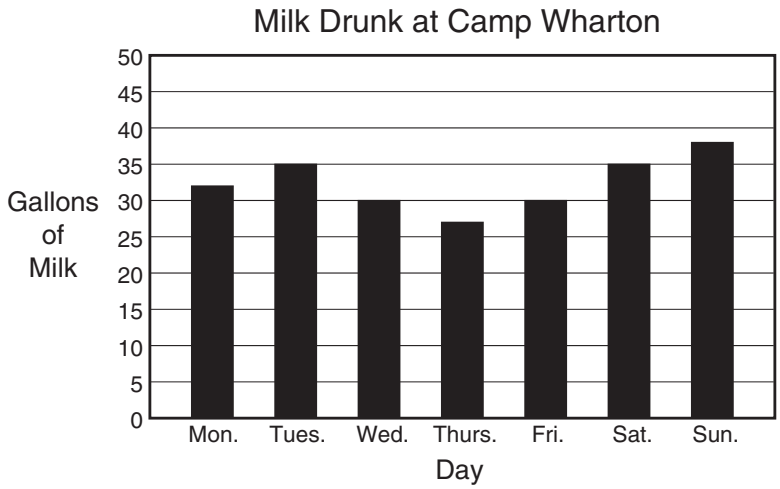
C $c = 2.5n + 4.80$

D $c = 0.24n \cdot 25$

Answer Key: page 240

Question 16

Camp Wharton serves milk to its campers at every meal. The graph below shows the number of gallons of milk the campers drank each day during one week.



Which table best represents the information in the graph?

A

Milk Drunk at Camp Wharton

Day	Milk (gallons)
Monday	32
Tuesday	35
Wednesday	30
Thursday	27
Friday	30
Saturday	35
Sunday	38

B

Milk Drunk at Camp Wharton

Day	Milk (gallons)
Monday	35
Tuesday	35
Wednesday	30
Thursday	30
Friday	30
Saturday	35
Sunday	40

C

Milk Drunk at Camp Wharton

Day	Milk (gallons)
Monday	30
Tuesday	35
Wednesday	30
Thursday	25
Friday	30
Saturday	35
Sunday	40

D

Milk Drunk at Camp Wharton

Day	Milk (gallons)
Monday	32
Tuesday	35
Wednesday	30
Thursday	27
Friday	30
Saturday	38
Sunday	35

Answer Key: page 240

Question 17

Linden Bank pays its customers interest on money kept in savings accounts. The table shows how much interest will be earned on $1,500 for different numbers of years the money is kept in the account.

Number of Years	2	3	4	5	11
Interest Earned	$195.00	$292.50	$390.00	$487.50	

Use the information in the table to determine how much interest in dollars and cents will be earned at Linden Bank in 11 years.

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.		
⓪	⓪	⓪	⓪		⓪	⓪
①	①	①	①		①	①
②	②	②	②		②	②
③	③	③	③		③	③
④	④	④	④		④	④
⑤	⑤	⑤	⑤		⑤	⑤
⑥	⑥	⑥	⑥		⑥	⑥
⑦	⑦	⑦	⑦		⑦	⑦
⑧	⑧	⑧	⑧		⑧	⑧
⑨	⑨	⑨	⑨		⑨	⑨

Answer Key: page 241

Question 18

A swim club charges its members a $25 annual membership fee plus $2 every time a member visits the pool. If Joanne spent a total of $365 last year in swim club charges, how many times did she visit the pool?

A 183

B 170

C 195

D 158

Answer Key: page 241

Question 19

Let n represent a term's position in a sequence. Which algebraic expression can be used to find the nth term of the sequence below?

4, 12, 24, 40, 60, . . .

A $2n(n + 1)$

B $4n$

C $4n^2$

D $n^2 + 3$

Answer Key: page 241

Question 20

A sequence of numbers was formed using the rule $\frac{n+1}{3n}$, where n represents the number's position in a sequence. Which sequence fits this rule?

A $\frac{2}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \frac{6}{18}, \ldots$

B $\frac{1}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \frac{6}{15}, \ldots$

C $0, \frac{2}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \ldots$

D $\frac{2}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \frac{6}{15}, \ldots$

Answer Key: page 241

Objective 3

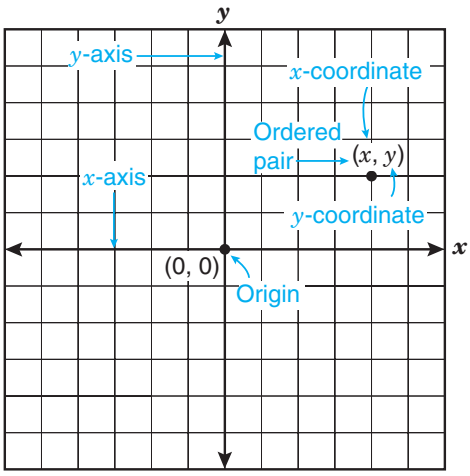
The student will demonstrate an understanding of geometry and spatial reasoning.

For this objective you should be able to

- use transformational geometry to develop spatial sense; and
- use geometry to model and describe the physical world.

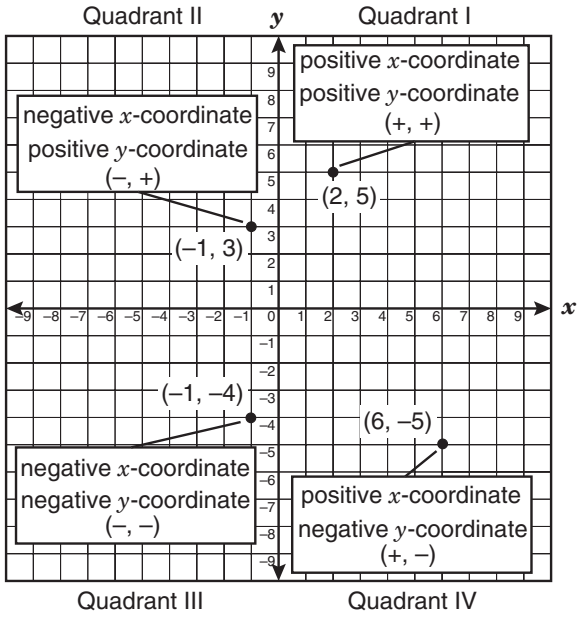
How Can You Locate and Name Points on a Coordinate Plane?

A **coordinate grid** is used to locate and name points on a plane. The coordinate grid is formed by two perpendicular number lines. A point is located by using an **ordered pair** of numbers. The two numbers that form the ordered pair are called **coordinates** of the point.



Do you see that . . .

The x-axis and y-axis divide the coordinate plane into 4 regions called **quadrants**. The quadrants are usually referred to by the Roman numerals I, II, III, and IV.



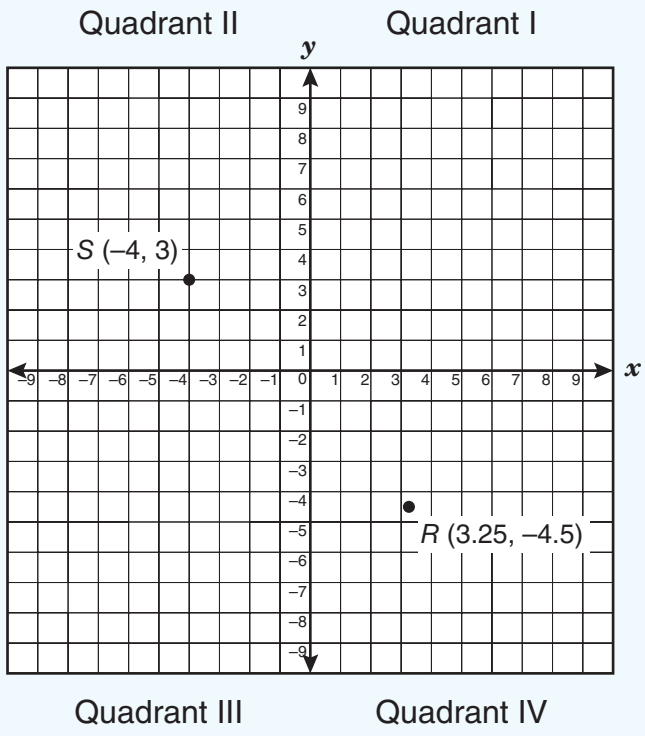
In which quadrants are points R (3.25, −4.5) and S (−4, 3) located?

For point R (3.25, −4.5):

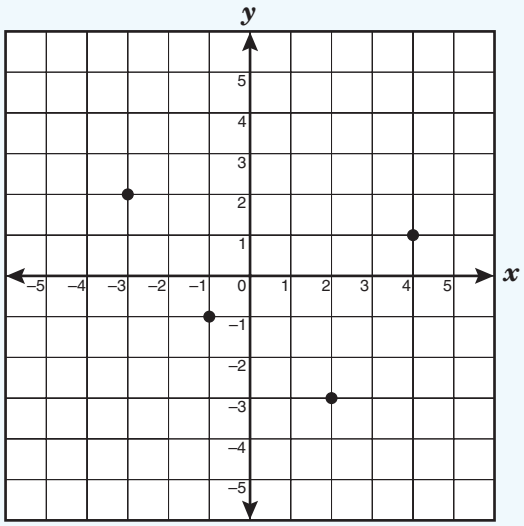
- The x-coordinate is 3.25, a positive value. The point is 3.25 units to the right of the origin, $\frac{1}{4}$ of the way between 3 and 4.
- The y-coordinate is −4.5, a negative value. The point is 4.5 units below the origin, halfway between −4 and −5.
- Point R (3.25, −4.5) is located in Quadrant IV because it has a positive x-coordinate and a negative y-coordinate.

For point S (−4, 3):

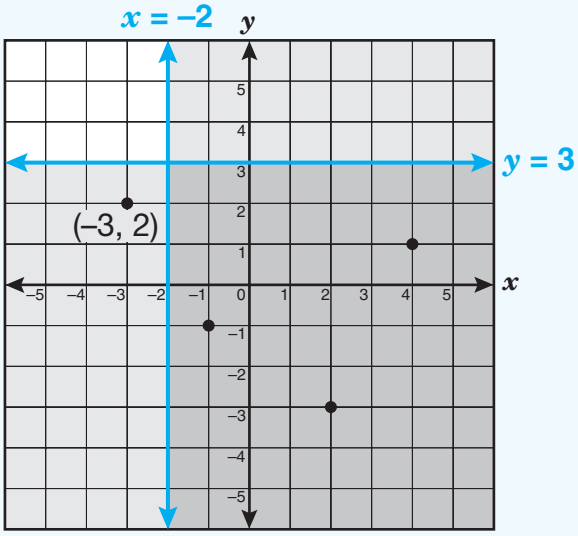
- The x-coordinate is −4, a negative value. The point is 4 units to the left of the origin.
- The y-coordinate is 3, a positive value. The point is 3 units above the origin.
- Point S (−4, 3) is located in Quadrant II because it has a negative x-coordinate and a positive y-coordinate.



Which point on the graph below does not meet the requirements $x \geq -2$ and $y \leq 3$?



- Draw a line through $x = -2$. All points to the right of this line have an x-coordinate greater than -2.
- Draw a line through $y = 3$. All points below this line have a y-coordinate less than 3.



Only the point with coordinates $(-3, 2)$ does not fall within the darker shaded region that satisfies the two inequalities.

Which of the three points below is on the line $y = 3x + 4$ and also in Quadrant II?

$A\ (-3, 5)$ $B\ (-1, 1)$ $C\ (1, 7)$

- Consider the coordinates of point A $(-3, 5)$. If x is replaced by -3 and y is replaced by 5, is the equation true?

$$y = 3x + 4$$
$$5 \stackrel{?}{=} 3(-3) + 4$$
$$5 \stackrel{?}{=} -9 + 4$$
$$5 \neq -5$$

No. Therefore, point $A\ (-3, 5)$ is not on the line $y = 3x + 4$.

- Consider the coordinates of point $B\ (-1, 1)$. If x is replaced by -1 and y is replaced by 1, is the equation true?

$$y = 3x + 4$$
$$1 \stackrel{?}{=} 3(-1) + 4$$
$$1 \stackrel{?}{=} -3 + 4$$
$$1 = 1$$

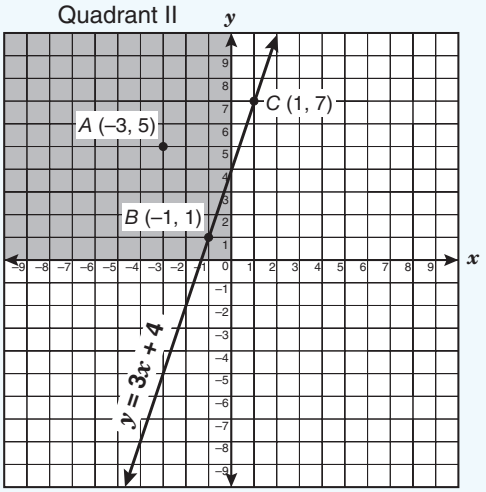
Yes. Therefore, point $B\ (-1, 1)$ is on the line $y = 3x + 4$.

Is point $B\ (-1, 1)$ in Quadrant II? Yes. It has a negative x-coordinate and a positive y-coordinate.

- Consider the coordinates of point $C\ (1, 7)$. If x is replaced by 1 and y is replaced by 7, is the equation true?

$$y = 3x + 4$$
$$7 \stackrel{?}{=} 3(1) + 4$$
$$7 \stackrel{?}{=} 3 + 4$$
$$7 = 7$$

Yes. Therefore, point $C\ (1, 7)$ is on the line $y = 3x + 4$.
Is point $C\ (1, 7)$ in Quadrant II? No. It has a positive x-coordinate and a positive y-coordinate, which means it is in Quadrant I.

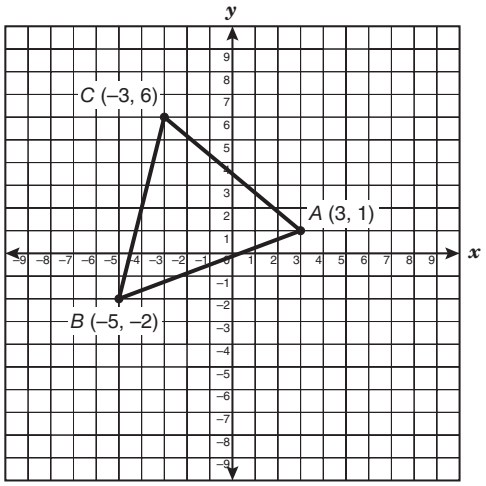


Only point B is on the line $y = 3x + 4$ and also in Quadrant II.

Are the x-coordinates and y-coordinates of points in Quadrant III positive or negative?

Try It

Which vertex of $\triangle ABC$ is in Quadrant III?



- For point A $(3, 1)$

  The x-coordinate is _____ , a positive value.

  The y-coordinate is _____ , a _____ value.

  The point is located in Quadrant _____ because it has a positive x-coordinate and a positive y-coordinate.

- For point B $(-5, -2)$

  The x-coordinate is _____ , a negative value.

  The y-coordinate is _____ , a _____ value.

The point is located in Quadrant _____ because it has a negative x-coordinate and a negative y-coordinate.

- For point C $(-3, 6)$

  The x-coordinate is _____ , a _____ value.

  The y-coordinate is _____ , a _____ value.

  The point is located in Quadrant _____ because it has a negative x-coordinate and a positive y-coordinate.

Point _____ is the only vertex of $\triangle ABC$ that is in Quadrant III.

For point A (3, 1)
The x-coordinate is 3, a positive value. The y-coordinate is 1, a positive value. The point is located in Quadrant I because it has a positive x-coordinate and a positive y-coordinate.

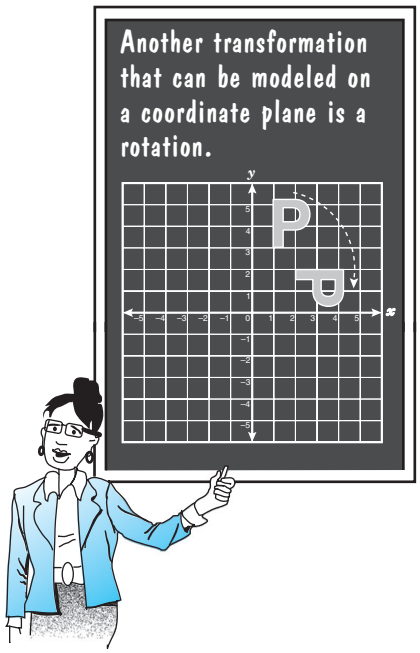
For point B $(-5, -2)$
The x-coordinate is -5, a negative value. The y-coordinate is -2, a negative value. The point is located in Quadrant III because it has a negative x-coordinate and a negative y-coordinate.

For point C $(-3, 6)$
The x-coordinate is -3, a negative value. The y-coordinate is 6, a positive value. The point is located in Quadrant II because it has a negative x-coordinate and a positive y-coordinate.

Point B is the only vertex of $\triangle ABC$ that is in Quadrant III.

How Can You Show Transformations on a Coordinate Plane?

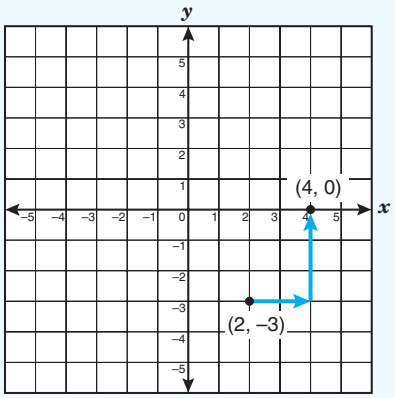
Translations, reflections, and dilations can be modeled on a coordinate plane. A figure has been translated or reflected if it has been moved without changing its shape or size. A figure has been dilated if its size has been changed proportionally.



Translations

A **translation** of a figure is a movement of the figure along a line. It can be described by stating how many units to the left or right the figure is moved and how many units up or down it is moved. A figure and its translated image are always congruent.

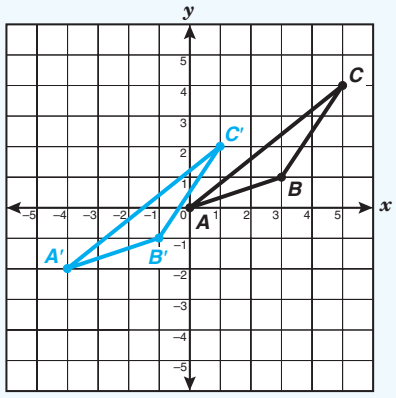
If the point (2, –3) is translated 2 units to the right and 3 units up, what are the coordinates of the new point?



- The x-coordinate increases by 2. Because $2 + 2 = 4$, the new x-coordinate is 4.
- The y-coordinate increases by 3. Because $-3 + 3 = 0$, the new y-coordinate is 0.

The coordinates of the new point are (4, 0).

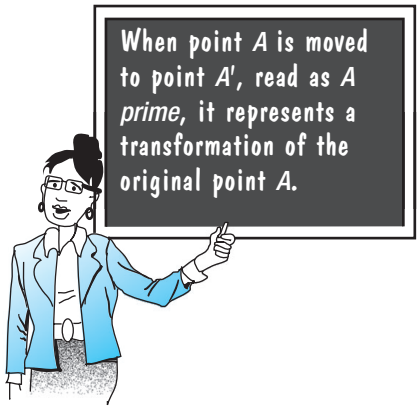
If $\triangle ABC$ is translated 4 units to the left and 2 units down, what will be the coordinates of the vertices of the translated triangle?



The vertices of $\triangle ABC$ are A (0, 0), B (3, 1), and C (5, 4).

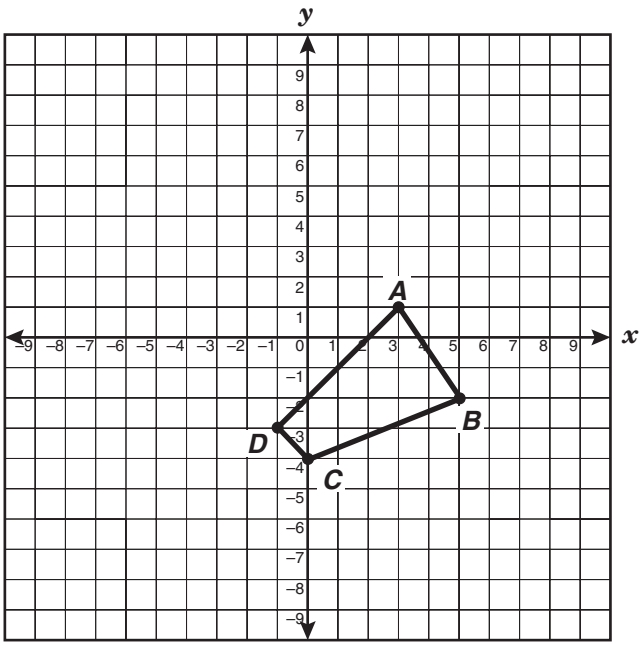
- The triangle is translated 4 units to the left, so subtract 4 from the x-coordinate of each vertex.
- The triangle is translated 2 units down, so subtract 2 from the y-coordinate of each vertex.

The vertices of $\triangle A'B'C'$ are A' $(-4, -2)$, B' $(-1, -1)$, and C' (1, 2).



Try It

Quadrilateral *ABCD* is translated to a new location. If point *A* is translated to (6, −1), what are the coordinates of points *B′*, *C′*, and *D′*?



The *x*-coordinate of point *A* was translated from 3 to 6.

Point *A* was moved _____ units to the right.

The *y*-coordinate of point *A* was translated from _____ to _____ .

Point *A* was moved _____ units _____ .

Translate the remaining points _____ units to the _____ and _____ units _____ .

The coordinates of vertex *B′* are (_____ , _____).

The coordinates of vertex *C′* are (_____ , _____).

The coordinates of vertex *D′* are (_____ , _____).

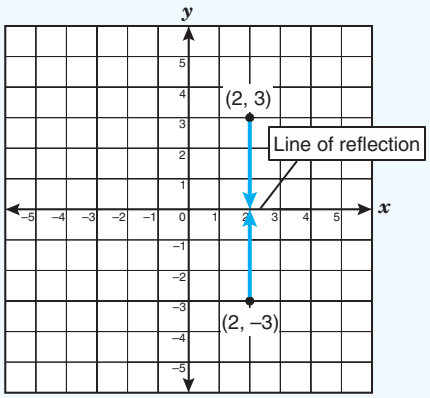
Point *A* was moved 3 units to the right. The *y*-coordinate of point *A* was translated from 1 to −1. Point *A* was moved 2 units down. Translate the remaining points 3 units to the right and 2 units down. The coordinates of vertex *B′* are (8, −4). The coordinates of vertex *C′* are (3, −6). The coordinates of vertex *D′* are (2, −5).

Reflections

A **reflection** of a figure is its mirror image. A figure is reflected across a line called the **line of reflection**. The line of reflection serves as the mirror on which the figure is reflected. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but it is on the opposite side of the line of reflection.

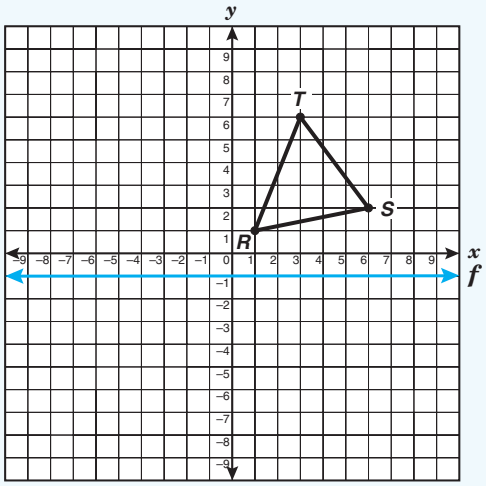
If the point $(2, -3)$ is reflected across the x-axis, what will its new coordinates be?



- The x-coordinate of the point will be unchanged because the point is being reflected across the x-axis. The new point will have an x-coordinate of 2.
- The y-coordinate of the point is 3 units below the x-axis, so the y-coordinate of the new point will be 3 units above the x-axis. The new point will have a y-coordinate of 3.

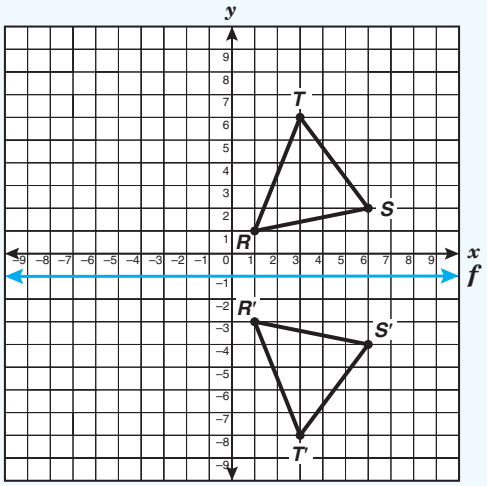
The coordinates of the new point will be $(2, 3)$. The point $(2, -3)$ and its image $(2, 3)$ are equally distant from the line of reflection, the x-axis.

Triangle *RST* has vertices *R* (1, 1), *S* (6, 2), and *T* (3, 6). Find the coordinates of its reflection across line *f*.



Each vertex of the original triangle and the reflected triangle must be the same distance from line *f*.

- Vertex *R* (1, 1) is 2 units above the line of reflection. The vertex of the reflection must be 2 units below the line. The coordinates of the reflected vertex *R'* are (1, −3).
- Vertex *S* (6, 2) is 3 units above the line of reflection. The vertex of the reflection must be 3 units below the line. The coordinates of the reflected vertex *S'* are (6, −4).
- Vertex *T* (3, 6) is 7 units above the line of reflection. The vertex of the reflection must be 7 units below the line. The coordinates of the reflected vertex *T'* are (3, −8).



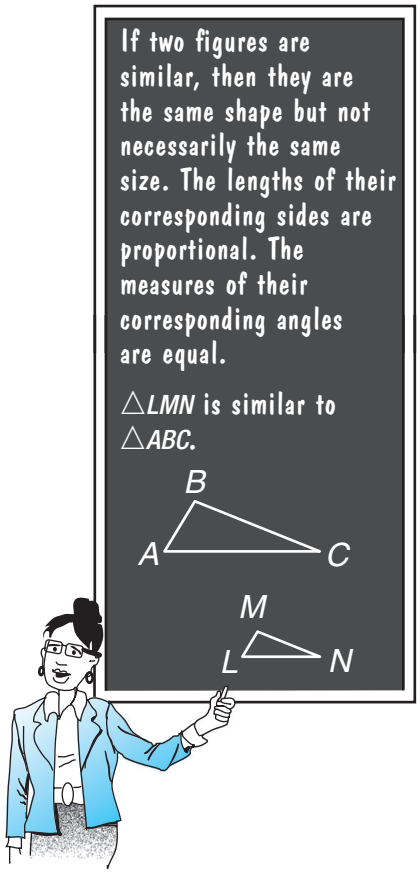
The vertices of the reflected triangle are *R'* (1, −3), *S'* (6, −4), and *T'* (3, −8).

Dilations

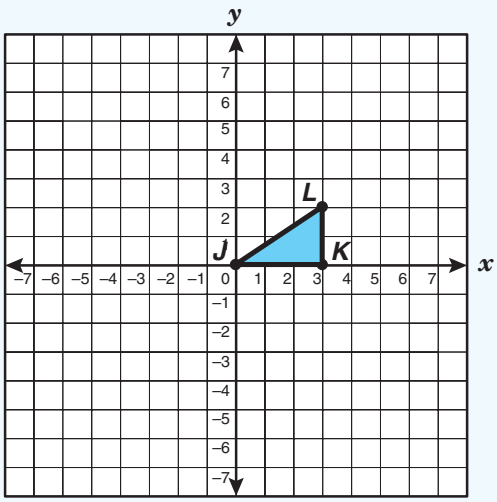
A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor > 1. This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor < 1. This is called a **reduction**.

A figure and its dilated image are always similar.

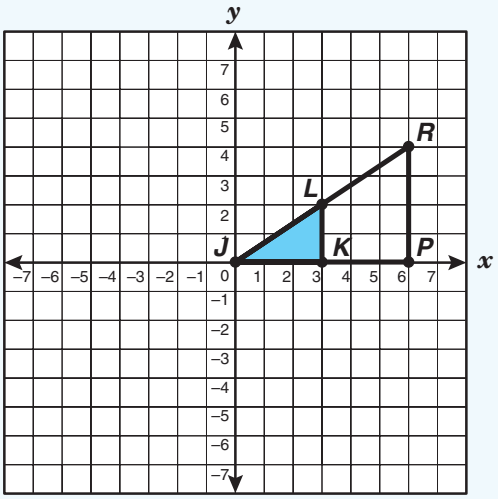


If $\triangle JKL$ is dilated by a factor of 2 using the origin as the center of dilation, and point J is not moved, what will be the coordinates of the dilation, $\triangle JPR$?



If the triangle is dilated by a factor of 2, each of the new triangle's line segments will be twice as long as the original triangle's line segments.

- The new triangle is shown on the coordinate grid below.

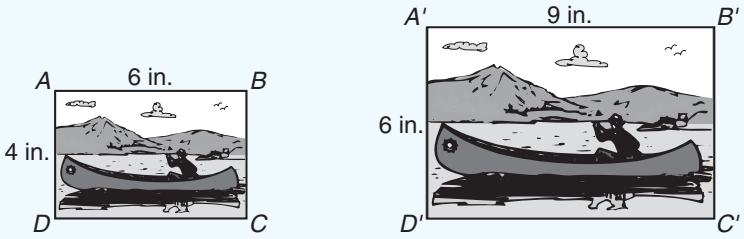


- $\overline{JP}$ will be twice as long as $\overline{JK}$. So the x-coordinate of point P will be 6, and the coordinates of point P will be (6, 0).
- Similarly, $\overline{PR}$ will be twice as long as $\overline{KL}$, so the coordinates of point R will be (6, 4).

The coordinates of $\triangle JPR$ will be J (0, 0), P (6, 0), and R (6, 4).

To find the scale factor of a dilation, compare the lengths of a pair of corresponding sides of the two figures. The ratio of their lengths is equal to the scale factor of the dilation.

If a 4 × 6 inch picture was enlarged so that the new picture measures 6 × 9 inch, by what scale factor was the picture enlarged?



- Write the ratios of the lengths of the corresponding sides of the dilated figure and the original figure.

$$\frac{\text{large}}{\text{small}} \quad \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{C'D'}{CD} = \frac{A'D'}{AD}$$

- Substitute the known measures.

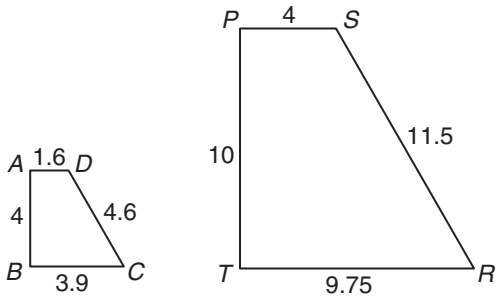
$$\frac{A'B'}{AB} = \frac{9}{6} = \frac{3}{2} \qquad \frac{B'C'}{BC} = \frac{6}{4} = \frac{3}{2}$$

$$\frac{C'D'}{CD} = \frac{9}{6} = \frac{3}{2} \qquad \frac{A'D'}{AD} = \frac{6}{4} = \frac{3}{2}$$

The scale factor is $\frac{3}{2}$, or 1.5. Each dimension of the enlarged picture is 1.5 times its corresponding measure in the smaller picture.

Try It

Trapezoid *PTRS* is a dilation of trapezoid *ABCD*. What is the scale factor of the dilation?



To find the scale factor, find the ratios of corresponding sides.

$\overline{PT}$ corresponds to _____ .

$$\frac{\text{large}}{\text{small}} \quad \frac{PT}{\square} = \frac{10}{\square}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is _____ .

$\overline{PS}$ corresponds to _____ .

$$\frac{\text{large}}{\text{small}} \quad \frac{PS}{\square} = \frac{\square}{\square}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is _____ .

The scale factor of the dilation is _____ .

$\overline{PT}$ corresponds to $\overline{AB}$.

$$\frac{PT}{AB} = \frac{10}{4}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is 2.5. $\overline{PS}$ corresponds to $\overline{AD}$.

$$\frac{PS}{AD} = \frac{4}{1.6}$$

Written as a decimal, the ratio of the lengths of these corresponding sides is 2.5. The scale factor of the dilation is 2.5.

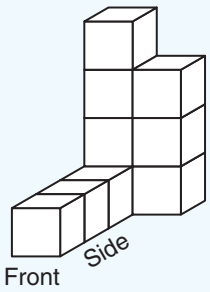
How Do You Recognize a Solid from Different Perspectives?

Given a drawing of a 3-dimensional figure, a solid, you should be able to recognize other drawings that represent the same figure from a different perspective.

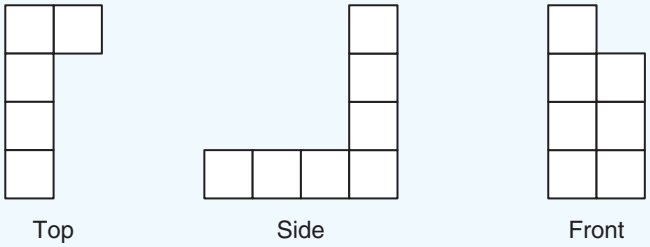
A 3-dimensional figure can be represented by drawing the figure from three different views: front, top, and side.

To recognize the solid from different perspectives, you must visualize what the solid would look like if you were seeing it from above, from one side, and from the front.

The solid below is made up of equal-sized cubes. Can you visualize what it would look like from above, from the side, and from the front?



These are the top, side, and front views of the solid.

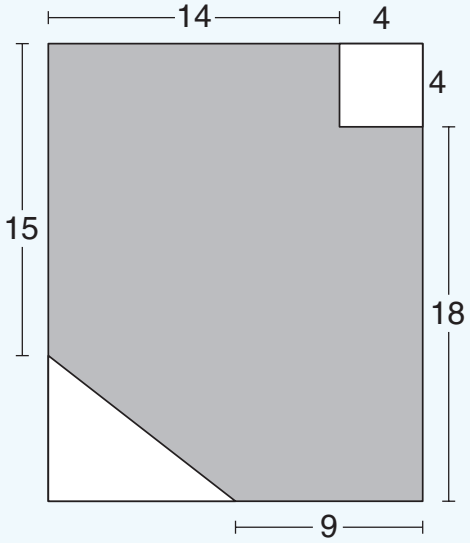


What Kinds of Problems Can You Solve with Geometry?

You can solve many types of problems using geometry, including problems involving these geometric concepts:

- the area or perimeter of figures;
- the measure of the sides or angles of polygons;
- the surface area and volume of solid figures;
- the ratios of the sides of similar figures; and
- the relationship among the sides of a right triangle.

The shaded part of the rectangular room in the drawing below is to be carpeted. The given dimensions are in feet.



How many square feet of carpet will be required?

- The room is a rectangle with two pieces that will not be carpeted: a square in the upper right corner and a triangle in the lower left corner. Use subtraction to remove the area of the two unshaded parts from the area of the whole.
- The area of a rectangle is $A = lw$, where l stands for the length of the rectangle and w stands for the width.

  The length of the rectangle is $18 + 4 = 22$ ft.
  The width of the rectangle is $14 + 4 = 18$ ft.
  The area of the rectangle is $22 \cdot 18 = 396$ ft^2.
- The area of the square is $A = s^2$, where s stands for the length of a side of the square.

  The length of a side of the square is 4 ft.
  The area of the square is $4 \cdot 4 = 16$ square ft^2.
- The area of the triangle is $A = \frac{1}{2}bh$, where b stands for the length

of the base of the triangle and h stands for the height of the triangle.

The base of the triangle is $18 - 9 = 9$ ft.
The height of the triangle is $22 - 15 = 7$ ft.

The area of the triangle is $\frac{1}{2} \cdot 9 \cdot 7 = 31.5$ ft^2.

- Subtract to find the area of the carpet needed to cover the floor.

$$396 - 16 - 31.5 = 348.5 \text{ ft}^2$$

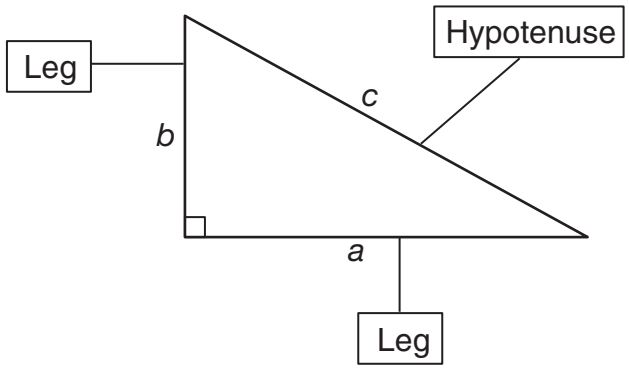
Covering the floor will require 348.5 square feet of carpet.

What Is the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. This relationship applies only to right triangles.

The sides of a right triangle have special names.

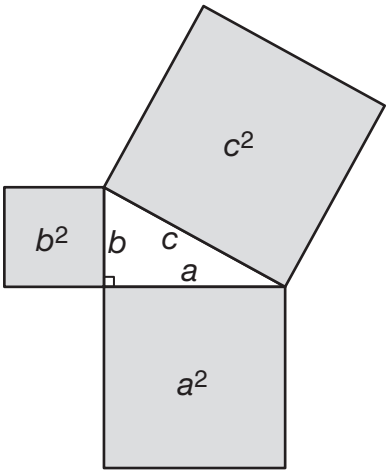
- The **hypotenuse** of a right triangle is the longest side of the triangle. The hypotenuse is always opposite the right angle in the triangle. In the diagram below, the length of the hypotenuse is represented by c.
- The **legs** of a right triangle are the two sides that form the right angle. In the diagram below, the lengths of the legs are represented by a and b.



The Pythagorean Theorem can be stated algebraically or verbally, or interpreted with a geometric model.

Algebraic	Verbal
In any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.	In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Geometric Model

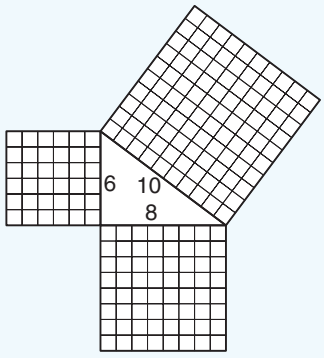


In the geometric model, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

You can use the Pythagorean Theorem to determine whether a triangle is a right triangle.

A triangle has side lengths of 6 units, 8 units, and 10 units. Is the triangle a right triangle?

- Identify the length of the longest side, 10 units. Substitute this value for c.
- The lengths of the other two sides are 6 units and 8 units. Substitute these values for a and b.
- Then see whether $a^2 + b^2 = c^2$.



$$6^2 + 8^2 \stackrel{?}{=} 10^2$$

$$36 + 64 \stackrel{?}{=} 100$$

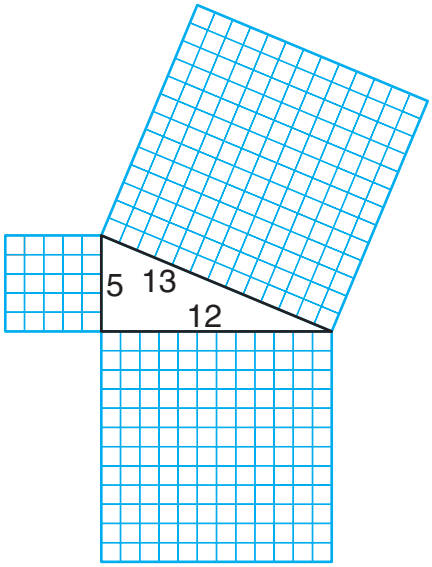
$$100 = 100$$

Since the Pythagorean Theorem is true for the lengths of the sides of this triangle, this triangle is a right triangle.

Try It

Mark cut two pieces of wood, one 5 feet long and the other 12 feet long. If the third piece he cuts is 13 feet long, could the three pieces form a right triangle?

The piece that is _____ feet long should be the hypotenuse of the triangle because it is the _____ side. The other two pieces should be the legs of the triangle.



The square formed by the longest side measures _____ feet by _____ feet.

It has an area of _____ • _____ = _____ square feet.

The square formed by the shortest side measures _____ feet by _____ feet.

It has an area of _____ • _____ = _____ square feet.

The square formed by the remaining side measures _____ feet by _____ feet.

It has an area of _____ • _____ = _____ square feet.

Since _____ = _____ + _____ , the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

The three pieces of wood could form a _____ triangle.

The piece that is 13 feet long should be the hypotenuse of the triangle because it is the longest side. The square formed by the longest side measures 13 feet by 13 feet. It has an area of $13 \cdot 13 = 169$ square feet. The square formed by the shortest side measures 5 feet by 5 feet. It has an area of $5 \cdot 5 = 25$ square feet. The square formed by the remaining side measures 12 feet by 12 feet. It has an area of $12 \cdot 12 = 144$ square feet. Since $169 = 25 + 144$, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. The three pieces of wood could form a right triangle.

Now practice what you've learned.

Question 21

A circle has a radius of 4 inches. The circle is dilated by reducing its radius by 60%. What is the radius of the reduced circle?

A 6.4 in.

B 2.4 in.

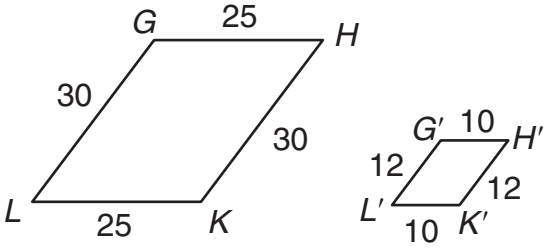
C 2 in.

D 1.6 in.

Answer Key: page 241

Question 22

The smaller parallelogram is a dilation of the larger one.



What scale factor was used to reduce the larger parallelogram?

A 0.3

B 2.5

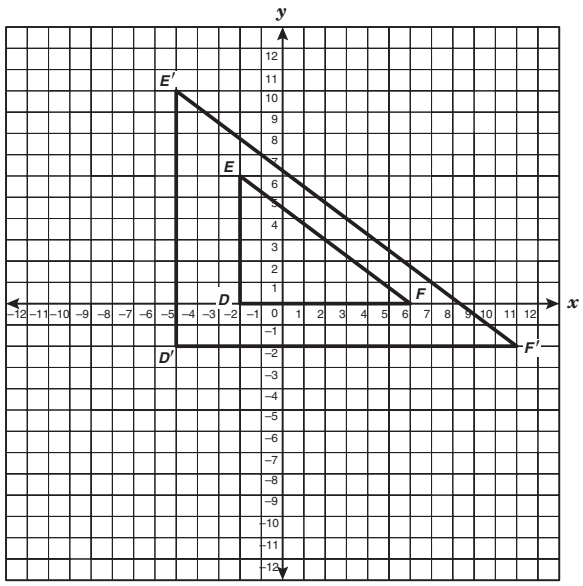
C 3.0

D 0.4

Answer Key: page 242

Question 23

The drawing shows $\triangle DEF$ and its enlargement, $\triangle D'E'F'$. By what scale factor was $\triangle DEF$ enlarged?



A $\frac{1}{4}$

B 4

C 2

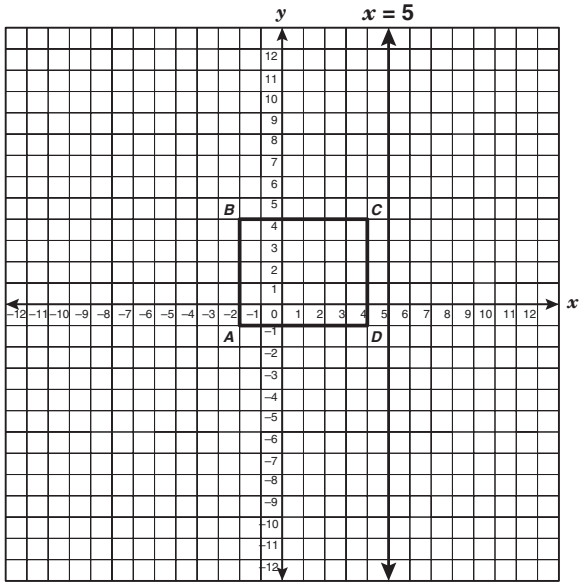
D $\frac{1}{2}$

Answer Key: page 242

MATHEMATICS

Question 24

If rectangle *ABCD* is reflected across the line $x = 5$, what will be the coordinates of the image of point *B*?



A (12, 4)

B (2, 4)

C (−2, 6)

D (11, −2)

Answer Key: page 242

Question 25

Triangle *ABC* is reflected across a line of reflection to form triangle *A'B'C'*. Triangle *ABC* has coordinates *A* (1, 2), *B* (4, 5), and *C* (5, 3). Its reflected image has coordinates *A'* (1, −2), *B'* (4, −5), and *C'* (5, −3). Which line is the line of reflection for this transformation?

A *y*-axis

B *x*-axis

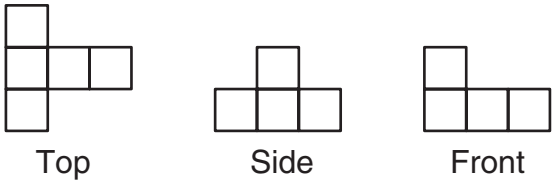
C $x = 1$

D $y = 2$

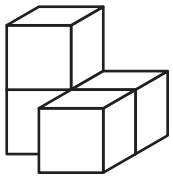
Answer Key: page 242

Question 26

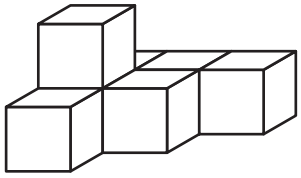
Which solid figure has the top, side, and front views that are shown below?



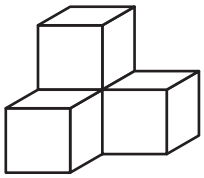
A



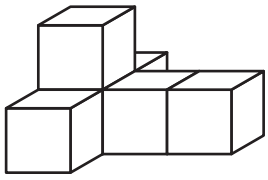
B



C



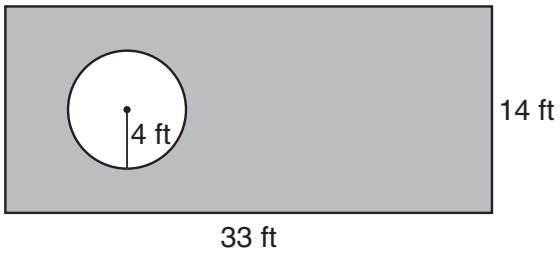
D



Answer Key: page 242

Question 27

Mr. Wythe uses a machine to polish the lobby floor in an office building. The floor is a rectangle with a circular fountain, as shown below. Mr. Wythe cannot polish the area covered by the fountain. Which of the following is closest to the number of square feet of floor Mr. Wythe polishes?



A 512.24 ft^2

B 462 ft^2

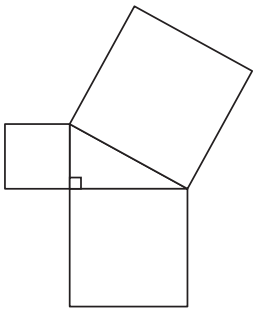
C 411.76 ft^2

D 436.88 ft^2

Answer Key: page 242

Question 28

A right triangle is shown below.



Which set of numbers represents the lengths of the sides of a right triangle?

A 12, 4, 14

B 5, 3, 4

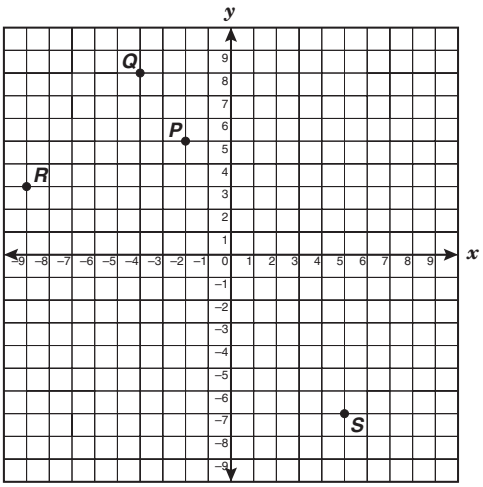
C 6, 8, 11

D 2, 9, 8

Answer Key: page 242

Question 29

Which of the following points on the coordinate grid below satisfies the conditions $x < -2.5$ and $y > 3$?



A Point P

B Point Q

C Point R

D Point S

Answer Key: page 243

Question 30

Which point lies on the line $y = -x$ and is in Quadrant IV?

A $(-7, 7)$

B $(5, 5)$

C $(4, -4)$

D $(1, -2)$

Answer Key: page 243

Objective 4

The student will demonstrate an understanding of the concepts and uses of measurement.

For this objective you should be able to

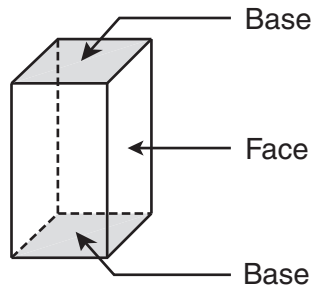
- use procedures to determine the measures of solids;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measures.

How Do You Find the Surface Area of Solids?

You can use models or formulas to find the surface area of prisms, cylinders, and other solids.

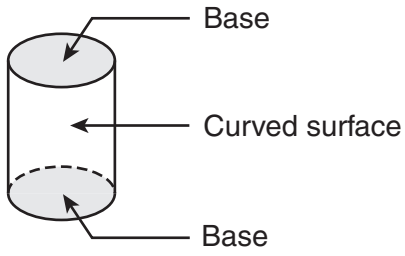
- A **prism** is a solid figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangles. The prism is named by the shape of its bases. For example, a square prism has two squares as its bases.

Square Prism



- A **cylinder** is a solid figure with two congruent circular bases and a curved surface.

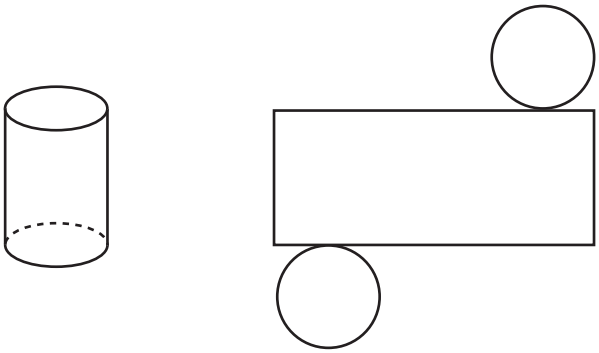
Cylinder

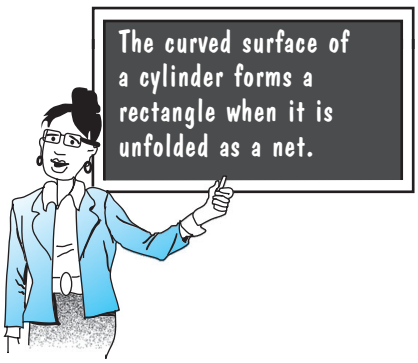


Like the area of a plane figure, the surface area of a solid figure is measured in square units.

- The **total surface area** of a solid figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a solid figure is equal to the sum of the areas of all its faces and curved surfaces. It does not include the areas of the figure's bases.

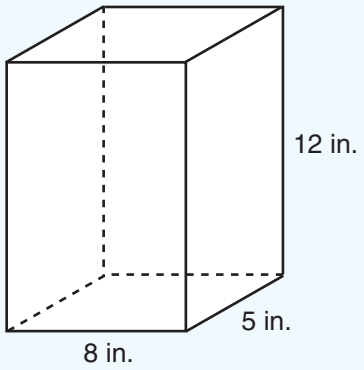
One way to compute the surface area of a solid figure is to use a net of the figure. A **net** of a three-dimensional figure is a two-dimensional drawing that shows what the figure would look like if it were unfolded, with all its surfaces laid flat. Use the net to find the area of each surface.



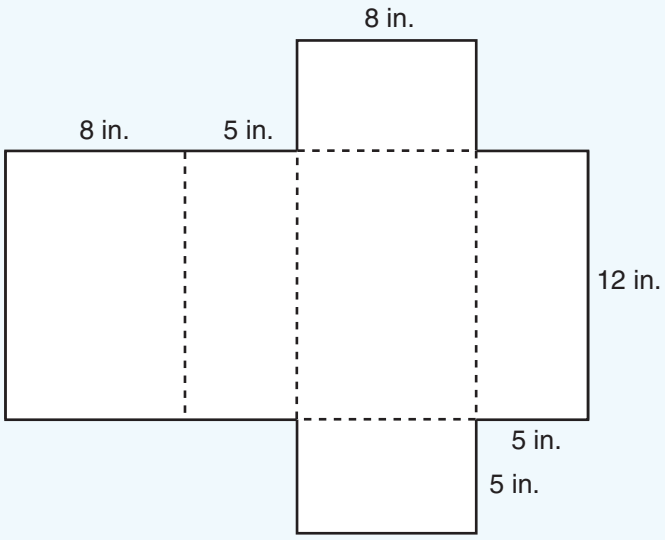


You can also find the surface area of a solid figure by using a formula. Substitute the dimensions of the figure in the formula and compute its surface area. The formulas for the total surface area and lateral surface area of several types of solid figures are included in the Mathematics Chart.

A gift box has dimensions of 5, 8, and 12 inches. Find its surface area in square inches.



The box, a rectangular prism, can be unfolded to make a net that shows the prism's faces. To find the surface area of a rectangular prism, find the sum of the areas of the six rectangular faces.



Find the areas of the six faces.

- There are two rectangular faces that each measure 5 by 8 inches.

  Each of these faces has an area of $5 \cdot 8 = 40$ in.^{2}

  The two faces have a total area of $2 \cdot 40 = 80$ in.^{2}
- There are two rectangular faces that each measure 5 by 12 inches.

  Each of these faces has an area of $5 \cdot 12 = 60$ in.^{2}

  The two faces have a total area of $2 \cdot 60 = 120$ in.^{2}
- There are two rectangular faces that each measure 8 by 12 inches.

  Each of these faces has an area of $8 \cdot 12 = 96$ in.^{2}

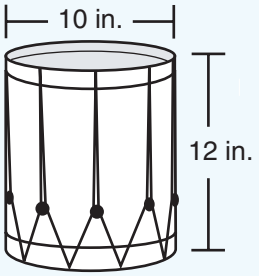
  The two faces have a total area of $2 \cdot 96 = 192$ in.^{2}

The surface area of the prism is the sum of the areas of all the faces.

$$80 \text{ in.}^2 + 120 \text{ in.}^2 + 192 \text{ in.}^2 = 392 \text{ in.}^2$$

The surface area of the gift box is 392 square inches.

A drum maker manufactures a cylindrical drum with a diameter of 10 inches and a height of 12 inches. The top and bottom of the drum are made of leather, but the curved surface of the drum is made of wood. Approximately how many square inches of wood is required to form the curved surface of the drum?

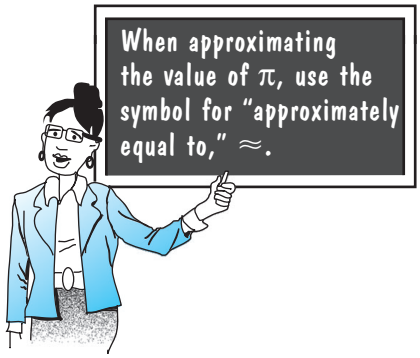


The drum is shaped like a cylinder. The area of the curved surface of the drum is its lateral surface area. Use the formula for the lateral surface area of a cylinder in the Mathematics Chart, $S = 2\pi rh$.

- The diameter of the drum is 10 inches, but the formula for the lateral surface area requires that you know r, the radius of the drum. The radius of a circle is $\frac{1}{2}$ the diameter. The radius of the drum is 5 inches. Substitute 5 for r in the formula.
- Substitute 12, the height of the drum, for h in the formula.
- Use 3.14 as an approximate value of π.

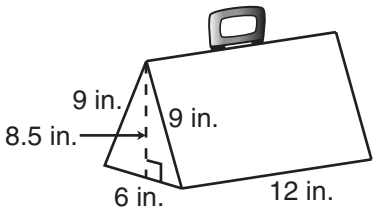
$$S = 2\pi rh$$
$$S \approx 2(3.14)(5)(12)$$
$$S \approx 376.8 \text{ in.}^2$$

About 377 square inches of wood is required to form the curved surface of the drum.



Try It

Maria is building a carrying case for a musical instrument. The case will have the shape of a triangular prism. She wants to know how much material will be needed to build it. The case will have the dimensions given in the diagram. What is the surface area of the case in square inches?



The case has the shape of a _____.

The surface area of a prism is equal to the _____ of the areas of the prism's surfaces.

There are _____ rectangular surfaces that each measure 9 by 12 inches. Each of these surfaces has an area of _____ • _____ = _____ $in.^2$ The two surfaces have a total area of 2 • _____ = _____ $in.^2$

There is _____ rectangular surface that measures 6 by 12 inches. This surface has an area of _____ • _____ = _____ $in.^2$

There are _____ triangular surfaces. They each have a base of _____ inches and a height of _____ inches. The formula for the area of a triangle is _____ .

Each of these surfaces has an area of _____ • _____ • _____ = _____ $in.^2$

The two surfaces have a total area of 2 • _____ = _____ $in.^2$

Find the sum of the areas of the prism's surfaces.

_____ + _____ + _____ = _____ $in.^2$

The surface area of the case is _____ square inches.

The case has the shape of a triangular prism. The surface area of a prism is equal to the sum of the areas of the prism's surfaces.

There are two rectangular surfaces that each measure 9 by 12 inches. Each of these surfaces has an area of $9 \cdot 12 = 108$ $in.^2$ The two surfaces have a total area of $2 \cdot 108 = 216$ $in.^2$

There is one rectangular surface that measures 6 by 12 inches. This surface has an area of $6 \cdot 12 = 72$ $in.^2$

There are two triangular surfaces. They each have a base of 6 inches and a height of 8.5 inches. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Each of these surfaces has an area of $\frac{1}{2} \cdot 6 \cdot 8.5 = 25.5$ $in.^2$ The two surfaces have a total area of $2 \cdot 25.5 = 51$ $in.^2$

Find the sum of the areas of the prism's surfaces: $216 + 72 + 51 = 339$ $in.^2$

The surface area of the case is 339 square inches.

What Is Volume?

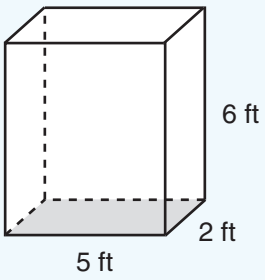
The **volume** of a solid is a measure of the space it occupies. Volume is measured in cubic units.

You can find the volume of a solid figure by using the appropriate volume formula. The formulas for computing the volume of several solid figures are given in the Mathematics Chart.

When using a formula to find the volume of a solid, follow these guidelines.

- Identify the solid figure with which you are working. This will help you select the correct volume formula.
- You may wish to use a model to help you visualize the solid and correctly assign the variables in the volume formula.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Compute the volume and state your answer in cubic units.

Find the volume of a rectangular prism that is 6 feet tall and has a base that measures 5 feet by 2 feet.



- The formula for the volume of a prism is found by multiplying the area of its base, B, by its height, h.

$$V = Bh$$

- The base of this prism is a rectangle. The area of a rectangle is equal to its length times its width.

$$B = 5 \text{ ft} \cdot 2 \text{ ft}$$

$$B = 10 \text{ ft}^2$$

- Substitute 10 square feet for B and 6 feet for h in the formula for the volume of a prism.

$$V = Bh$$

$$V = 10 \text{ ft}^2 \cdot 6 \text{ ft}$$

$$V = 60 \text{ ft}^3$$

The volume of the rectangular prism is 60 cubic feet.

Do you see that . . .

Try It

What is the approximate volume of a cylinder with a radius of 1.5 inches and a height of 4.25 inches?

The formula for the volume of a cylinder is _____ .

In the formula, B represents the _____ of the base of a cylinder.

The base of a cylinder is a _____ .

Its area, B, is equal to _____ .

Use 3.14 as an estimate of the value of _____ .

Substitute _____ inches for r, the _____ of the cylinder's base.

$$B \approx \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$B \approx \_\_\_\_\_ \text{ in.}^2$$

Substitute _____ square inches for B in the formula for the volume of a cylinder.

Substitute _____ inches for h, the _____ of the cylinder.

$$V = Bh$$

$$V \approx \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$V \approx \_\_\_\_\_ \text{ in.}^3$$

The cylinder has a volume of approximately _____ cubic inches.

The formula for the volume of a cylinder is $V = Bh$. In the formula, B represents the area of the base of a cylinder. The base of a cylinder is a circle. Its area, B, is equal to πr^2. Use 3.14 as an estimate of the value of π. Substitute 1.5 inches for r, the radius of the cylinder's base.

$$B \approx 3.14 \cdot 1.5^2$$

$$B \approx 7.07 \text{ in.}^2$$

Substitute 7.07 square inches for B in the formula for the volume of a cylinder. Substitute 4.25 inches for h, the height of the cylinder.

$$V = Bh$$

$$V \approx 7.07 \cdot 4.25$$

$$V \approx 30.05 \text{ in.}^3$$

The cylinder has a volume of approximately 30 cubic inches.

Héctor can buy either a rectangular box of oats that measures 8 by 1.5 by 10 inches or a cylindrical container of oats that has a radius of 2.5 inches and a height of 8 inches. Which container has the greater volume?

- Find the volume of the box, which is a rectangular prism.

  The formula for the volume of a rectangular prism is $V = Bh$. Find B, the area of the base of the prism. The base of the prism is a rectangle. Its area is equal to lw. Substitute 8 for l and 1.5 for w.

  $$B = 8 \cdot 1.5$$
  $$B = 12 \text{ in.}^2$$

  Substitute 12 for B and 10 for h in the formula for the volume of a prism.

  $$V = Bh$$
  $$V = 12 \cdot 10$$
  $$V = 120 \text{ in.}^3$$

  The volume of the rectangular box is 120 cubic inches.

- Find the volume of the cylindrical container.

  The formula for the volume of a cylinder is $V = Bh$. Find B, the area of the base of the cylinder. The base of the cylinder is a circle. Its area is equal to πr^2. Substitute 2.5 for r and use 3.14 as an approximate value of π.

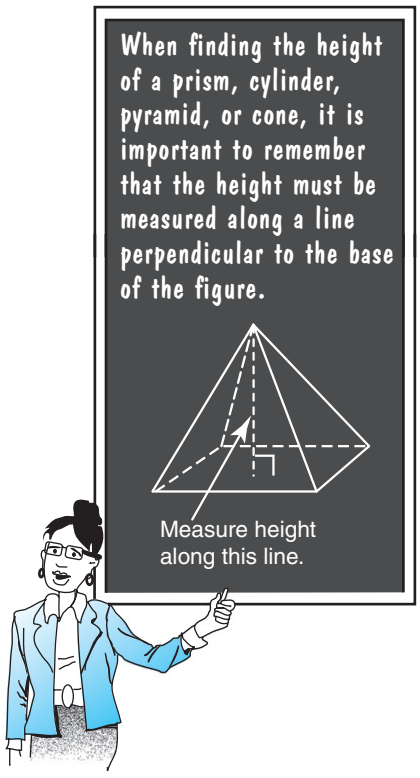
  $$B \approx 3.14 \cdot 2.5^2$$
  $$B \approx 3.14 \cdot 6.25$$
  $$B \approx 19.6 \text{ in.}^2$$

  Substitute 19.6 for B and 8 for h in the formula for the volume of a cylinder.

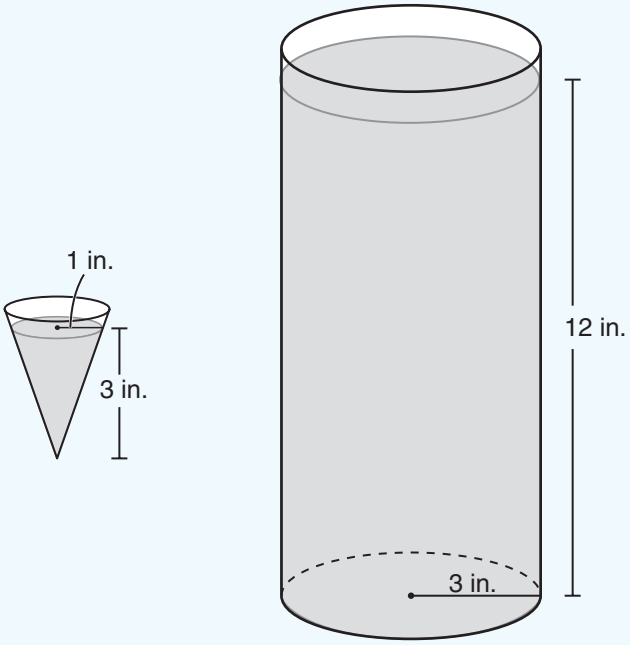
  $$V = Bh$$
  $$V \approx 19.6 \cdot 8$$
  $$V \approx 156.8 \text{ in.}^3$$

  The volume of the cylindrical container is about 157 cubic inches.

Since $157 \text{ in.}^3 > 120 \text{ in.}^3$, the cylindrical container has a greater volume than the rectangular box.



A cylindrical container with a 3-inch radius is filled with juice to a height of 12 inches. How many cone-shaped paper cups can be filled with juice from the container? Assume that a full paper cup has a height of 3 inches and a radius of 1 inch.



To find the number of paper cups that can be filled, first find the volume of one full paper cup. Then find the volume of juice in the container and divide it by the volume of one full paper cup.

- Find the volume of one full paper cup, which is a cone.

  The formula for the volume of a cone is $V = \frac{1}{3}Bh$.

  Find B, the area of the base of the cone. The base of the cone is a circle. Its area is equal to πr^2. Substitute 1 for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot 1^2$$
$$B \approx 3.14 \cdot 1$$
$$B \approx 3.14 \text{ in.}^2$$

  Substitute 3.14 for B and 3 for h in the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$
$$V \approx \frac{1}{3} \cdot 3.14 \cdot 3$$
$$V \approx 3.14 \text{ in.}^3$$

  The volume of one full paper cup is about 3.14 cubic inches.

- Find the volume of juice in the container, which is a cylinder.

  The formula for the volume of a cylinder is $V = Bh$.
  Find B, the area of the base of the cylinder. The base of the cylinder is a circle. Its area is equal to πr^2. Substitute 3 for r and use 3.14 as an approximate value of π.

  $$B \approx 3.14 \cdot 3^2$$
  $$B \approx 3.14 \cdot 9$$
  $$B \approx 28.26 \text{ in.}^2$$

  Substitute 28.26 for B and 12 for h in the formula for the volume of a cylinder.

  $$V = Bh$$
  $$V \approx 28.26 \cdot 12$$
  $$V \approx 339.12 \text{ in.}^3$$

  The volume of juice in the container is about 339.12 cubic inches.

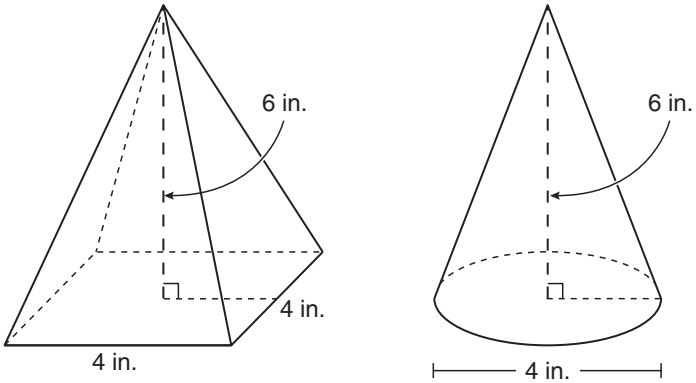
- Divide to find the number of paper cups that can be filled from the juice container.

  $$339.12 \div 3.14 = 108$$

About 108 paper cups can be filled from the juice container.

Try It

Which has the greater volume, a 4-inch-square pyramid that is 6 inches tall or a cone that is 6 inches tall and has a 4-inch diameter?



The formula for the volume of a pyramid is _____ .

Find B, the area of the _____ of a pyramid. The base of the pyramid is a _____ . Its area is equal to _____ . Substitute _____ for s.

$$B = \_\_\_\_\_$$

$$B = \_\_\_\_\_ \text{ in.}^2$$

Substitute _____ for B and _____ for h in the formula for the volume of a pyramid.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3} \cdot \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$V = \_\_\_\_\_ \text{ in.}^3$$

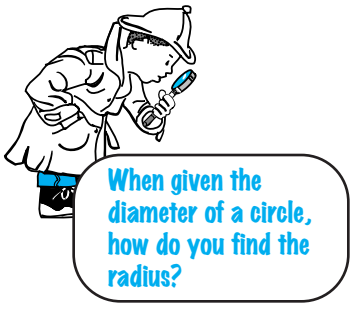
The volume of the pyramid is _____ cubic inches.

The formula for the volume of a cone is _____ .

Find B, the area of the _____ of a cone. The base of the cone is a _____ . Its area is equal to _____ . Substitute _____ for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot \_\_\_\_\_$$

$$B \approx 3.14 \cdot \_\_\_\_\_$$



$$B \approx \_\_\_\_\_ \text{ in.}^2$$

Substitute _____ for B and _____ for h in the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$

$$V \approx \frac{1}{3} \cdot \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$V \approx \_\_\_\_\_ \text{ in.}^3$$

The volume of the cone is about _____ cubic inches.

The _____ has the greater volume.

The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$. Find B, the area of the base of a pyramid. The base of the pyramid is a square. Its area is equal to s^2. Substitute 4 for s.

$$B = 4^2$$

$$B = 16 \text{ in.}^2$$

Substitute 16 for B and 6 for h in the formula for the volume of a pyramid.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3} \cdot 16 \cdot 6$$

$$V = 32 \text{ in.}^3$$

The volume of the pyramid is 32 cubic inches.

The formula for the volume of a cone is $V = \frac{1}{3}Bh$. Find B, the area of the base of a cone. The base of the cone is a circle. Its area is equal to πr^2. Substitute 2 for r and use 3.14 as an approximate value of π.

$$B \approx 3.14 \cdot 2^2$$

$$B \approx 3.14 \cdot 4$$

$$B \approx 12.56 \text{ in.}^2$$

Substitute 12.56 for B and 6 for h in the formula for the volume of a cone.

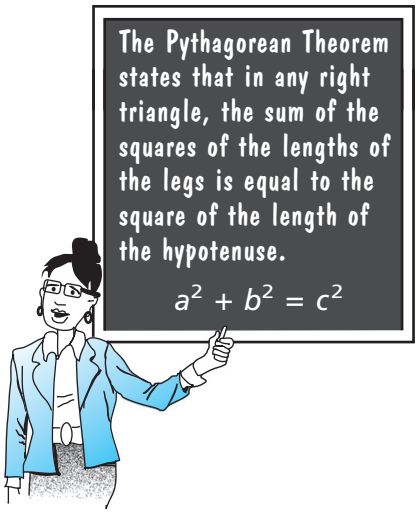
$$V = \frac{1}{3}Bh$$

$$V \approx \frac{1}{3} \cdot 12.56 \cdot 6$$

$$V \approx 25.12 \text{ in.}^3$$

The volume of the cone is about 25.12 cubic inches.

The pyramid has the greater volume.



How Can You Solve Problems Using the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. The Pythagorean Theorem applies only to right triangles.

You can use the Pythagorean Theorem to determine whether a triangle is a right triangle.

- Identify which side could be the hypotenuse, c. It must be the longest side.
- See whether $a^2 + b^2 = c^2$.
- If the lengths of the sides satisfy the Pythagorean Theorem, then the triangle is a right triangle.

Any set of three whole numbers that satisfies the Pythagorean Theorem is called a **Pythagorean triple**.

- The numbers 3, 4, and 5 form a Pythagorean triple because they satisfy the equation $a^2 + b^2 = c^2$.

$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

A triangle with side lengths of 3, 4, and 5 units is a right triangle.

Do you see that . . .

- Any multiple of a Pythagorean triple is also a Pythagorean triple.

Because the numbers 3, 4, and 5 form a Pythagorean triple, it is also true that these numbers multiplied by 3 form a Pythagorean triple: 9, 12, and 15.

$$9^2 + 12^2 = 15^2$$
$$81 + 144 = 225$$
$$225 = 225$$

A triangle with side lengths of 9, 12, and 15 units is also a right triangle.

A triangle has one leg that is 15 meters long and another leg that is 20 meters long. For the triangle to be a right triangle, what must the length of the hypotenuse be?

The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 15 for a and 20 for b in the equation.

$$a^2 + b^2 = c^2$$
$$15^2 + 20^2 = c^2$$
$$225 + 400 = c^2$$
$$625 = c^2$$
$$\sqrt{625} = \sqrt{c^2}$$
$$c = 25$$

The hypotenuse must be 25 meters long.

The hypotenuse of a right triangle is 17 centimeters long. One leg of the triangle is 15 centimeters long. Find the length of the other leg.

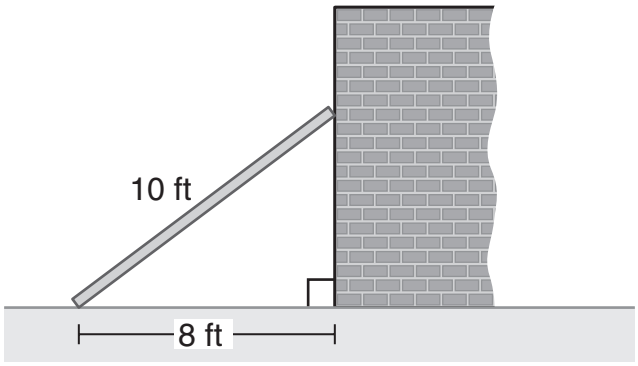
The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 17 for c and 15 for a in the equation.

$$a^2 + b^2 = c^2$$
$$15^2 + b^2 = 17^2$$
$$225 + b^2 = 289$$
$$b^2 = 64$$
$$b = 8$$

The other leg of the triangle is 8 centimeters long.

Try It

A 10-foot-long piece of lumber is leaning against a wall. The bottom of the piece of lumber is 8 feet from the base of a wall. How high up the wall does the piece of lumber reach?



The side lengths of a right triangle must satisfy the

_____.

Substitute _____ for c and _____ for a in the equation.

$$a^2 + b^2 = c^2$$
$$\_\_\_\_\_ + b^2 = \_\_\_\_\_$$
$$\_\_\_\_\_ + b^2 = \_\_\_\_\_$$
$$b^2 = \_\_\_\_\_$$
$$b = \_\_\_\_\_$$

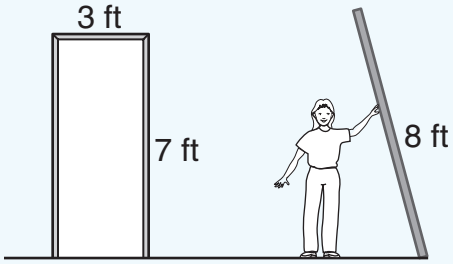
The piece of lumber reaches _____ feet up the wall.

The side lengths of a right triangle must satisfy the Pythagorean Theorem. Substitute 10 for c and 8 for a in the equation.

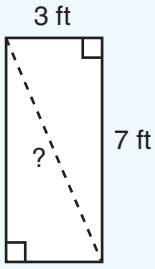
$$8^2 + b^2 = 10^2$$
$$64 + b^2 = 100$$
$$b^2 = 36$$
$$b = 6$$

The piece of lumber reaches 6 feet up the wall.

Shirley is trying to roll a circular folding table with an 8-foot diameter through a doorway that measures 7 feet by 3 feet. She plans to lean the table over and roll it through the diagonal of the doorway. Will the table fit through the doorway?



You can model this problem by drawing a picture of the diagonal of a rectangle.



The rectangle represents the doorway, and the dashed line represents the diagonal of the doorway.

The triangles formed by the width, height, and diagonal of the doorway are right triangles. The diagonal is the hypotenuse of the triangles.

Find the length of the hypotenuse and compare it to 8 feet. If the hypotenuse is greater than 8 feet, the table will fit through the doorway. If the hypotenuse is less than 8 feet, the table will not fit.

Use the Pythagorean Theorem to find the length of the diagonal. Substitute 3 and 7 for the legs, a and b.

$$a^2 + b^2 = c^2$$
$$3^2 + 7^2 = c^2$$
$$9 + 49 = c^2$$
$$58 = c^2$$
$$\sqrt{58} = c$$

Because $7^2 = 49$ and $8^2 = 64$, $\sqrt{58}$ is between 7 and 8. The diagonal of the doorway measures less than 8 feet.

Since the diameter of the table, 8 feet, is greater than the diagonal of the doorway, the table will not fit through the doorway.

Try It

Television sizes are described by the diagonal measurement across the screen. The rectangular screen of John's television set measures 12 inches by 16 inches. What is the size of his television to the nearest inch?

©Photodisc

Model the television screen by drawing a _____ .

The size of the television screen is the length of the _____ .

The diagonal divides the rectangle into two right _____ .

Use the _____ to find the length of the diagonal.

The diagonal is opposite the right angle, so it is the _____ of the right triangle.

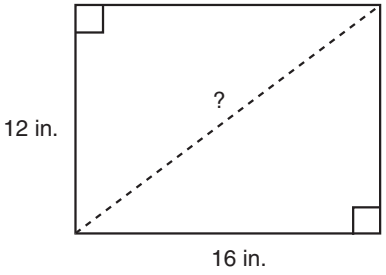
Represent the diagonal with c. Substitute _____ and _____ for a and b, the two legs of the triangle.

$$a^2 + b^2 = c^2$$
$$\_\_\_\_\_ + \_\_\_\_\_ = c^2$$
$$\_\_\_\_\_ + \_\_\_\_\_ = c^2$$
$$\_\_\_\_\_ = c^2$$
$$\_\_\_\_\_ = c$$

John has a _____-inch television set.

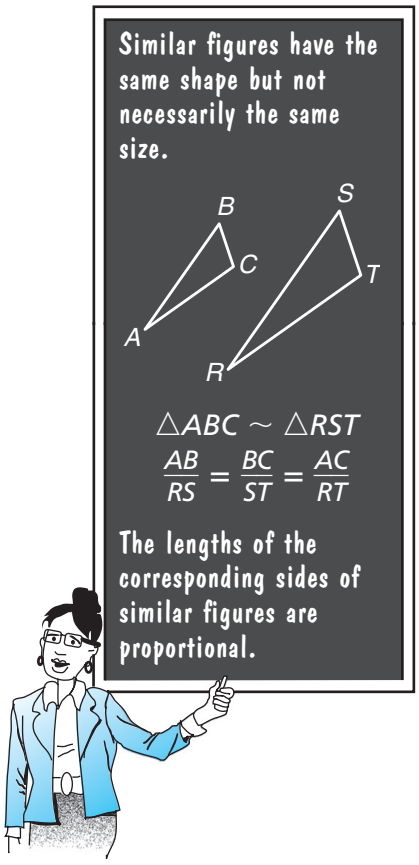
Model the television screen by drawing a rectangle.

The size of the television screen is the length of the diagonal. The diagonal divides the rectangle into two right triangles. Use the Pythagorean Theorem to find the length of the diagonal. The diagonal is opposite the right angle, so it is the hypotenuse of the right triangle. Represent the diagonal with c. Substitute 12 and 16 for a and b, the two legs of the triangle.



$$a^2 + b^2 = c^2$$
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$20 = c$$

John has a 20-inch television set.



How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines.

- Identify the corresponding sides of similar figures.
- Write and solve a proportion by using cross products.
- Answer the question asked.

A sign company sells advertising banners that are 20 feet long by 4 feet wide. A customer asked the company to increase a banner's size proportionally so it would be 25 feet long. How wide will the banner be when it is enlarged?

The original banner and the new banner are similar rectangles. You can use proportional relationships to find missing side lengths in similar figures.

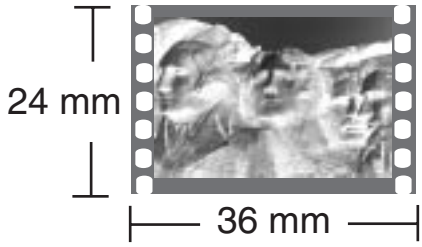
Write a proportion. Let x represent the width of the enlarged banner.

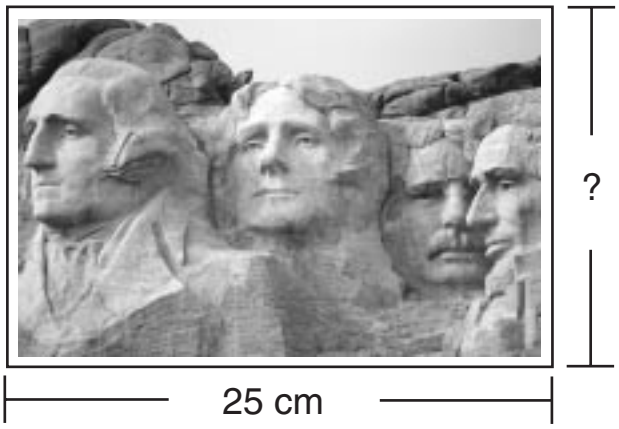
$$\frac{\text{original}}{\text{new}} \quad \frac{20}{25} = \frac{4}{x}$$
$$20x = 25 \cdot 4$$
$$20x = 100$$
$$x = 5$$

The new banner will be 5 feet wide.

Try It

A 35-millimeter film negative is approximately 24 millimeters tall by 36 millimeters wide. Chris wants to use the negative to make a print that is proportional in size to the negative.





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If the print will be 25 centimeters wide, how tall must it be, to the nearest centimeter?

The negative and the print are _____ figures because the lengths of their sides are proportional.

Identify corresponding sides.

The width of the negative corresponds to the _____ of the print.

The height of the negative corresponds to the _____ of the print.

Let h represent the _____ of the print.

Write a proportion.

$$\frac{\text{negative}}{\text{print}} \qquad \frac{36}{\square} = \frac{\square}{h}$$

Use cross products to solve the proportion.

$$\_\_\_\_\_\ h = \_\_\_\_\_ \cdot \_\_\_\_\_$$

$$\_\_\_\_\_\ h = \_\_\_\_\_$$

$$h \approx \_\_\_\_\_$$

The print will be about _____ centimeters tall.

The negative and the print are similar figures because the lengths of their sides are proportional. The width of the negative corresponds to the width of the print. The height of the negative corresponds to the height of the print. Let h represent the height of the print.

$$\frac{36}{25} = \frac{24}{h}$$

$$36h = 25 \cdot 24$$

$$36h = 600$$

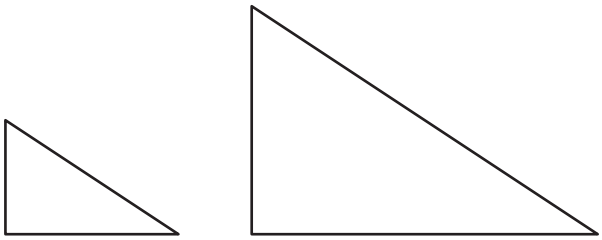
$$h \approx 16.67$$

The print will be about 17 centimeters tall.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

Do you see that . . .

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. For example, the triangle on the left has been dilated by a scale factor of 2 to form the triangle on the right. The perimeter of the dilated figure will change by the same scale factor.



If a triangle with side lengths of 3, 7, and 10 inches is dilated by a scale factor of 2, its new dimensions will be 6, 14, and 20 inches. Each dimension has been multiplied by 2. What effect will this dilation have on the triangle's perimeter?

- Find the perimeter of the original triangle.

$$P = 3 + 7 + 10 = 20 \text{ inches}$$

- Find the perimeter of the dilated triangle.

$$P = 6 + 14 + 20 = 40 \text{ inches}$$

The perimeter of the dilated triangle has also increased by a factor of 2, because the perimeter of a dilated figure changes by the same scale factor as the side lengths of the figure.

A rectangle with a perimeter of 10 inches is dilated by a scale factor of $\frac{1}{3}$. What is the perimeter of the new rectangle?

The perimeter of the original rectangle is 10 inches. The perimeter should change by the same scale factor, $\frac{1}{3}$. Multiply the original perimeter by the scale factor.

$$P = 10\left(\frac{1}{3}\right) = \frac{10}{3} = 3\frac{1}{3} \text{ in.}$$

The perimeter of the new rectangle is $3\frac{1}{3}$ inches.

Try It

A triangle with a perimeter of 128.5 feet is enlarged by a scale factor of 10.5. What is the perimeter of the new triangle?

The perimeter of the original triangle is _____ feet.

The perimeter should change by a scale factor of _____ .

Multiply the original perimeter by the same scale factor.

$$P = \_\_\_\_\_ \cdot \_\_\_\_\_ = \_\_\_\_\_$$

The perimeter of the new triangle is _____ feet.

The perimeter of the original triangle is 128.5 feet. The perimeter should change by a scale factor of 10.5.

$$P = 128.5 \cdot 10.5 = 1{,}349.25$$

The perimeter of the new triangle is 1,349.25 feet.

MATHEMATICS

Do you see that . . .

How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.

Marsha put a 5-by-9-inch picture on a photocopier and dilated it by a scale factor of $\frac{1}{3}$. How is the area of the picture affected by the reduction?

The area of the picture should change by the square of the scale factor, which is $\left(\frac{1}{3}\right)^2$, or $\frac{1}{9}$.

Check to see if this is true.

- Find the area of the original picture.

$$A = lw = 5 \cdot 9 = 45 \text{ in.}^2$$

- Find the dimensions of the dilated picture.

$$\text{new length} = \text{scale factor} \cdot \text{original length} = \left(\frac{1}{3}\right)5 = \frac{5}{3} \text{ in.}$$

$$\text{new width} = \text{scale factor} \cdot \text{original width} = \left(\frac{1}{3}\right)9 = 3 \text{ in.}$$

- Find the area of the dilated picture.

$$A = \left(\frac{5}{3}\right)3 = 5 \text{ in.}^2$$

Since $\frac{1}{9}$ of 45 is 5, the area of the dilated picture is $\frac{1}{9}$ the area of the original picture.

A hexagon with an area of 125 cm^2 is dilated by a scale factor of 5. What is the area of the new hexagon?

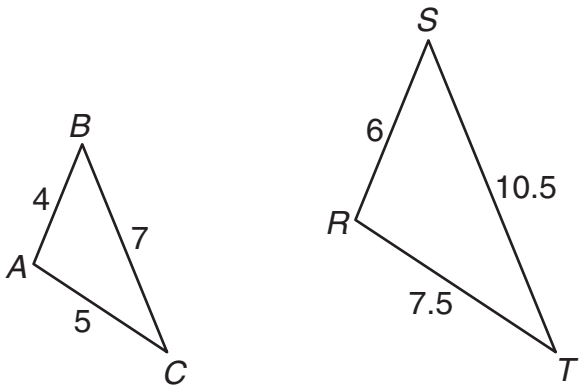
The area should change by the square of the scale factor, which is $5^2 = 25$. Multiply the area of the original hexagon by the square of the scale factor.

$$A = 125 \cdot 25 = 3{,}125 \text{ cm}^2$$

The area of the new hexagon is 3,125 square centimeters.

Try It

$\triangle ABC$ is similar to $\triangle RST$.



What is the ratio of the area of the larger triangle to the area of the smaller triangle?

Find the scale factor.

$\overline{AB}$ corresponds to _____.

$RS =$ _____ units

$AB =$ _____ units

$$\frac{RS}{AB} = \frac{6}{\square} = \frac{\square}{\square}$$

The scale factor is $\frac{\square}{\square}$.

The area of the new triangle will increase by the _____ of the scale factor.

The square of the scale factor is $\left(\frac{\square}{\square}\right)^2 = \frac{\square^2}{\square^2} = \frac{\square}{\square}$.

The ratio of the area of the larger triangle to the area of the smaller triangle is $\frac{\square}{\square}$.

$\overline{AB}$ corresponds to $\overline{RS}$.

$RS = 6$ units

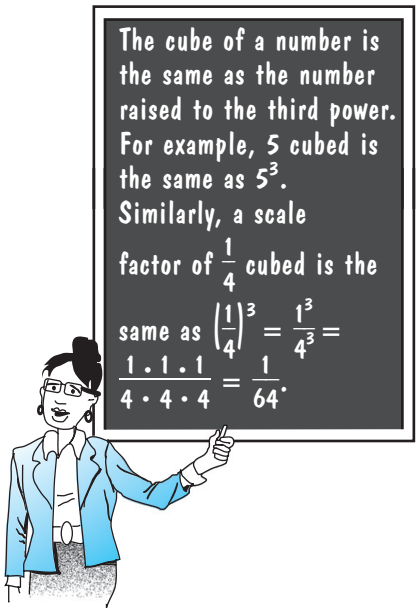
$AB = 4$ units

$$\frac{RS}{AB} = \frac{6}{4} = \frac{3}{2}$$

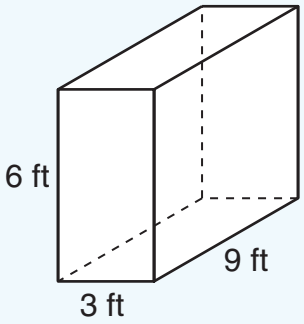
The scale factor is $\frac{3}{2}$. The area of the new triangle will increase by the square of the scale factor. The square of the scale factor is $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$. The ratio of the area of the larger triangle to the area of the smaller triangle is $\frac{9}{4}$.

How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.



The dimensions of a rectangular prism are 9 by 3 by 6 feet. If the dimensions of the prism are dilated by a scale factor of $\frac{1}{3}$, how is the volume of the prism affected?



The volume of the prism should change by the cube of the scale factor, which is $\left(\frac{1}{3}\right)^3 = \frac{1^3}{3^3} = \frac{1}{27}$.

Check to see if this is true.

- Find the volume of the original prism.

$$V = Bh$$
$$V = (9 \cdot 3) \cdot 6$$
$$V = 27 \cdot 6$$
$$V = 162 \text{ ft}^3$$

- Find the dimensions of the dilated prism.

new length = scale factor • original length = $\left(\frac{1}{3}\right)9 = 3$ ft

new width = scale factor • original width = $\left(\frac{1}{3}\right)3 = 1$ ft

new height = scale factor • original height = $\left(\frac{1}{3}\right)6 = 2$ ft

- Find the volume of the dilated prism.

$$V = Bh$$
$$V = (3 \cdot 1) \cdot 2$$
$$V = 3 \cdot 2$$

$$V = 6 \text{ ft}^3$$

Since $\frac{1}{27}$ of 162 is 6, the volume of the dilated prism is $\frac{1}{27}$ the volume of the original prism.

A cone with a volume of 75 cubic meters is dilated by a scale factor of 1.5. What is the volume of the new cone?

The volume should change by the cube of the scale factor.

$$(1.5)^3 = (1.5)(1.5)(1.5) = 3.375$$

Multiply the original volume by the cube of the scale factor.

$$V = 75 \cdot 3.375 = 253.125 \text{ m}^3$$

The volume of the new cone is 253.125 cubic meters.

Try It

A hexagonal prism has a volume of 80 cubic inches. The dimensions of the prism are enlarged by a scale factor of $\frac{3}{2}$ to produce a new prism. What is the volume of the enlarged hexagonal prism?

The dimensions of the prism were enlarged by a scale factor of $\frac{\square}{\square}$.

The volume of the new prism will increase by the cube of $\frac{\square}{\square}$.

$$\left(\frac{3}{2}\right)^3 = \frac{\square^3}{\square^3} = \frac{\square}{\square}$$

Multiply the original volume by the cube of the scale factor.

$$\_\_\_\_\_ \cdot \frac{\square}{\square} = \_\_\_\_\_ \text{ in.}^3$$

The volume of the enlarged prism is _____ cubic inches.

The dimensions of the prism were enlarged by a scale factor of $\frac{3}{2}$. The volume of the new prism will increase by the cube of $\frac{3}{2}$.

$$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Multiply the original volume by the cube of the scale factor.

$$80 \cdot \frac{27}{8} = 270 \text{ in.}^3$$

The volume of the enlarged prism is 270 cubic inches.

Now practice what you've learned.

Question 31

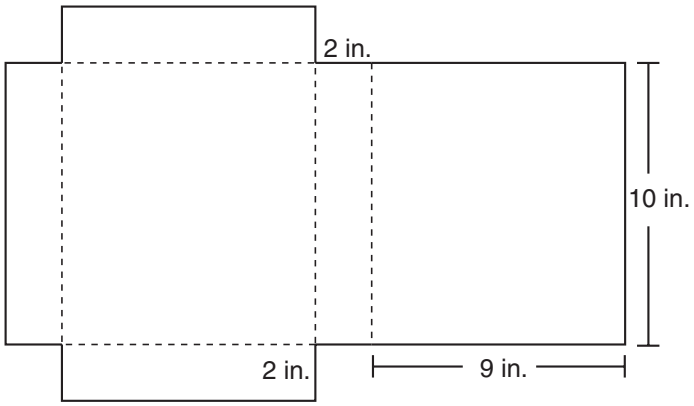
What is the surface area of the rectangular prism that can be formed from the net shown at the right?

A 128 square inches

B 360 square inches

C 256 square inches

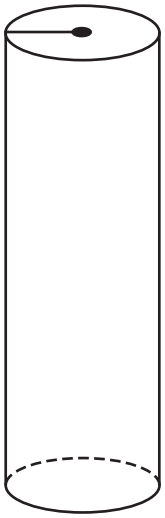
D 42 square inches



Answer Key: page 243

Question 32

A small candy container is shaped like a cylinder. The manufacturer wraps a label around the curved surface of the can. Use the ruler on the Mathematics Chart to measure the height and radius of the cylinder in centimeters. Which of the following is closest to the area of the label in square centimeters?



A 19 cm^2

B 6 cm^2

C 38 cm^2

D 44 cm^2

Answer Key: page 243

Question 33

Gerard is building a rectangular patio measuring 12.5 feet long by 9 feet wide. If the floor will be a cement slab 4 inches thick, how many cubic feet of cement will it take to build the slab?

A 37.5 ft^3

B 450 ft^3

C 375 ft^3

D 4,500 ft^3

Answer Key: page 243

Question 34

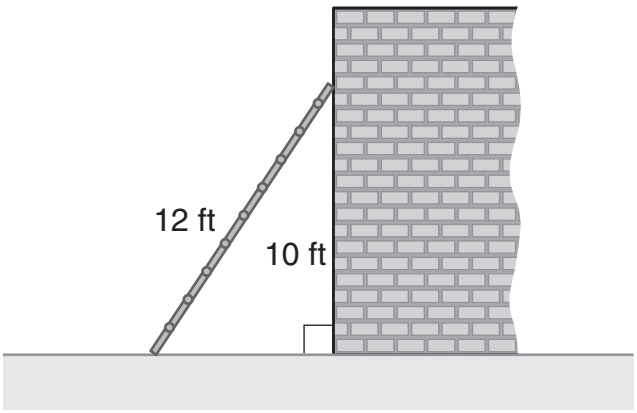
A 12-foot ladder is leaning against the side of a building. The top of the ladder reaches 10 feet up the side of the building. Approximately how far is the bottom of the ladder from the base of the building?

A 2 ft

B 15.6 ft

C 6.6 ft

D 1.4 ft



Answer Key: page 244

Question 35

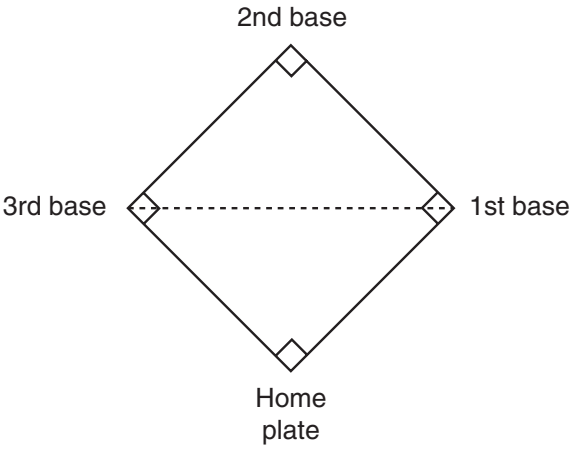
A baseball diamond is a square with a side length of 90 feet. To the nearest foot, what is the distance between first base and third base?

A 810 ft

B 90 ft

C 180 ft

D 127 ft



Answer Key: page 244

Question 36

An architect built a scale model of a house to show his client. The length of the model is 12.5 inches, and its width is 7.75 inches. If the actual house will be 64 feet long, how wide will it be to the nearest foot?

A 84 ft

B 40 ft

C 60 ft

D 103 ft

Answer Key: page 244

Question 37

A rectangle has a perimeter of 39 centimeters. The rectangle is dilated by a scale factor of $\frac{1}{2}$ to produce a new rectangle. What is the perimeter of the new rectangle?

A 19.5 cm

B 9.75 cm

C 78 cm

D 156 cm

Answer Key: page 244

Question 38

A triangle has an area of 6 square feet. The triangle is dilated by a scale factor of 5 to produce a new triangle. What is the area of the new triangle?

A $\frac{6}{5}$ ft^2

B 30 ft^2

C 150 ft^2

D 180 ft^2

Answer Key: page 244

Question 39

A cylinder has a volume of 3,600 cubic centimeters. If the cylinder is dilated by a factor of $\frac{1}{4}$, what is the volume of the new cylinder?

A 14,400 cm^3

B 900 cm^3

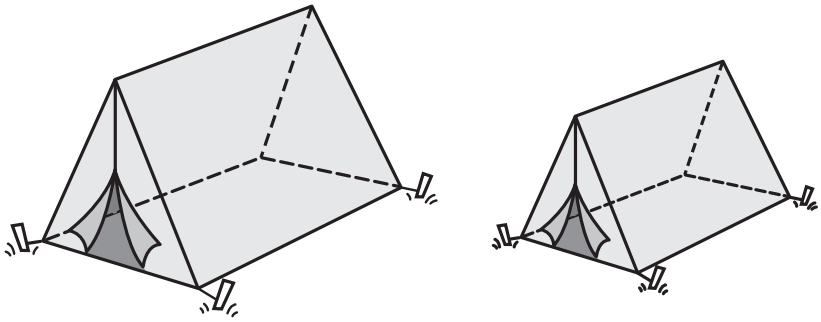
C 225 cm^3

D 56.25 cm^3

Answer Key: page 244

Question 40

A tent has dimensions that are $\frac{3}{4}$ of the dimensions of the tent next to it. If the volume of the smaller tent is 1,000 cubic feet, what is the volume of the larger tent, to the nearest cubic foot?



A 1,777 ft^3

B 2,370 ft^3

C 750 ft^3

D 1,333 ft^3

Answer Key: page 245

Objective 5

The student will demonstrate an understanding of probability and statistics.

For this objective you should be able to

- apply theoretical and experimental probability concepts to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

What Is Probability?

Probability is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of possible outcomes. In a probability experiment, favorable outcomes are the outcomes that you are interested in.

The probability, P, of an event occurring is from 0 to 1.

- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

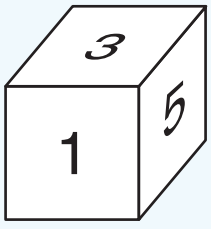
For example, the probability of drawing a white marble from a bag containing 3 red marbles and 1 white marble is the ratio of the number of favorable outcomes, 1 white marble, to the number of possible outcomes, 4 marbles. The probability of drawing a white marble is $\frac{1}{4}$.

This is often written as $P(\text{white}) = \frac{1}{4}$.

Do you see that . . .

Drawing a white marble from a bag of colored marbles is a **simple event**. A simple event in a probability experiment is determined by the outcome of one trial in the experiment. An event that is made up of a sequence of simple events is called a **compound event**. For example, drawing a white marble first and drawing a red marble second is a compound event.

What is the probability of rolling a 1 or a 2 on a fair number cube numbered 1 through 6?



The sample space for this experiment is {1, 2, 3, 4, 5, 6}. There are 6 possible outcomes.

There are 2 favorable outcomes for this experiment: rolling a 1 or a 2.

The probability of rolling a 1 or a 2 is the ratio of the number of favorable outcomes to the number of possible outcomes:

2 out of 6, or $\frac{1}{3}$.

The probability of rolling a 1 or a 2 on a fair number cube is $\frac{1}{3}$. This can also be written as $P(1 \text{ or } 2) = \frac{1}{3}$.

How Do You Find the Probability of Compound Events?

One way to find the probability of compound events is to multiply the probabilities of the simple events that make up the compound event.

If $P(A)$ represents the probability of event A and $P(B)$ represents the probability of event B, then the probability of the compound event (A and B) can be represented algebraically.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

For example, the probability of tossing a head on a fair coin toss is $P(H) = \frac{1}{2}$. The probability of tossing heads twice on two fair coin tosses is

$$P(H \text{ and } H) = P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

A probability experiment consists of rolling a fair number cube numbered 1 through 6 and then spinning a spinner with two equally likely outcomes, red or blue. Find the probability of rolling a 4 on the number cube and spinning blue on the spinner.

- The probability of rolling a 4 on the number cube is
$$P(4) = \frac{1}{6}$$
- The probability of spinning blue on the spinner is
$$P(\text{blue}) = \frac{1}{2}$$
- The probability of rolling a 4 on the number cube and spinning blue on the spinner is
$$P(4 \text{ and blue}) = P(4) \cdot P(\text{blue})$$
$$= \frac{1}{6} \cdot \frac{1}{2}$$
$$= \frac{1}{12}$$

Another way to find the probability of this compound event is to look at the sample space for the experiment and identify the favorable outcomes. This method works only if the outcomes are all equally likely. The following table lists all the possible outcomes. The one favorable outcome is shaded.

Sample Space

Number Cube	Spinner
1	red
1	blue
2	red
2	blue
3	red
3	blue
4	red
4	**blue**
5	red
5	blue
6	red
6	blue

There are 12 possible outcomes; only 1 of them is favorable. The probability of rolling a 4 on the number cube and spinning blue on the spinner is $\frac{1}{12}$.

This matches the result obtained from using the rule for the probability of a compound event.

$$P(4 \text{ and blue}) = \frac{1}{12}$$

Do you see that . . .

What Is the Difference Between Dependent and Independent Events?

In a compound event, if the outcome of the first event affects the possible outcomes of the second event, the events are called **dependent events**. If the outcome of the first event does not affect the possible outcomes of the second event, the events are called **independent events**.

When finding the probability of a compound event, first determine whether the simple events included are dependent or independent.

A bag contains 6 blue marbles, 3 red marbles, and 1 green marble. Lilah draws one marble from the bag, records its color, and then puts it back in the bag. Then she draws another marble and records its color. What is the probability of drawing 2 blue marbles?

This is an example of a compound event made up of two independent events. The same number and color of marbles are in the bag for each draw. The outcome of the first marble drawn does not affect the possible outcomes of the second marble drawn. Therefore, the events are independent.

- Find the probability of getting a blue marble on the first draw.

  The probability of drawing 1 of the 6 blue marbles out of the 10 marbles in the bag is $\frac{6}{10}$.

$$P(\text{blue}) = \frac{6}{10}$$

- Find the probability of getting a blue marble on the second draw.

  After the first draw the marble was returned to the bag. There are still 6 blue marbles out of the 10 marbles in the bag.

  The probability of getting a blue marble on the second draw is $\frac{6}{10}$.

$$P(\text{blue}) = \frac{6}{10}$$

- The probability of drawing 2 blue marbles is

$$\begin{aligned} P(\text{blue and blue}) &= P(\text{blue}) \cdot P(\text{blue}) \\ &= \frac{6}{10} \cdot \frac{6}{10} \\ &= \frac{36}{100} \\ &= \frac{9}{25} \end{aligned}$$

The probability of drawing 2 blue marbles when the first marble is replaced in the bag is $\frac{9}{25}$.

A bag contains 6 blue marbles, 3 red marbles, and 1 green marble. One marble is drawn from the bag, and its color is recorded. Another marble is drawn, and its color is also recorded. What is the probability of drawing 2 blue marbles if the first marble is not returned to the bag before the second marble is drawn?

This is an example of a compound event made up of dependent events. On the first draw there are 10 marbles in the bag, but on the second draw there are only 9 marbles in the bag. The outcome of the first marble drawn affects the possible outcomes of the second marble drawn. Therefore, the events are dependent.

- Find the probability of getting a blue marble on the first draw.

  The probability of drawing 1 of the 6 blue marbles out of the 10 marbles in the bag is $\frac{6}{10}$.

$$P(\text{blue}_{\text{first}}) = \frac{6}{10}$$

- Find the probability of getting a blue marble on the second draw. Assume that the first marble drawn was blue and that it was not returned to the bag. This means that there are now only 9 marbles in the bag and that only 5 of them are blue.

  The probability of drawing 1 of the 5 blue marbles out of the 9 marbles in the bag is $\frac{5}{9}$.

$$P(\text{blue}_{\text{second}}) = \frac{5}{9}$$

- Find the probability of drawing 2 blue marbles.

$$P(\text{blue}_{\text{first}} \text{ and blue}_{\text{second}}) = P(\text{blue}_{\text{first}}) \cdot P(\text{blue}_{\text{second}})$$

$$= \frac{6}{10} \cdot \frac{5}{9}$$

$$= \frac{30}{90}$$

$$= \frac{1}{3}$$

The probability of drawing 2 blue marbles when the first one is not replaced in the bag is $\frac{1}{3}$.

Do you see that . . .

The probabilities of drawing 2 blue marbles are different in these two examples. When the compound events are independent, the probability is $\frac{9}{25}$. When they are dependent, the probability is $\frac{1}{3}$.

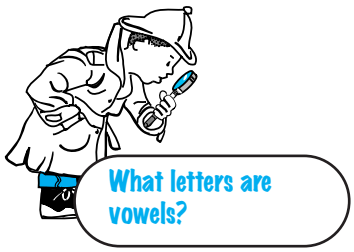
Try It

A bag contains 10 tiles numbered 1 through 10. Another bag contains 5 tiles with the letters *A*, *E*, *U*, *X*, and *Z* written on them. One tile is drawn from each bag. What is the probability of drawing both an even number and a vowel?

The outcome from the first event does not affect the possible outcomes of the second event. These events are _____ events. The probability of drawing an even number from the first bag is the ratio of the number of _____ outcomes to the number of _____ outcomes. There are _____ favorable outcomes. There are a total of _____ possible outcomes.

$$P(\text{even number}) = \frac{\square}{\square}$$

MATHEMATICS



The probability of drawing a vowel from the second bag is the ratio of the number of _____ outcomes to the number of _____ outcomes. There are _____ favorable outcomes. There are a total of _____ possible outcomes.

$$P(\text{vowel}) = \frac{\square}{\square}$$

The probability of drawing both an even number and a vowel is

$$P(\text{even number and vowel}) = \frac{\square}{\square} \cdot \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$$

These events are independent events. The probability of drawing an even number from the first bag is the ratio of the number of favorable outcomes to the number of possible outcomes. There are 5 favorable outcomes. There are a total of 10 possible outcomes.

$$P(\text{even number}) = \frac{5}{10}$$

The probability of drawing a vowel from the second bag is the ratio of the number of favorable outcomes to the number of possible outcomes. There are 3 favorable outcomes. There are a total of 5 possible outcomes.

$$P(\text{vowel}) = \frac{3}{5}$$

$$P(\text{even number and vowel}) = \frac{5}{10} \cdot \frac{3}{5} = \frac{15}{50} = \frac{3}{10}$$

What Is the Difference Between Theoretical and Experimental Probability?

The **theoretical probability** of an event occurring is the ratio comparing the number of ways the favorable outcome should occur to the number of possible outcomes. If you toss a coin, theoretically it should land on heads $\frac{1}{2}$ of the time.

$$P(H) = \frac{1}{2} = 0.5$$

The **experimental probability** of an event occurring is the ratio of the actual number of times the favorable outcome occurs in a series of repeated trials of an experiment to the total number of trials in the experiment. If you toss a coin 100 times, it is possible that the coin will land on heads 48 times and tails 52 times. The experimental probability of the coin landing on heads in this situation is $\frac{48}{100}$.

$$P(H) = \frac{48}{100} = 0.48$$

The two types of probabilities, theoretical and experimental, are not always equal. In this case, the theoretical probability is 0.5, but the experimental probability is 0.48.

For a given situation, the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.

Do you see that . . .

A cube has the letters *A* through *F* written on its faces, with one letter on each face. Find the experimental and theoretical probabilities of rolling the letter *B*.

- The theoretical probability of rolling the letter *B* is $\frac{1}{6}$. There is 1 face with a *B* on it out of the 6 faces on the cube.
- The experimental probability is determined by conducting repeated trials of rolling the cube and recording how many times the letter *B* is actually rolled.

  Suppose you roll the cube 30 times and get the results listed below. How many times did you actually roll the letter *B*?

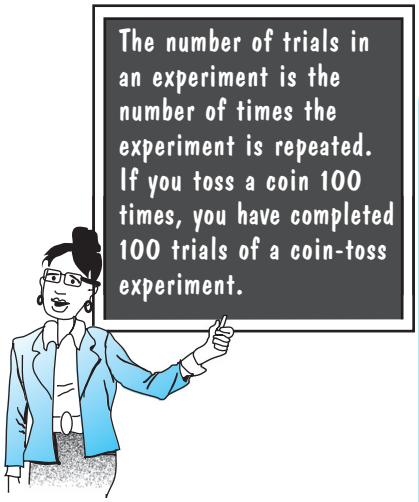
Rolls of a Fair Cube

Outcome	Frequency
A	5
B	**4**
C	5
D	6
E	4
F	6
Total	30

In this experiment the letter *B* was rolled 4 times out of 30 rolls of the cube.

The experimental probability of rolling the letter *B* is the number of times the letter *B* was actually rolled compared to the total number of trials: $\frac{4}{30}$, or $\frac{2}{15}$.

For this experiment the two probabilities, theoretical and experimental, are not equal. The theoretical probability was $\frac{1}{6}$, but the experimental probability was $\frac{2}{15}$.



Try It

A bag contains 10 marbles: 5 red, 3 blue, and 2 green. Wendy draws a marble from the bag, records the color, and then puts the marble back in the bag. She repeats this for a total of 100 trials. The table below summarizes the results of her experiment.

Draws of a Marble

Outcome	Frequency
Red	48
Blue	27
Green	25
Total	100

How does the experimental probability of drawing a blue marble compare to the theoretical probability of drawing a blue marble?

The experimental probability of drawing a blue marble is the number of times Wendy actually drew a _____ marble compared to the total number of trials.

Experimental probability: $P(\text{blue}) = \frac{\square}{\square} = 0.27$.

The theoretical probability of drawing a blue marble from the bag is the number of _____ outcomes compared to the number of _____ outcomes. There are _____ blue marbles out of a total of _____ marbles in the bag.

Theoretical probability: $P(\text{blue}) = \frac{\square}{\square} = 0.30$.

Since $0.27 < 0.30$, the experimental probability of drawing a blue marble is slightly _____ than the theoretical probability of drawing a blue marble.

The experimental probability of drawing a blue marble is the number of times Wendy actually drew a blue marble compared to the total number of trials. Experimental probability: $P(\text{blue}) = \frac{27}{100} = 0.27$. The theoretical probability of drawing a blue marble from the bag is the number of favorable outcomes compared to the number of possible outcomes. There are 3 blue marbles out of a total of 10 marbles in the bag. Theoretical probability: $P(\text{blue}) = \frac{3}{10} = 0.30$. Since $0.27 < 0.30$, the experimental probability of drawing a blue marble is slightly less than the theoretical probability of drawing a blue marble.

How Can You Use Probability to Make Predictions and Decisions?

You can use theoretical and experimental probabilities to make predictions. If you know the probability of an event occurring and you know the total number of trials, then you can use a proportion to predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the total number of trials.
- Write a proportion by setting the two ratios equal to each other.
- Solve the proportion by using cross products.

The speeds of 10 pitches thrown by a major-league baseball pitcher were measured using a radar device. The speeds, measured in miles per hour, were 88, 89, 92, 92, 93, 94, 94, 95, 96, and 99 mph. If the pitcher throws 50 more pitches, what is the best prediction of the number of these pitches that will be thrown at a speed greater than 90 mph?

This is an example of using an experimental probability to make a prediction.

- Find the experimental probability of throwing a pitch faster than 90 mph. The pitcher threw 8 pitches at a speed greater than 90 mph. He threw a total of 10 pitches.

  The experimental probability of the pitcher throwing a pitch at a speed greater than 90 mph is $\frac{8}{10}$, or $\frac{4}{5}$.
- The pitcher is going to throw 50 more pitches. The predicted number of pitches that he will throw at a speed greater than 90 mph can be found by solving the proportion $\frac{4}{5} = \frac{x}{50}$.

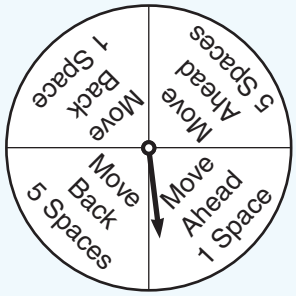
$$\frac{4}{5} = \frac{x}{50}$$

$$200 = 5x$$

$$x = 40$$

The pitcher can be expected to throw 40 of the next 50 pitches at a speed greater than 90 mph.

A game uses a spinner to determine the players' next moves.



If the spinner is spun 60 times, what is the best prediction of the number of times it will land on the section marked "Move back 5 spaces"?

This is an example of using a theoretical probability to make a prediction.

- Find the theoretical probability of spinning "Move back 5 spaces."

  The spinner is divided into 4 equal sections.

  The probability of landing on "Move back 5 spaces" is $\frac{1}{4}$.

- The spinner will be spun 60 times.

  Write a proportion.

$$\frac{1}{4} = \frac{x}{60}$$

- Solve the proportion for x, the predicted number of times the spinner will land on "Move back 5 spaces."

$$\frac{1}{4} = \frac{x}{60}$$

$$4x = 60$$

$$x = 15$$

The spinner can be expected to land on "Move back 5 spaces" 15 out of the next 60 times it is spun.

Try It

A newspaper surveyed 100 residents in a city and found that 63 of them planned to vote on election day. If there are 14,500 residents in the city, what is the best estimate of the number of residents who will vote on election day?

Find the _____ probability of a resident voting on election day.

Of the residents surveyed, _____ plan to vote on election day.

The newspaper surveyed _____ residents.

The ratio of residents who plan to vote to the total number of residents surveyed is $\frac{\square}{\square}$.

The experimental probability of a resident voting is $\frac{\square}{\square}$.

Use this probability to predict the number of residents who will vote.

There are _____ residents in the city.

Write the proportion. $\frac{\square}{100} = \frac{x}{\square}$

_____ = _____ x

_____ = x

An estimate of the number of residents in the city who should be expected to vote on election day is _____ .

Find the experimental probability of a resident voting on election day. Of the residents surveyed, 63 plan to vote on election day. The newspaper surveyed 100 residents. The ratio of residents who plan to vote to the total number of residents surveyed is $\frac{63}{100}$. The experimental probability of a resident voting is $\frac{63}{100}$.

There are 14,500 residents in the city.

$$\frac{63}{100} = \frac{x}{14{,}500}$$

$$913{,}500 = 100x$$

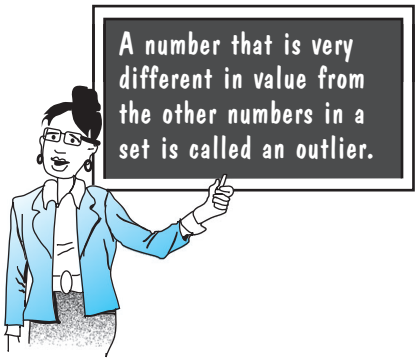
$$9{,}135 = x$$

An estimate of the number of residents in the city who should be expected to vote on election day is 9,135.

How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, mean, and range are all called **measures of central tendency**.

Mode	The **mode** of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and occur more often than all the other numbers in the set, those numbers are all modes for the data set. If each number in the set occurs the same number of times, the set of data has no mode.	Use the mode to show which value in a set of data occurs most often. For the set {1, 1, 2, 3, 5, 6, 10}, the mode is 1 because it occurs most frequently.
Median	The **median** of a set of data describes what value is in the middle if the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half of the values are greater than the median, and half of the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier.	Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set {1, 1, 2, 3, 5, 6, 10}, the median is 3 because it is in the middle when the numbers are listed in order.
Mean	The **mean** of a set of data describes their average. To find the mean, add all of the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier. The mean is a good measure of central tendency to use when a set of data does not have any outliers.	Use the mean to show the numerical average of a set of data. For the set {1, 1, 2, 3, 5, 6, 10}, the mean is the sum, 28, divided by the number of items, 7. The mean is $28 \div 7 = 4$.
Range	The **range** of a set of data describes how big a spread there is from the largest value in the set to the smallest value.	Use the range to show how much the numbers vary. For the set {1, 1, 2, 3, 5, 6, 10}, the range is $10 - 1 = 9$.



To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*

Sometimes you need to describe how a change in data affects one or more measures of the data set.

Mr. Jonas owns an ice-cream store. The table below shows his sales for several months, to the nearest five dollars.

Ice-Cream Sales

Month	Sales
April	$610
May	$560
June	$660
July	$625

In August the store was closed for 2 weeks, and the sales for the month were only $275. Which measure of central tendency for these 5 months would change the most because of the store's low sales in August?

- The mean of a set of data is the average of the set's values. The value $275 is very different in size from the other values, so it should have a significant impact on the mean.

  The mean for April through July:

  $$(610 + 560 + 660 + 625) \div 4 = 2{,}455 \div 4 = \$613.75$$

  The mean for April through August:

  $$(610 + 560 + 660 + 625 + 275) \div 5 = 2{,}730 \div 5 = \$546.00$$

  The difference in the means is $613.75 – 546.00 = $67.75.

- The median of a set of data is the middle value when the values are listed in order. Even though $275 is very different from the other values, it should not have a significant impact on the median.

  The median for April through July:

  $$(610 + 625) \div 2 = \$617.50$$

  The median for April through August is $610.00.

  The difference between the medians is 617.50 – 610.00 = $7.50.

- The mode of a set of data is the value occurring most frequently. No values are repeated in either of these sets, so neither set has a mode.

The measure of central tendency that changed the most because of the store's low sales in August was the mean.

Try It

Mrs. Jenkins recorded the scores for all the eighth-grade students who took a test. The table below summarizes the measures of the data.

Test Scores

Measure	Value (percent)
Mean	85
Median	80
Mode	83
Range	37

If 55% is the lowest score on the test, which measure of data would be best for finding the highest score?

The _____ of a set of data is the average of the scores. It does not help you find the greatest or least score.

The _____ of a set of data is the middle score when the scores are listed in order. It does not help you find the highest score because it tells you the _____ score rather than the greatest or least score.

The _____ of a set of data is the value that occurs most _____. It does not help you find the highest score because it tells you the most _____ score rather than the greatest or least score.

The _____ of a set of data is the difference between the highest and _____ scores. If _____ % is the lowest score, then the highest score is _____ + _____ = _____%.

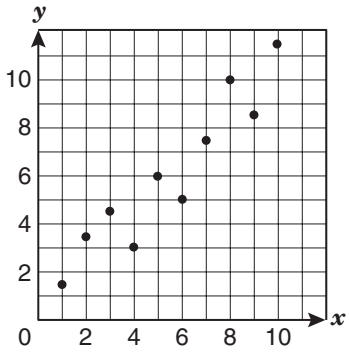
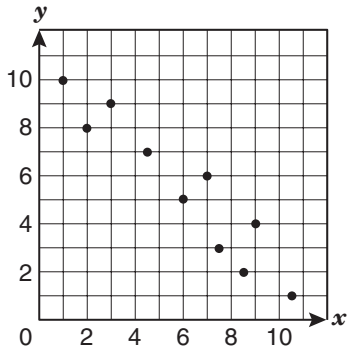
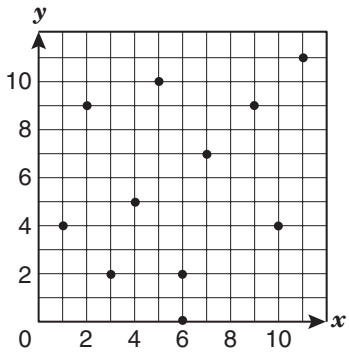
Use the _____ of the values to find the highest score.

The mean of a set of data is the average of the scores. The median of a set of data is the middle score when the scores are listed in order. It does not help you find the highest score because it tells you the middle score rather than the greatest or least score. The mode of a set of data is the value that occurs most frequently. It does not help you find the highest score because it tells you the most frequent score rather than the greatest or least score. The range of a set of data is the difference between the highest and lowest scores. If 55% is the lowest score, then the highest score is 55 + 37 = 92%. Use the range of the values to find the highest score.

How Do You Use a Scatterplot to Draw Conclusions and Make Predictions?

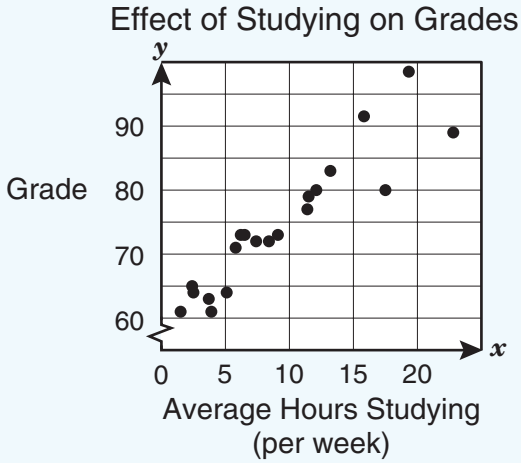
A **scatterplot** is a type of graph that can be used to show whether there is a relationship between two quantities.

To make predictions using a scatterplot, look for correlations in the data or a pattern in the data points. Patterns in a scatterplot are usually called trends. The trend may not be true for every point, but look for the overall pattern the data seem to fit.

As you move from left to right on the graph, if the data points …	as shown in this scatterplot …	they show this type of correlation:
move up		positive correlation
move down		negative correlation
show no pattern		no correlation

The following scatterplot shows the relationship between the average number of hours spent studying per week and the grade for the last semester for a group of 20 students.

What trend do these data show?



Look for a pattern.

The average number of hours spent studying per week increases as you move from left to right on the *x*-axis.

Do the corresponding *y*-coordinates for the points on the graph increase or decrease?

In general, they appear to increase. This is not true for every point, but the data seem to fit into the overall pattern.

When one value increases as the other increases, the pattern is a positive trend.

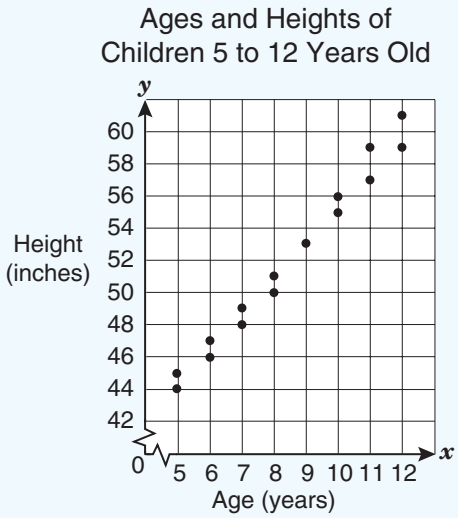
These data show a positive trend between the average number of hours spent studying per week and the grade earned.

Suppose you had a set of data that listed the heights and ages of a group of people. Would a scatterplot of those data show a positive trend between height and age?

Not necessarily. The results would depend on the sample of people chosen.

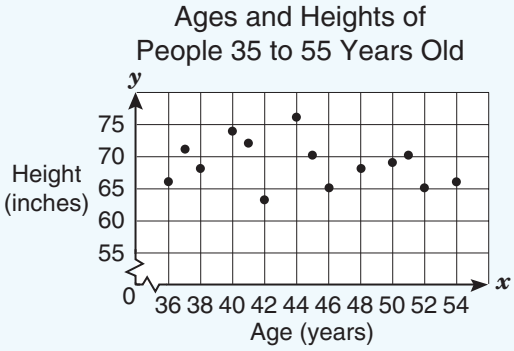
- The scatterplot would show a positive trend if people's heights increased with their ages. The older they were, the taller they would be.

  This relationship would be likely if the people in the sample data were young and still growing.



- The scatterplot would show no trend if people's heights neither increased nor decreased in any pattern with age. If one person from the group were older than another, there would be no way to predict who was taller or shorter.

  This relationship would be likely if the people in the sample data were all full-grown adults.

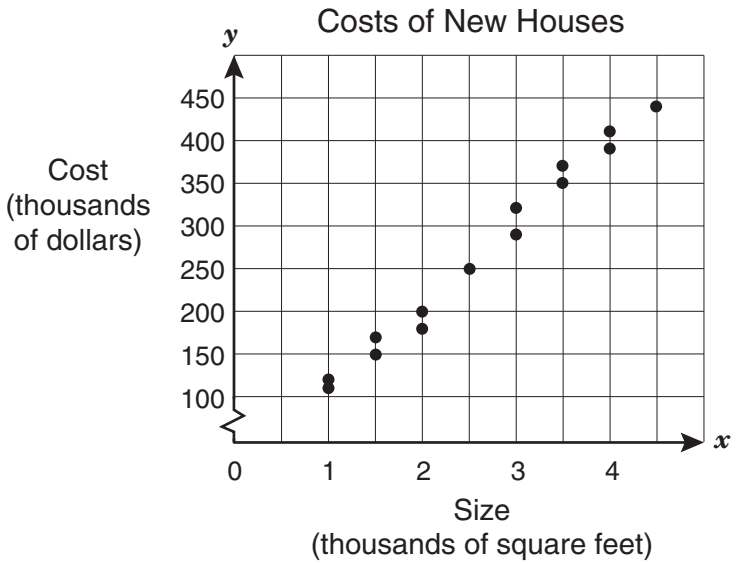


- The scatterplot would show a negative trend if people's heights decreased while their ages increased.

  This type of relationship is unlikely in this case, but it is quite possible in a context other than one involving age and height.

Try It

The scatterplot below shows the costs of new houses compared to their size measured in square feet.



Based on the information in the scatterplot, what conclusion can you draw about the relationship between the cost of a house and its size in square feet?

As you move from left to right on the *x*-axis, the *y*-coordinates of the points generally _____.

This means that there is a _____ trend in the data.

The graph shows that the cost of a new house _____ as its size increases.

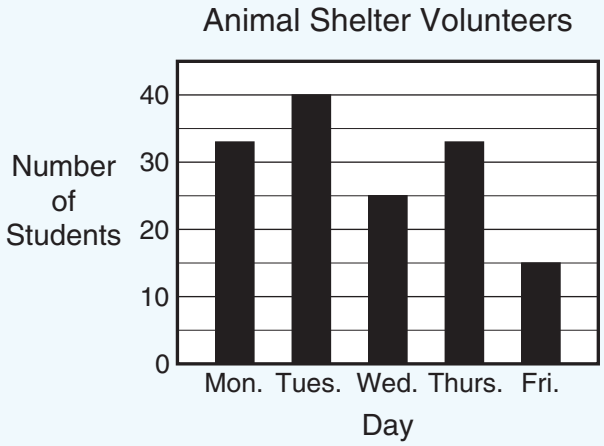
As you move from left to right on the *x*-axis, the *y*-coordinates of the points generally increase. This means that there is a positive trend in the data. The graph shows that the cost of a new house increases as its size increases.

How Do You Use Graphs to Represent Data?

There are many ways to represent data graphically. Bar graphs, histograms, and circle graphs are three types of graphs used to display data. Graphical representations of data often make it easier to see relationships in the data. However, in order to draw valid conclusions, you must read and interpret the data from the graph accurately.

A **bar graph** uses bars of different heights or lengths to show the relationship between different categories of data.

This bar graph shows the number of students who volunteered at an animal shelter each weekday.



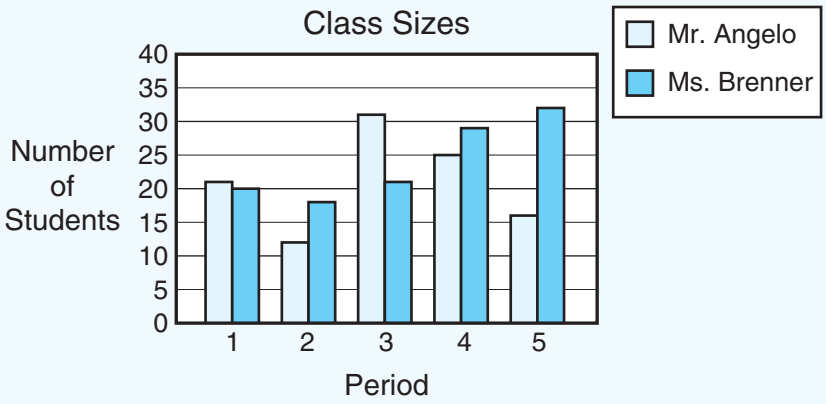
What conclusions can you draw about the number of students who volunteered on various days?

- The graph shows that more students volunteered on Tuesday than on any other day of the week.
- The day on which the least number of students volunteered was Friday.
- Approximately the same number of students volunteered on Monday as on Thursday.

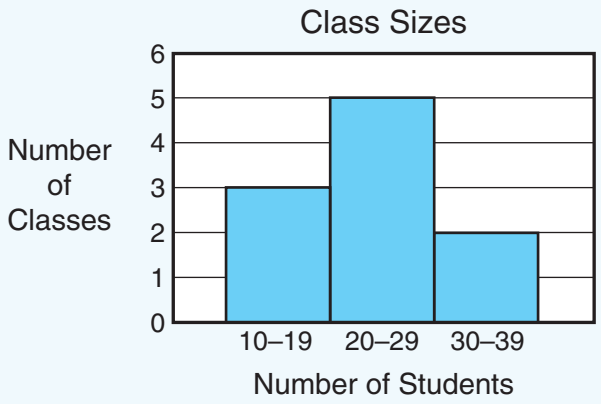
A **histogram** is a special kind of bar graph. A histogram shows the number of data points that fall within specific intervals of values.

Mr. Angelo and Ms. Brenner both teach math classes at Longmont Middle School. The graphs below show the number of students in their classes, but the two graphs do so in different ways.

- The first graph is a double bar graph. It compares the number of students in each teacher's classes, period by period.



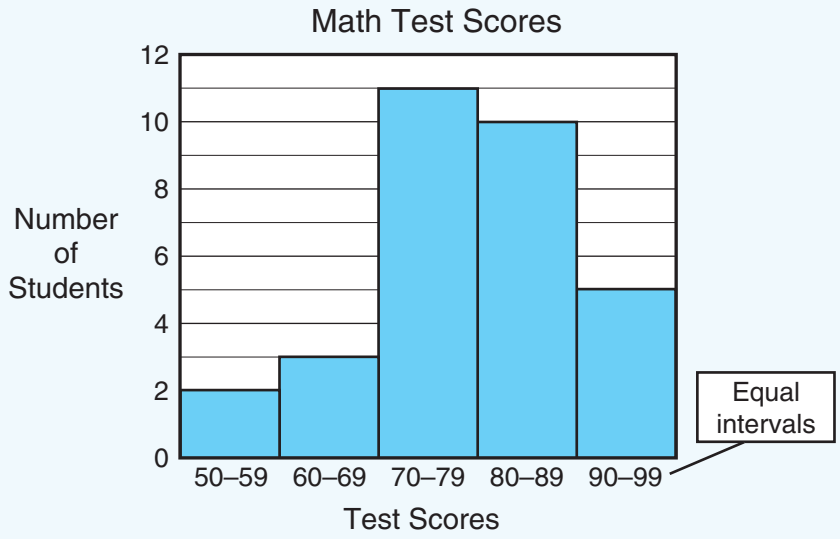
- The second graph shows their class sizes using a histogram. It shows how many classes in different size ranges they both teach.



In a histogram the data's range should be divided into equal intervals. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

Do you see that . . .

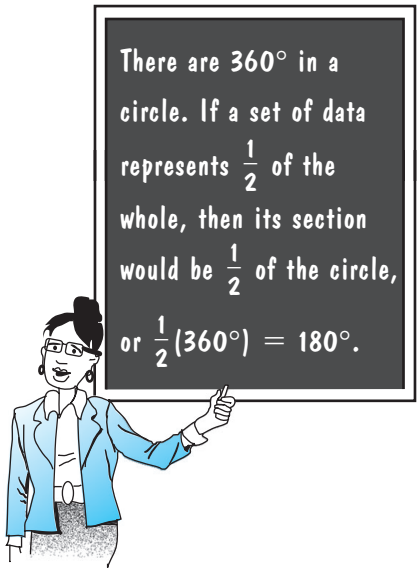
This histogram shows how students scored on a math test last week.



What conclusions can you draw about student scores from the graph?

- The greatest number of students scored between 70 and 79 on the test.
- The smallest number of students scored between 50 and 59 on the test.
- Fewer students scored 90–99 than 80–89 on the test.

A **circle graph** compares the numbers in a set of data by showing the relative sizes of the parts that make up a whole. The circle represents the whole, which is made up of all the data elements. Each section of the circle represents part of the whole.

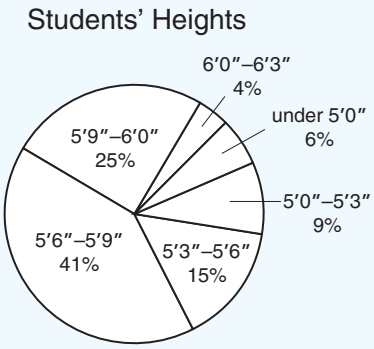


The circle graph below compares the height of 100 students in a school. If 25 out of 100 students are between 5′9″ and 6′ tall, then $\frac{25}{100} = 25\%$ are between 5′9″ and 6′ tall.
The same fraction of the circle was used to represent that height.

$$0.25(360°) = \frac{1}{4}(360°) = 90°$$

Students' Heights

Height	Number of Students
Under 5′0″	6
5′0″–under 5′3″	9
5′3″–under 5′6″	15
5′6″–under 5′9″	41
5′9″–under 6′0″	25
6′0″–under 6′3″	4
Total	100



The greatest number of students are between 5′6″ and 5′9″ tall, and the greatest part of the circle is used to represent that height.

This circle graph compares the number of different types of TV shows that were televised one week.

What conclusions can you draw about the different types of TV shows televised that week?

- The circle graph shows that the most common type of TV show televised was comedy. The section representing comedy has the greatest area.
- The circle graph shows that news and documentary programs make up approximately $\frac{1}{4}$ of the total TV shows. The sections representing news and documentaries together form an angle that is approximately $\frac{1}{4}$ of 360°, or 90°.

Try It

On a walk Matthew counted the number of different kinds of trees that he saw. Matthew saw 40 oak trees, 5 dogwood trees, 20 maple trees, 10 cedar trees, and 5 pine trees. If he wants to represent these data in a circle graph, what size angle should he use to form the section labeled "cedar trees"?

First find the fraction of the total number of trees that were cedar trees.

Matthew saw _____ cedar trees.

Matthew saw a total of _____ trees.

The fraction of the trees that were cedar trees is $\frac{\square}{\square}$, or $\frac{\square}{\square}$.

Then find the size of the angle that should be used to form the section for cedar trees.

The sum of all the angles in a circle graph is _____ °.
The part of the circle that should be used to represent cedar trees should be the _____ as the fraction of cedar trees that Matthew saw. The angle for cedar trees should be

$$\frac{\square}{\square} \cdot \_\_\_\_\_^\circ = \_\_\_\_\_^\circ.$$

A _____ ° angle should be used to form the section labeled "cedar trees."

Matthew saw 10 cedar trees. Matthew saw a total of 80 trees. The fraction of the trees that were cedar trees is $\frac{10}{80}$, or $\frac{1}{8}$. The sum of all the angles in a circle graph is 360°. The part of the circle that should be used to represent cedar trees should be the same as the fraction of cedar trees that Matthew saw. The angle for cedar trees should be $\frac{1}{8} \cdot 360^\circ = 45^\circ$.

A 45° angle should be used to form the section labeled "cedar trees."

How Do You Know Whether a Sample Is Representative of a Larger Group?

There are times when you want to know something about a large group of people but it is impractical to ask every member of the group. Instead, you ask a smaller group, a sample, and apply your conclusions to the entire population. This process is known as **sampling**. Sampling is frequently used in science experiments, social studies, and in surveys.

From a survey you can draw conclusions about a total population based on information you have about a few of its members. Those conclusions are valid only if the sample is representative of the total population.

A sample will be representative of a total population if these guidelines are followed.

- The group of people who are surveyed should be selected at random. This is called taking a **random sample**. The sample should not be selected in a way that might bias the results.

Suppose you want to know the favorite restaurant of the residents of a certain town. To find out, you stand outside a particular restaurant in that town and ask the people entering, *What is your favorite restaurant?* Would this method of selecting people generate a random sample?

No. This method of sampling is biased because it surveys people who probably prefer that restaurant. You need to select the sample more randomly. You could, for example, open the town's phone book and call every hundredth person. Using a list of names drawn randomly from the phone book would ensure that your survey was not biased.

- The group surveyed must be varied enough to be representative of the total population.

Suppose you want to know the favorite teacher of all the students in a school. Would it be valid to ask only students in Spanish classes who their favorite teacher is?

No. This method of sampling is biased because the opinions might not be representative of the opinions of the total student population. Nor would it be valid to ask only freshmen, who would probably name only teachers who teach freshmen. Your sample should model the total population with respect to gender, race, age, grades earned, etc. Your sample needs to have a mix of students proportional to the mix in the total student population.

- The size of the group surveyed must be large enough to be representative of the total population.

If only 5 people in a city of 100,000 are in a survey sample, would the conclusions reached be valid for the entire city's population?

No. The survey group of 5 is not a large enough sample to be representative of all the people in the city. A sample size can never be too large, but it can be too small. There is no simple formula for just how large a sample must be, but common sense will usually tell you whether a sample is too small.

Try It

Wellington Middle School has 965 students. Ms. Philips, the principal, wanted to know her students' preference for a new school mascot. To find out, she went to an eighth-grade English class one day and distributed a survey. Of the 35 students who completed the survey, 30 said they preferred a bulldog for the school mascot. Based on this survey, Ms. Philips concluded that most of the school's students wanted a bulldog for the new mascot. Why might her conclusion not be valid?

The sample of students surveyed was not representative of the school's population because the sample was not a

_____ sample. Only eighth-grade students were surveyed.

Only _____ out of 965 students were surveyed. The sample was not _____ to be representative.

The sample of students surveyed was not representative of the school's population because the sample was not a random sample. Only 35 out of 965 students were surveyed. The sample was not large enough to be representative.

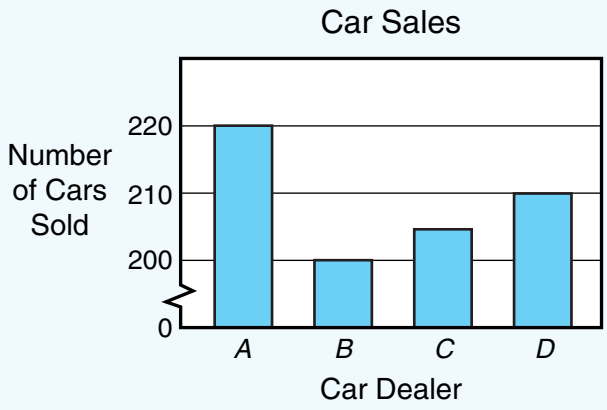
How Do You Know Whether Conclusions Drawn from Graphical or Numerical Information Are Valid?

To know whether conclusions drawn from graphical or numerical information are valid, you must determine whether you have read the graph accurately and analyzed the numerical information correctly.

Here are some guidelines to follow to ensure that the conclusions you draw from graphs and data are valid.

- Be certain that any pattern you find in the data is truly reflected by the data.
- Compare information presented in a table or graph accurately.
- Make sure the units used in a table or graph are consistent with the conclusions drawn.
- Be logical. Do not draw conclusions beyond those represented by the data.

The graph shows the total number of cars sold last month by four different car dealers.



Based on the graph, is it valid to infer that Car Dealer *A* sold more than twice as many cars last month as Car Dealer *B*?

- The bar for Dealer *A* appears to be more than twice as long as the bar for Dealer *B*. This might lead you to conclude that Dealer *A* sold more than twice as many cars as Dealer *B*.
- The vertical axis representing the number of cars sold is broken to show that part of the bars have been omitted.
- Read the values from the graph for the number of cars sold.

  Dealer *A* sold 220 cars. Dealer *B* sold 200 cars.
- Compare the numerical values, not the lengths of the bars.

  Dealer *A* sold 20 more cars than Dealer *B*.

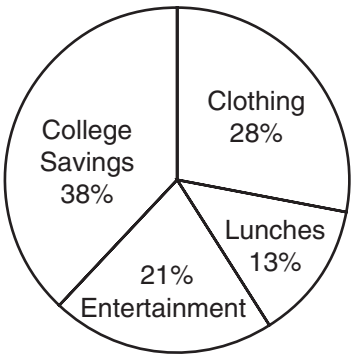
Dealer *A* sold more cars than Dealer *B*, but not twice as many. Dealer *B* sold 200 cars. Twice as many cars would be 400 cars. Dealer *A* did not sell 400 cars.

Do you see that . . .

Try It

Jorge has a part-time job after school. The graph shows how he spends his earnings in a typical month.

Jorge's Monthly Spending



If Jorge earned $265 last month, does the graph support the conclusion that he spent a little more than $90 on lunches and entertainment last month?

Find the percent spent on lunches and entertainment combined.

Of Jorge's earnings, _____% is spent on lunches, and _____% on entertainment.

The combined total of his spending on these two items is _____%.

Jorge earned $_____ last month, and _____ % of $_____

is $_____.

Jorge spent $_____ on lunches and entertainment last month.

The graph does support the conclusion that he spent a little more than $90 on lunches and entertainment last month.

Of Jorge's earnings, 13% is spent on lunches, and 21% on entertainment. The combined total of his spending on these two items is 34%. Jorge earned $265 last month, and 34% of $265 is $90.10. Jorge spent $90.10 on lunches and entertainment last month.

Now practice what you've learned.

Question 41

A spinner for a board game has 3 red sections, 2 blue sections, 2 green sections, and 1 white section. The sections are all of equal size. What is the probability of spinning red on the first spin and green on the second spin?

A $\frac{9}{64}$

B $\frac{3}{32}$

C $\frac{1}{16}$

D $\frac{6}{56}$

Answer Key: page 245

Question 42

Erica is playing a game using tiles with the following letters written on them: 4 tiles with *E*, 3 tiles with *A*, 3 tiles with *T*, and 5 tiles with *R*. Erica draws the first tile and records the letter. She does not replace the tile. She draws a second tile and records the letter. What is the probability that Erica draws two vowels?

A $\frac{14}{15}$

B $\frac{7}{15}$

C $\frac{49}{225}$

D $\frac{1}{5}$

Answer Key: page 245

Question 43

Of the 30 eighth-grade students surveyed, 5 said they would vote for Nancy as their representative on the student council. If the results of this survey are used to predict the election outcome and 332 eighth-grade students vote, which answer is closest to the number of votes Nancy should get?

A 199

B 53

C 55

D 553

Answer Key: page 245

Question 44

Mark has participated in 8 track meets so far this season. His running times for the 440-meter race have been 70, 63, 68, 65, 69, 61, 66, and 64 seconds. To qualify for the state finals, Mark must run each of his next two 440-meter races in less than 65 seconds. Based only on his times for the first 8 meets, what is the probability that Mark will run each of his next two races in less than 65 seconds?

A $\frac{3}{8}$

B $\frac{9}{64}$

C $\frac{7}{16}$

D $\frac{3}{4}$

Answer Key: page 245

Question 45

The students in a science lab followed the directions in the lab manual. They combined two chemicals and then measured the mass of the mixture. The students recorded their data and then determined the mean, median, mode, and range. The table below summarizes the measures of data for the experiment.

Chemical Mixture

Measure	Mass (grams)
Mean	2.96
Median	2.97
Mode	3.01
Range	0.17

Which measure of the data could best be used to find the greatest mass the students recorded if the smallest mass recorded was 2.87 grams?

A Mean

B Median

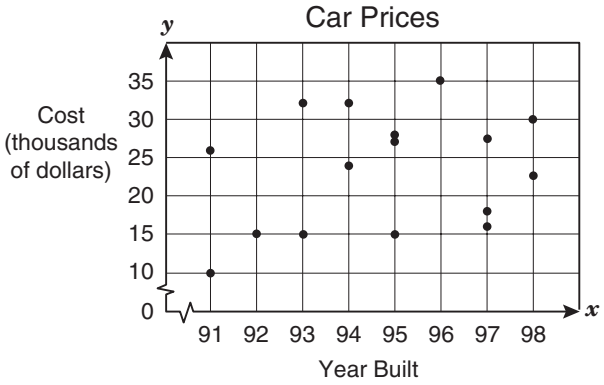
C Mode

D Range

Answer Key: page 246

Question 46

A car dealer put a sticker in the window of each used car in his lot to show the sale price of the vehicle and the year in which it was built. The scatterplot below shows the data.



Which description best represents the relationship in the recorded data?

A No trend

B Positive trend

C Negative trend

D Not Here

Answer Key: page 246

Question 47

The school cafeteria served sandwiches for lunch. The table shows the types of sandwiches selected by students.

Sandwich Choices

Sandwich	Number of Students
Grilled cheese	30
Tuna salad	40
Roast beef	75
Turkey	35

If the data are displayed in a circle graph, what size angle should form the section of the circle for tuna salad?

A 90°

B 40°

C 180°

D 80°

Answer Key: page 246

Question 48

Last week Janis surveyed customers leaving Food Superstore. Of the 500 people surveyed, 447 said that Food Superstore was their favorite grocery store. From these survey results Janis concluded that Food Superstore was the favorite grocery store among all the people in her town. Which is the best explanation for why her conclusion might not be valid?

A The sample may not have been representative of all the people in Janis's town.

B She asked every customer coming out of the store rather than asking every fifth customer who left the store.

C The survey she used did not ask how old the customers were.

D The sample size was too small.

Answer Key: page 246

Question 49

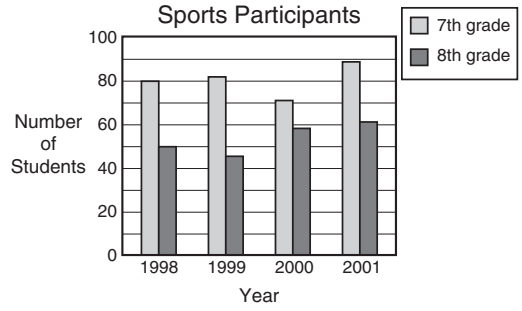
This table shows the number of 7th and 8th graders who played sports during the past 4 years.

Sports Participants

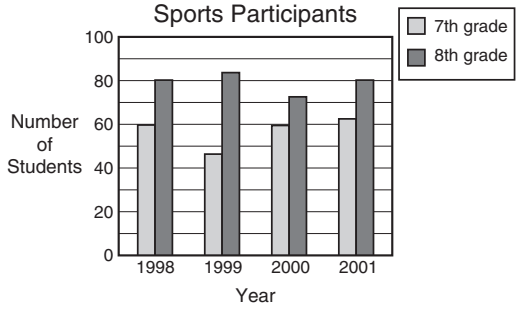
Year	7th Grade	8th Grade
1998	50	80
1999	46	83
2000	59	72
2001	62	90

Which bar graph best represents the data in the table?

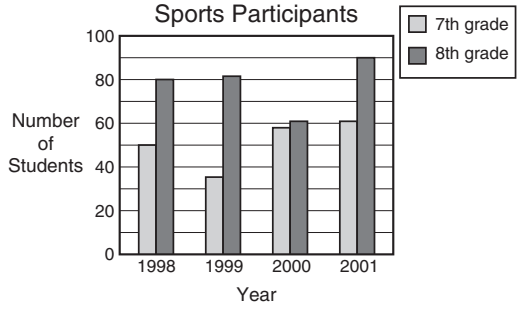
A



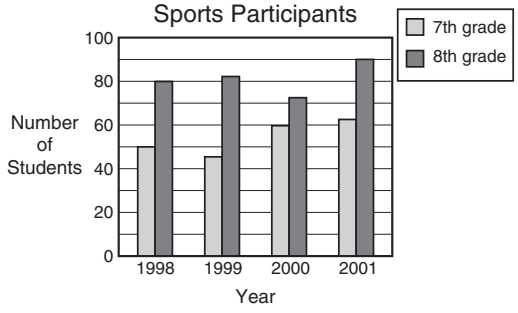
C



B



D



Answer Key: page 246

Question 50

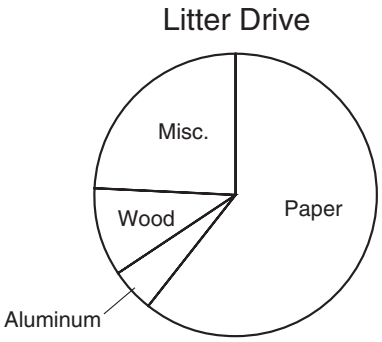
The eighth-grade class at Barryville Middle School conducted a litter drive as a community-service project. The table below shows the number of pounds of different types of litter they collected.

Litter Drive

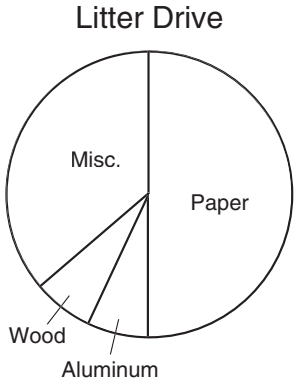
Type of Litter	Number of Pounds
Paper	305
Aluminum	25
Wood	51
Miscellaneous	126

Which graph best represents these data?

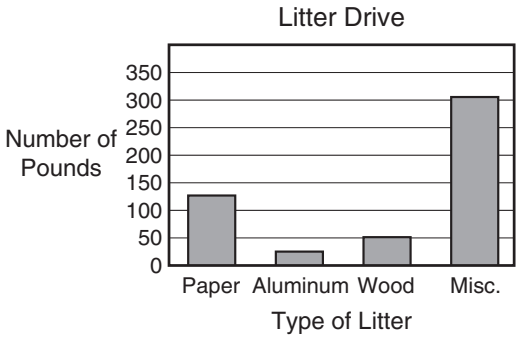
A



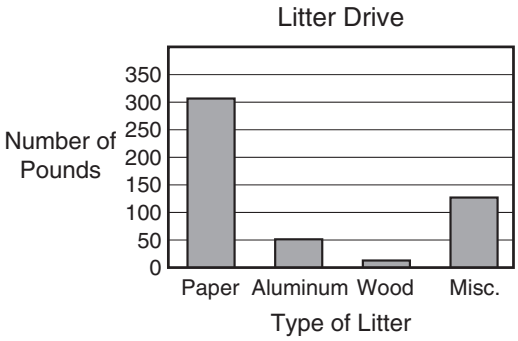
B



C



D



Answer Key: page 246

Objective 6

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Can You Use Mathematics to Solve Everyday Problems?

Many situations in everyday life involve mathematics. For example, you might want to compute the likelihood of your favorite team winning its next game based on its win-loss record, or you might need to estimate the area of a room based on its dimensions. Solving everyday problems often requires the use of mathematics.

Solving problems involves more than just numerical computation; logical reasoning and careful planning also play important roles. Here are some steps to follow when solving problems:

- Understand the problem. Organize the information you are given and identify exactly what you must find. You may need information that is not given in the problem, such as a formula. You may be given information that is not needed in order to solve the problem.
- Make a plan. After you have organized the information, decide how to use this information to find an answer. Think about the math concepts that apply to the situation. Identify the order in which you will find new information and the formulas or equations you will use to find it.
- Carry out the plan. After you have chosen a problem-solving strategy, use the strategy to work toward a solution to the problem. Go step-by-step through your plan, writing down important information at each step.
- Check to see whether your answer is reasonable. Check to see whether your answer makes sense. Does it answer the question asked? Is it stated in the correct units? Is it reasonable? You can estimate the solution and then compare the estimate to your calculated answer. They should be approximately equal.

Andy bought a $50 portable radio on sale for 15% off the list price. When he went to pay for his purchase, the clerk reminded him that he must also pay sales tax. If he paid for the radio with a $50 bill, how much change should he have received?

What additional information is needed to find the amount of change Andy should have received?

Understand the problem.

- What information do you already have?

  The list price of the radio was $50.

  The list price was reduced by 15%.

  There was sales tax on the purchase.

  Andy paid for the radio with a $50 bill.

- What do you want to know?

  How much change should Andy have received from the $50 bill?

Make a plan.

- Find the amount by which the sale reduced the list price.
- Subtract the reduction from the list price to find the sale price.
- Find the sales tax on the sale price.
- Add the sales tax to the sale price to find the total purchase price.
- Subtract the total purchase price from $50 to find the amount of change Andy should have received.

Carry out the plan.

- You can complete the first two steps of this problem.

  Write a proportion to find 15% of $50.

$$\frac{15}{100} = \frac{x}{50}$$
$$100x = 750$$
$$x = 7.5$$

  The list price was reduced by $7.50.

$$\$50.00 - \$7.50 = \$42.50$$

  The sale price is $42.50.

- You cannot find the sales tax, because you do not know the sales tax rate.

The missing piece of information is the sales tax rate.

Try It

A bag contains a number of colored marbles. Jay draws a marble from the bag, records its color, and returns the marble to the bag. He repeats this process 100 times. The table below shows the experimental results of the 100 trials.

Experimental Results

Color	Frequency
Red	35
Blue	20
White	45

Based on the experimental results, how many more red marbles than blue marbles can Jay expect to draw in 250 trials?

The experimental probability of drawing a red marble is $\frac{\square}{\square}$.

Write a proportion that can be used to predict r, the number of times Jay can expect to draw a red marble in 250 trials.

$$\frac{35}{100} = \frac{r}{\square}$$

$$100r = 35 \cdot \_\_\_\_\_$$

$$100r = \_\_\_\_\_$$

$$r = \_\_\_\_\_$$

Jay can expect to draw a red marble about _____ times in 250 trials.

The experimental probability of drawing a blue marble is $\frac{\square}{\square}$.

Write a proportion that can be used to predict b, the number of times Jay can expect to draw a blue marble in 250 trials.

$$\frac{20}{100} = \frac{b}{\square}$$

$$100b = 20 \cdot \_\_\_\_\_$$

$$100b = \_\_\_\_\_$$

$$b = \_\_\_\_\_$$

Jay can expect to draw a blue marble about _____ times in 250 trials.

To find the difference between the number of red marbles and the number of blue marbles Jay can expect to draw, subtract.

_____ – _____ = _____

In 250 trials Jay can expect to draw about _____ more red marbles than blue marbles.

The experimental probability of drawing a red marble is $\frac{35}{100}$.

$$\frac{35}{100} = \frac{r}{250}$$

$$100r = 35 \cdot 250$$

$$100r = 8{,}750$$

$$r = 87.5$$

Jay can expect to draw a red marble about 88 times in 250 trials.

The experimental probability of drawing a blue marble is $\frac{20}{100}$.

$$\frac{20}{100} = \frac{b}{250}$$

$$100b = 20 \cdot 250$$

$$100b = 5{,}000$$

$$b = 50$$

Jay can expect to draw a blue marble about 50 times in 250 trials.

$$88 - 50 = 38$$

In 250 trials Jay can expect to draw about 38 more red marbles than blue marbles.

What Is a Problem-Solving Strategy?

A **problem-solving strategy** is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backwards.

One way to solve a problem is to draw a picture. Drawing a picture can help you organize the information you need to solve the problem.

Each tire on Phil's bike has a diameter of 27 inches. The post office is 2.5 miles from Phil's house. If Phil rides his bike directly from his house to the post office, how many complete rotations will each of his bike tires make?

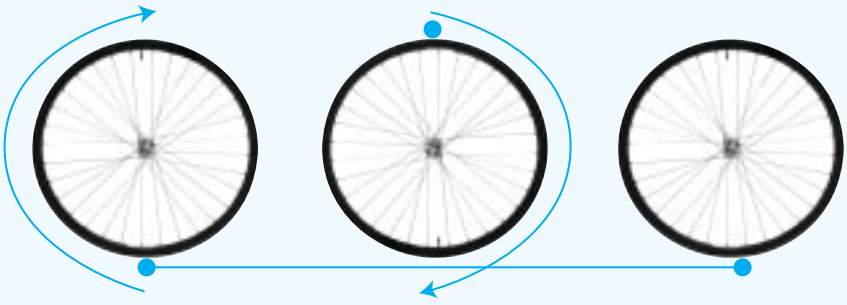
- What information do you already have?

  The diameter of each tire is 27 inches.

  The post office is 2.5 miles from Phil's house.
- What do you want to know?

  The distance each tire travels in each rotation

  The number of times each tire rotates in 2.5 miles
- To figure out how far each tire travels in one rotation, you could draw a picture.



  In one rotation the tire will have traveled the same distance on the ground as the circumference of the circle.
- To find the number of times the tire rotates, divide the number of feet in 2.5 miles by the number of feet covered in each rotation of the tire.

- Find the circumference of the tire.

$$C = \pi d$$

Use 3.14 as an approximate value of π. The diameter of the tire is 27 inches.

$$C \approx 3.14 \cdot 27 \approx 84.78 \text{ in.}$$

The circumference of the tire is about 84.78 inches. In one rotation the tire will travel about 84.78 inches on the ground.

- Divide by 12 to convert the circumference from inches to feet.

$$84.78 \div 12 = 7.065 \text{ ft}$$

The tire travels about 7.065 feet per rotation.

- Find the number of feet in 2.5 miles. There are 5,280 feet in 1 mile. Multiply by 5,280 to convert from miles to feet.

$$2.5 \cdot 5{,}280 = 13{,}200 \text{ ft}$$

There are 13,200 feet in 2.5 miles.

- To find the number of times the tire will make a complete rotation in 2.5 miles, divide 13,200 feet by 7.065 feet per rotation.

$$13{,}200 \div 7.065 \approx 1{,}868 \text{ rotations}$$

Each of Phil's bike tires makes approximately 1,868 complete rotations as he rides the 2.5 miles from his house to the post office.

Try It

A right triangle with leg lengths of 9 and 12 units is dilated by a scale factor of 2.5 to produce a new right triangle. What is the perimeter of the new triangle? You may want to draw a picture to help you solve this problem.

The perimeter should change by the _____.
First find the hypotenuse of the original triangle. The numbers 9 and 12 form the first two terms of a Pythagorean triple. Multiply the Pythagorean triple 3, 4, and 5 by _____ to get the new triple _____, _____, and _____.

The hypotenuse of the original triangle is _____.

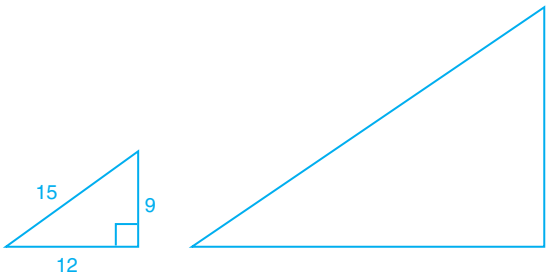
Find the perimeter of the original triangle.

P = _____ + _____ + _____ = _____ units

Find the perimeter of the dilated triangle. Multiply the original perimeter by the _____.

_____ • 2.5 = _____

The perimeter of the dilated triangle is _____ units.



The perimeter should change by the scale factor. Multiply the Pythagorean triple 3, 4, and 5 by 3 to get the new triple 9, 12, and 15. The hypotenuse of the original triangle is 15.

$P = 9 + 12 + 15 = 36$ units

Multiply the original perimeter by the scale factor.

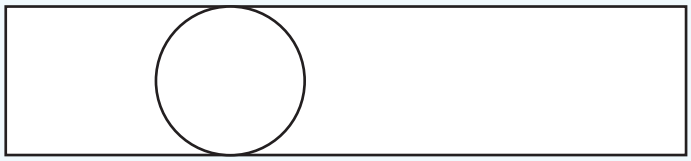
$36 \cdot 2.5 = 90$

The perimeter of the dilated triangle is 90 units.

How Do You Change Words into Math Language and Symbols?

It is important to be able to rewrite a problem using mathematical language and symbols. The words used in the problem will give you clues about what operations to use. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or a graph.

In the figure below, the length of the rectangle is 5 centimeters more than twice its width. The radius of the circle is 1 centimeter.



Write a ratio that compares the area of the rectangle to the area of the circle.

To find the area of the rectangle, you must know its length and width.

- Find the width of the rectangle.

  The width of the rectangle is equal to the diameter of the circle. If the radius of the circle is 1 centimeter, its diameter is 2 centimeters. Therefore, the width of the rectangle is 2 centimeters.

- Find the length of the rectangle.

  The length is 5 more than twice the width.

  $l = 2w + 5$ Substitute 2 for w in the expression.

  $l = 2(2) + 5$

  $l = 9$ cm

  The length of the rectangle is 9 centimeters.

- Find the area of the rectangle.

  The formula for the area of a rectangle is $A = lw$. Substitute 9 for l and 2 for w.

  $A = lw$

  $A = 9 \cdot 2 = 18 \text{ cm}^2$

  The area of the rectangle is 18 square centimeters.

- Find the area of the circle.

  The formula for the area of a circle is $A = \pi r^2$. Use 3.14 as an approximate value of π and substitute 1 for r.

$$A = \pi r^2$$
$$A \approx 3.14 \cdot 1^2$$
$$A \approx 3.14 \cdot 1$$
$$A \approx 3.14\,\text{cm}^2$$

  The area of the circle is approximately 3.14 square centimeters.

- Write the ratio of the area of the rectangle to the area of the circle.

$$\frac{\text{rectangle}}{\text{circle}} \qquad \frac{18}{3.14}$$

The ratio of the area of the rectangle to the area of the circle is $\frac{18}{3.14}$.

Try It

Mr. and Mrs. Braun go to a restaurant. Mr. Braun's meal costs \$15.40, and his wife's meal costs \$12.35. Before giving them their check, the server adds 7% tax to it. Describe the steps you would take to find the amount of change the Brauns should receive if they pay the check with two \$20 bills.

To solve this problem, you must first _____ to find the total cost of the Brauns' meals before tax.

Then _____ the total cost of the meals by _____ to find the tax on the check.

Next _____ the tax to the cost of the meals to find the total amount of the check.

Finally, _____ the total amount of the check from

\$_____ to find their change.

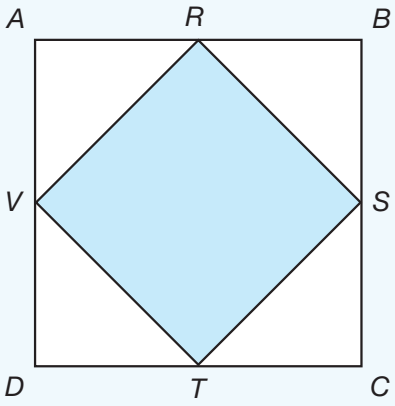
To solve this problem, you must first add to find the total cost of the Brauns' meals before tax. Then multiply the total cost of the meals by 0.07 to find the tax on the check. Next add the tax to the cost of the meals to find the total amount of the check. Finally, subtract the total amount of the check from \$40 to find their change.

How Can You Use Logical Reasoning as a Problem-Solving Tool?

Logical reasoning is thinking of something in a way that makes sense. Thinking about mathematics problems involves logical reasoning. You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions from the data that can be used to solve problems.

Finding patterns involves identifying characteristics that objects or numbers have in common. You can look for a pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be quadrilaterals, or they may all have right angles.

The midpoints of the sides of square *ABCD* are connected to form square *RSTV*.



The table below shows how the area of square *RSTV* will vary as the length of $\overline{AB}$ varies.

Length of $\overline{AB}$	Area of Square *RSTV*
1	0.5
2	2.0
4	8.0
6	18.0

If the pattern continued, what would be the area of square *RSTV* if $\overline{AB}$ measured 10 units?

To find the pattern, use the table to compare the lengths of $\overline{AB}$ with the areas of square *RSTV*. There is not an obvious pattern.

Since the problem asks you to find the area of square *RSTV*, use the length of $\overline{AB}$ to calculate the area of square *ABCD*. Then compare the area of square *ABCD* to the area of square *RSTV*.

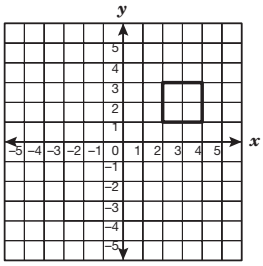
Length of $\overline{AB}$	Area of Square *ABCD*	Area of Square *RSTV*
1	1	0.5
2	4	2.0
4	16	8.0
6	36	18.0

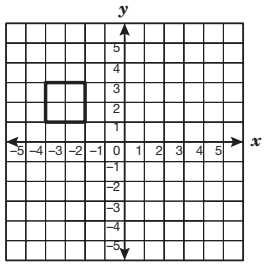
With the new data in the table, it is easier to see the pattern. The area of square *RSTV* is $\frac{1}{2}$ the area of square *ABCD*.

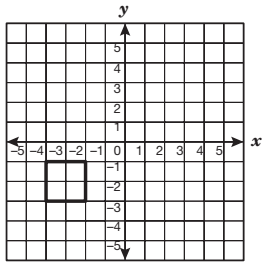
When $\overline{AB}$ is 10 units long, the area of square *ABCD* is 100 square units. Therefore, the area of square *RSTV* would be $\frac{1}{2} \cdot 100 = 50$ square units.

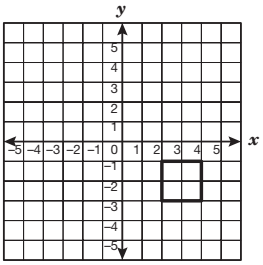
Try It

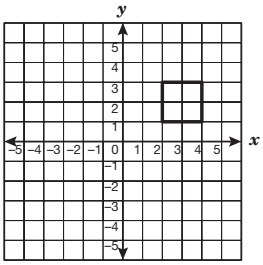
The graphs below have a repeating pattern.

1 

2 

3 

4 

5 

Draw the eighth graph in this pattern.

In the second graph, the original square has been reflected across the _____-axis.

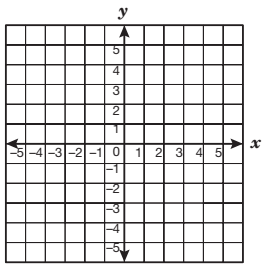
In the third graph, the previous square has been reflected across the _____-axis.

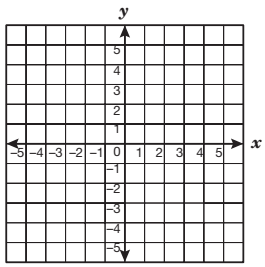
In the fourth graph, the previous square has been reflected across the _____-axis.

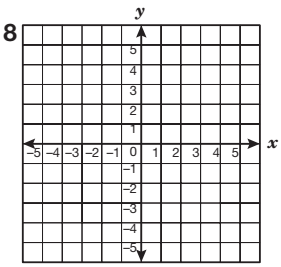
In the fifth graph, the previous square has been reflected across the _____-axis.

The pattern is a series of _____.

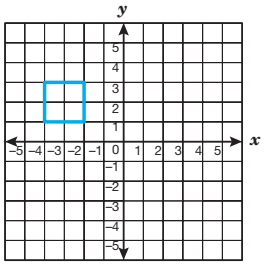
Continue the pattern for three more graphs to find the _____ graph.

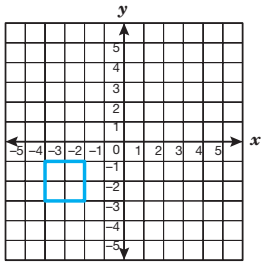
6 

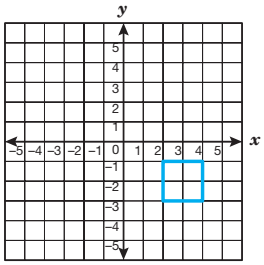
7 

8 

In the second graph, the original square has been reflected across the *y*-axis. In the third graph, the previous square has been reflected across the *x*-axis. In the fourth graph, the previous square has been reflected across the *y*-axis. In the fifth graph, the previous square has been reflected across the *x*-axis. The pattern is a series of reflections. Continue the pattern for three more graphs to find the eighth graph.

6 

7 

8 

The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing conclusions, and you should be able to explain that reason.

In math class Karl is working with the equation of the line $x + 2y = 6$. What is wrong with Karl's argument below, in which he attempts to justify that the point $(4, -2)$ is a point on the line $x + 2y = 6$?

Karl's Work

Step #1: To show that the point $(4, -2)$ is a point on the line, substitute its coordinates in the equation and see whether they make the equation true.

Step #2: To check the point $(4, -2)$, substitute -2 for x and 4 for y.

Step #3:
$$x + 2y = 6$$
$$-2 + 2(4) \stackrel{?}{=} 6$$
$$-2 + 8 \stackrel{?}{=} 6$$
$$6 = 6$$

Step #4: Since the equation is true, the point $(4, -2)$ is a point on the line $x + 2y = 6$.

Why is this conclusion not valid?

Is Karl's arithmetic with integers correct?

Yes, the calculation $-2 + 2(4) = -2 + 8 = 6$ is correct.

Is Karl's reasoning correct?

Yes, a point is on a graph if its coordinates satisfy the graph's equation.

Did Karl substitute the coordinates of the point correctly?

No, he switched the x-coordinate and y-coordinate for the point $(4, -2)$. The x-coordinate is the first number in the ordered pair $(4, -2)$. The x-coordinate is 4. The y-coordinate is the second number in the ordered pair $(4, -2)$. The y-coordinate is -2.

Karl substituted the wrong values for the x-coordinate and y-coordinate.

Now practice what you've learned.

Question 51

Which of these situations requires calculating the area of a figure?

A Consuelo wants to mail a package shaped like a rectangular prism. The post office requires that the sum of the three dimensions of a package be smaller than 42 inches. She needs to know whether her package can be mailed.

B Mrs. Snow wants to cover a section of her living room floor with tiles. She has two choices for the shape of the section, a circle with a 6-foot radius or a square with a side of 10 feet. She needs to know which shape will require the greater number of tiles.

C Mr. Carr is drawing the boundary lines of a soccer field with a machine that requires 1 ounce of chalk powder for every 175 feet of line it draws. He needs to know how much chalk powder the machine will use.

D Dave wants to store a quantity of sugar. He has two storage containers from which to choose, one shaped like a cylinder and the other shaped like a rectangular prism. He needs to find which of the containers will hold more sugar.

Answer Key: page 246

Question 52

Tom is twice Celia's age. Jack is two-thirds of Mary's age. Celia is 15 years old. Mary is 6 years younger than Tom. How much older is Jack than Celia?

A 1 year

B 11 years

C 9 years

D 14 years

Answer Key: page 246

Question 53

Phil wants to buy a pair of shoes originally priced at $34.80. The shoes are now on sale for 25% off the original price. There is a 6% sales tax on the sale price. If Phil has only $25, about how much more money does he need in order to buy the shoes?

A $2.17

B $59.19

C $33.50

D $2.67

Answer Key: page 246

Question 54

Mr. Wilson plans to cover his 15-by-6-foot rectangular patio with square tiles that measure 9 inches on each side. If the tiles are sold in boxes of 12, how many boxes of tiles will he need to buy to cover the patio?

A 2

B 13

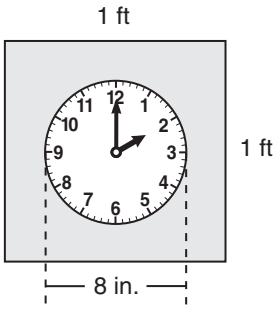
C 14

D 160

Answer Key: page 247

Question 55

A circular clock with a diameter of 8 inches is mounted in a 1-foot-square frame.



Which expression can be used to find the area of the shaded region in square inches?

A $1^2 - 8^2\pi$

B $12^2 - 4^2\pi$

C $8^2\pi - 12^2$

D $1^2 - 4^2\pi$

Answer Key: page 247

Question 56

A ladder is leaning against the side of a building. The building forms one side of a right triangle. The ground beneath the ladder forms another side of the triangle. The ladder forms the third side. Which statement about the triangle must be true?

A The longest leg of the triangle is the leg formed by the ground beneath the ladder.

B The height of the triangle is equal to the combined squares of the lengths of the legs.

C The sum of the lengths of the legs is equal to the length of the hypotenuse.

D The sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Answer Key: page 247

Question 57

A company that makes jewelry boxes wants to compare the cost of manufacturing two sizes of boxes. One is a cubical box that will be covered with material that costs $0.12 per square foot. The other is a cubical box with dimensions that are twice as large as the smaller box. It will be covered with material that costs $0.04 per square foot. Which of the following statements is true?

A It will cost the same amount to cover both boxes.

B The smaller jewelry box will cost more to cover than the larger box.

C The larger jewelry box will cost more to cover than the smaller box.

D It is not possible to tell which box costs more to cover.

Answer Key: page 247

Question 58

Which statement best describes the pattern of the terms in this sequence?

$$\frac{1}{1}, \frac{2}{4}, \frac{3}{9}, \frac{4}{16}, \frac{5}{25}, \ldots$$

A The numerator equals the term's position in the sequence, and the denominator equals two times the term's position in the sequence.

B The numerators are factors of 5, and the denominators are factors of 25.

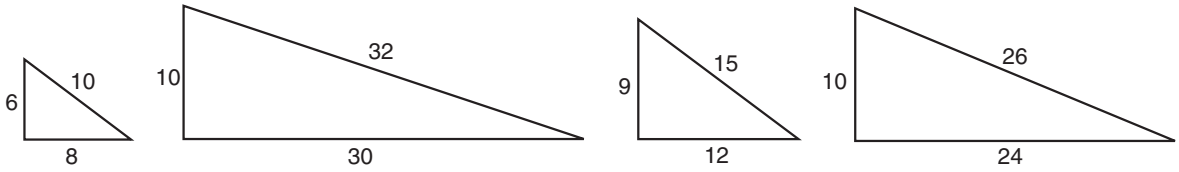
C The numerator equals the term's position in the sequence, and the denominator equals the square of the term's position in the sequence.

D The numerators increase by 1, and the denominators increase by a factor of 4.

Answer Key: page 247

Question 59

Which triangle does NOT belong in this group?



Triangle *R* Triangle *S* Triangle *T* Triangle *V*

A Triangle *R*

B Triangle *S*

C Triangle *T*

D Triangle *V*

Answer Key: page 248

Question 60

At Pablo's Pizza Pie, medium cheese pizzas cost $12.50, large cheese pizzas cost $15, and a 3-liter bottle of soda costs $2.50. Kevin purchased one bottle of soda and 5 pizzas for his party. If he spent exactly $70, what combination of medium and large pizzas did he purchase?

A 4 medium and 1 large

B 2 medium and 3 large

C 5 medium and 0 large

D 3 medium and 2 large

Answer Key: page 248

Mathematics Answer Key

Objective 1

Question 1 (page 113)

B Correct. To determine whether $\frac{22}{25}$ is greater than or equal to 82%, write both numbers in the same form. To convert $\frac{22}{25}$ to a decimal, divide 22 by 25.

$$22 \div 25 = 0.88$$

To convert 82% to a decimal, move the decimal point two places to the left and delete the percent sign.

$$82\% = 0.82$$

Compare 0.88 to 0.82. Since $0.88 > 0.82$, then $\frac{22}{25} > 82\%$. The fraction $\frac{22}{25}$ represents a quiz score high enough to ensure that Brian passes.

Question 2 (page 113)

D Correct. To find the tax on the meal, first convert 4% to a decimal.

$$4\% = 0.04$$

Multiply the cost of the meal by 0.04 to find the total tax.

$$42.25 \cdot 0.04$$

Divide the total tax by the number of friends, 5, to find the amount of tax that each of them paid.

$$(42.25 \cdot 0.04) \div 5$$

Question 3 (page 113)

C Correct. To approximate the value of $\sqrt{29}$, find two consecutive integers such that the first integer squared is less than 29 and the second integer squared is greater than 29. Since $5^2 < 29$ and $6^2 > 29$, the approximate value of $\sqrt{29}$ is between 5 and 6.

Question 4 (page 113)

B Correct. The number 384,000 is greater than 10, so the decimal point must be moved to the left to make the factor greater than or equal to 1 but less than 10. If the decimal point is moved five places to the left, the number will be 3.84. Since the decimal point was moved to the left, the exponent must be positive. The number 384,000 is written as 3.84×10^5 in scientific notation.

Question 5 (page 114)

C Correct. Multiply the number of pounds of beans by the price per pound to represent the cost of the beans.

$$1.19 \cdot 0.88$$

Divide \$1.48 by 2 to find the cost of 1 loaf of bread.

$$1.48 \div 2$$

Add to find t, the total cost of the beans and the bread.

$$t = (1.19 \cdot 0.88) + (1.48 \div 2)$$

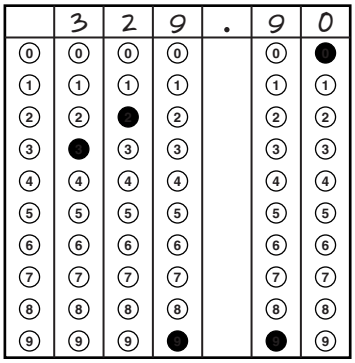
Question 6 (page 114)

The correct answer is 329.90. The company charges \$230.93 to rent a car for 7 days. To find the rate per day, divide \$230.93 by 7.

$$230.93 \div 7 = 32.99$$

The rate per day is \$32.99. To find the cost for 10 days, multiply the daily rental rate by 10.

$$32.99 \cdot 10 = 329.90$$



Question 7 (page 114)

C Correct. Carol made 1,215 minutes of local calls. The first 1,000 minutes cost \$29.50. The remaining 215 minutes cost \$0.04 per minute. Multiply to find the cost of the remaining local minutes.

$$\$0.04 \cdot 215 = \$8.60$$

Add to find the total cost for local calls.

$$\$29.50 + \$8.60 = \$38.10$$

Question 8 (page 114)

B Correct. Mario estimated the sum by adding the dollar part of each check and dropping the cents. The actual total will include the sum of the dollars and cents. Adding the cents will increase the actual total. The actual total is greater than the estimate because the values Mario added were all less than the actual amounts.

Question 9 (page 115)

C Correct. The formula for the area of a circle is $A = \pi r^2$. To see which answer is reasonable, estimate the area of the piece of glass. Use $\frac{1}{2}$ of 8.1, or about 4 centimeters, as an estimate of the radius of the piece of glass. Substitute the values for π and r in the formula.

$$A = \pi r^2.$$
$$A \approx 3.14 \cdot 4^2$$
$$A \approx 3.14 \cdot 16$$
$$A \approx 50.24$$

The area of the circle is approximately 50.24 cm^2, so 50 cm^2 is a reasonable value.

Question 10 (page 115)

D Correct. Use a proportion to compare the dollars earned, d, to the number of hours worked, 9.

$$\frac{\text{money}}{\text{hours}} \qquad \frac{78}{6} = \frac{d}{9}$$

Use cross products to solve the proportion.

$$6d = 78 \cdot 9$$

Objective 2

Question 11 (page 134)

B Correct. Find two ratios, each of which compares the number of words typed to the number of minutes required. Use a proportion to show the relationship between the ratios. The first ratio is $\frac{280 \text{ words}}{8 \text{ minutes}}$. The number of words that Sam can type in half an hour is represented by n. The second ratio is n words to $\frac{1}{2}$ hour. But ratios that are being compared must be expressed in the same units. Convert $\frac{1}{2}$ hour to 30 minutes. The second ratio is then $\frac{n}{30}$. Write a proportion. The proportion $\frac{280}{8} = \frac{n}{30}$ could be used to find n, the number of words Sam can type in half an hour.

Question 12 (page 134)

A Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice A compares 30 newspapers to $\frac{1}{2}$ hour. For ratios to be compared, they must be expressed in the same units; $\frac{1}{2}$ hour equals 30 minutes. The ratio in choice A compares 30 newspapers to 30 minutes: $\frac{30}{30}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{30}{30}$$
$$1{,}800 \neq 1{,}350$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 30 papers in $\frac{1}{2}$ hour is not equivalent to 60 papers in 45 minutes.

B Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice B compares 75 newspapers to 1 hour. For ratios to be compared, they must be expressed in the same units; 1 hour equals 60 minutes. The ratio in choice B compares 75 newspapers to 60 minutes: $\frac{75}{60}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{75}{60}$$
$$3{,}600 \neq 3{,}375$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 75 papers in 1 hour is not equivalent to 60 papers in 45 minutes.

C Correct. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice C compares 120 newspapers to $1\frac{1}{2}$ hours. For rates to be compared, they must be expressed in the same units; $1\frac{1}{2}$ hours equals 90 minutes. The ratio in choice C compares 120 newspapers to 90 minutes: $\frac{120}{90}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{120}{90}$$

$$5{,}400 = 5{,}400$$

Since the cross products are equal, the two ratios are proportional. A rate of 120 papers in $1\frac{1}{2}$ hours is equivalent to 60 papers in 45 minutes.

D Incorrect. The first ratio compares the number of newspapers delivered to the number of minutes required: $\frac{60}{45}$. The ratio in choice D compares 100 newspapers to 1 hour. For rates to be compared, they must be expressed in the same units; 1 hour equals 60 minutes. The ratio in choice D compares 100 newspapers to 60 minutes: $\frac{100}{60}$. The two ratios represent the same rate of delivering newspapers if they are proportional. Check to see whether their cross products are equal.

$$\frac{60}{45} \stackrel{?}{=} \frac{100}{60}$$

$$3{,}600 \neq 4{,}500$$

Since the cross products are not equal, the two ratios are not proportional. A rate of 100 papers in 1 hour is not equivalent to 60 papers in 45 minutes.

Question 13 (page 134)

D Correct. Write two ratios that compare the number of shoes boxed to the number of minutes required. The first ratio is 5 pairs of shoes every 3 minutes, $\frac{5}{3}$. Let x represent the number of shoes that can be placed in shoe boxes in 8 hours. The second ratio should compare x to the number of minutes in 8 hours.

There are $8 \cdot 60 = 480$ minutes in 8 hours. Write a proportion.

$$\frac{5}{3} = \frac{x}{480}$$

Then set the cross products equal to each other to solve the proportion.

$$3x = 5 \cdot 480$$

$$3x = 2{,}400$$

$$x = 800$$

During 8 hours of work, 800 pairs of shoes can be placed in shoe boxes.

Question 14 (page 134)

C Correct. Use a proportion to solve the problem. The ratio of students who ride the bus expressed as a percent is 78%, or 78 out of 100, which is equivalent to $\frac{78}{100}$. The ratio of students who ride the bus can also be expressed in terms of n, the total number of students in the school. That ratio is $\frac{975}{n}$. Write a proportion.

$$\frac{975}{n} = \frac{78}{100}$$

To solve, use cross products.

$$78n = 975 \cdot 100$$

$$78n = 97{,}500$$

$$n = 1{,}250$$

There are 1,250 students attending Cantor Middle School.

Question 15 (page 134)

B Correct. First find the additional fee per piece of lumber. Look for a pattern in the table. If 100 pieces of lumber cost an additional \$24, then each piece costs an additional \$0.24 to deliver.

$$24 \div 100 = 0.24$$

Check this unit fee for 20 pieces of lumber.

$$20 \cdot 0.24 = 4.80$$

This value agrees with the value in the table for 20 pieces of lumber. Check the other values. They all agree with a unit fee of \$0.24. There is a \$25 fee for any delivery. Use these facts to write an equation. The unit fee, \$0.24, times the number of pieces of lumber, n, plus the \$25 fee should equal the total charge, c. The equation $c = 0.24n + 25$ could be used to find the total cost for a delivery of n pieces of lumber.

Question 16 (page 135)

A Correct. In the graph, the lengths of the bars represent the number of gallons of milk drunk on a given day. The vertical scale of the graph is 0 to 50 gallons, with each mark standing for 5 gallons. Three of the bars do not end exactly on a mark. They are slightly above or below the mark. Estimate to find the values these bars represent: Monday (32 gallons), Thursday (27 gallons), and Sunday (38 gallons). The values in choice A match the estimates of the data represented in the graph.

Question 17 (page 136)

The correct answer is 1072.50. Look for a pattern in the table. The interest earned in 2 years is \$195.00.

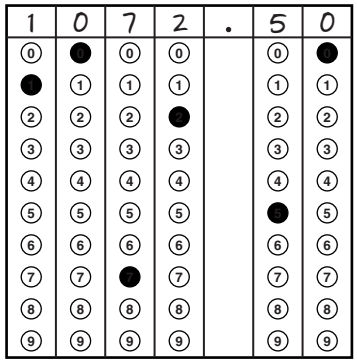
$$\$195 \div 2 = \$97.50 \text{ per year}$$

Check to see if this rate satisfies the remaining values in the table.

$$3 \cdot \$97.50 = \$292.50$$
$$4 \cdot \$97.50 = \$390.00$$
$$5 \cdot \$97.50 = \$487.50$$

Use this rate to find the interest earned in 11 years.

$$11 \cdot \$97.50 = \$1072.50$$



Question 18 (page 136)

B Correct. Write an equation that can be used to find the total annual cost of belonging to the swim club. Let n represent the number of times the pool is visited. If it costs \$2 every time a member visits the pool, then $2n$ represents the pool charges for the year. The annual membership charge is \$25. The total annual cost, c, is then $c = 2n + 25$. Find n when c is 365 by substituting values into the equation.

$$c = 2n + 25$$
$$365 = 2n + 25 \quad \text{Substitute.}$$
$$340 = 2n \quad \text{Subtract 25 from each side.}$$
$$170 = n \quad \text{Divide each side by 2.}$$

Joanne visited the pool 170 times last year.

Question 19 (page 136)

A Correct. Only the rule $2n(n + 1)$ produces the correct value for each term.

Position	$2n(n + 1)$	Value of Term
1	$2(1)(1 + 1) = 2 \cdot 2 = 4$	4
2	$2(2)(2 + 1) = 4 \cdot 3 = 12$	12
3	$2(3)(3 + 1) = 6 \cdot 4 = 24$	24
4	$2(4)(4 + 1) = 8 \cdot 5 = 40$	40
5	$2(5)(5 + 1) = 10 \cdot 6 = 60$	60

Question 20 (page 137)

D Correct. Only the sequence $\frac{2}{3}, \frac{3}{6}, \frac{4}{9}, \frac{5}{12}, \frac{6}{15}, \ldots$ has the correct value for each term.

Position	$\frac{n + 1}{3n}$	Value of Term
1	$\frac{1 + 1}{3(1)} = \frac{2}{3}$	$\frac{2}{3}$
2	$\frac{2 + 1}{3(2)} = \frac{3}{6}$	$\frac{3}{6}$
3	$\frac{3 + 1}{3(3)} = \frac{4}{9}$	$\frac{4}{9}$
4	$\frac{4 + 1}{3(4)} = \frac{5}{12}$	$\frac{5}{12}$
5	$\frac{5 + 1}{3(5)} = \frac{6}{15}$	$\frac{6}{15}$

Objective 3

Question 21 (page 157)

D Correct. The radius is reduced by 60%. The original radius is 4 inches.

$$60\% \text{ of } 4 \text{ inches} = 0.60 \cdot 4 = 2.4 \text{ inches}$$

The radius is reduced by 2.4 inches. The new radius is 4 inches − 2.4 inches = 1.6 inches.

Question 22 (page 157)

D Correct. The ratios of the corresponding sides are $\frac{12}{30}$ and $\frac{10}{25}$. Both fractions are equal to $\frac{2}{5}$ or 0.4, the scale factor that was used to reduce the larger parallelogram.

Question 23 (page 157)

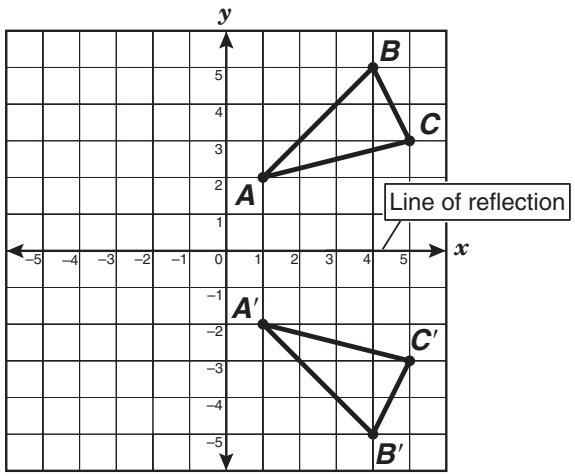
C Correct. The length of side DE is 6 units. The length of side $D'E'$ is 12 units. The ratio of these corresponding sides is $\frac{D'E'}{DE} = \frac{12}{6} = 2$. The lengths of sides DF and $D'F'$ are 8 units and 16 units, respectively. The ratio of these corresponding sides is $\frac{D'F'}{DF} = \frac{16}{8} = 2$. Triangle DEF was enlarged by a scale factor of 2.

Question 24 (page 158)

A Correct. Point B has the coordinates $(-2, 4)$. The x-coordinate of point B, -2, is 7 units to the left of the line of reflection, $x = 5$. The reflection of point B must be 7 units to the right of the line of reflection. The x-coordinate of the reflection of point B is $7 + 5 = 12$. The y-coordinate of the point is unchanged by the reflection. The y-coordinate is still 4. The coordinates of the reflection of point B are $(12, 4)$.

Question 25 (page 158)

B Correct. The x-axis is the line of reflection for this transformation.



Question 26 (page 158)

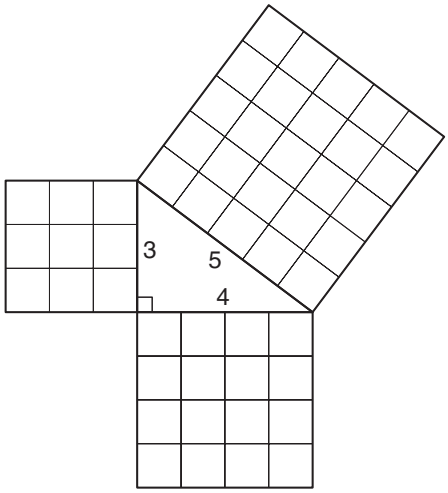
D Correct. When viewed from the top, this figure has a row of 3 blocks perpendicular to a column of three blocks in the middle. From the side you can see one row of 3 blocks with another block in the center of a second row. From the front you can see one row of 3 blocks with another block on the left of a second row. Only the figure in choice D has these three views.

Question 27 (page 159)

C Correct. The rectangular floor has an area of 33 feet • 14 feet = 462 ft^2. The circular fountain takes up an area of $\pi \cdot 4^2 \approx 3.14 \cdot 16 \approx 50.24$ ft^2. Subtract the area of the fountain from the area of the entire floor. The area that Mr. Wythe polishes is 462 ft^2 − 50.24 ft^2 ≈ 411.76 ft^2.

Question 28 (page 159)

B Correct. If the set 5, 3, and 4 satisfies $a^2 + b^2 = c^2$, then the set can form the sides of a right triangle. You can use a geometric model to see whether this is true. The longest side of this triangle is 5, so it would have to be the hypotenuse of the right triangle. The other two sides, 3 and 4, would be the legs of the right triangle.



The area of the square along one leg is $3 \cdot 3 = 9$ square units. The area of the square along the other leg is $4 \cdot 4 = 16$ square units. The area of the square along the hypotenuse is $5 \cdot 5 = 25$ square units. Since $9 + 16 = 25$, it follows that $3^2 + 4^2 = 5^2$. Therefore, the sides of the triangle satisfy the Pythagorean Theorem. A triangle with side lengths 5, 3, and 4 units is a right triangle.

Question 29 (page 159)

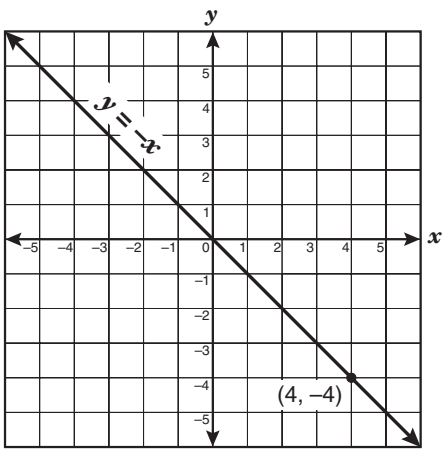
B Correct. Point Q has the coordinates $(-4, 8)$. The x-coordinate of point Q is -4, and $-4 < -2.5$. The y-coordinate of point Q is 8, and $8 > 3$. The coordinates of point Q satisfy both conditions, $x < -2.5$ and $y > 3$.

Question 30 (page 159)

A Incorrect. For a point to be in Quadrant IV, it must have an x-coordinate that is positive and a y-coordinate that is negative. The x-coordinate of point $(-7, 7)$ is -7. It is negative. The y-coordinate of point $(-7, 7)$ is 7. It is positive. The point $(-7, 7)$ is in Quadrant II, not Quadrant IV.

B Incorrect. For a point to be on the line $y = -x$, its coordinates must satisfy the equation. The x-coordinate of point $(5, 5)$ is 5. The y-coordinate of point $(5, 5)$ is 5. Substitute these values into the equation $y = -x$. Since $5 \neq -5$, the point is not on the line.

C Correct. For a point to be on the line $y = -x$, its coordinates must satisfy the equation. The x-coordinate of point $(4, -4)$ is 4. The y-coordinate of point $(4, -4)$ is -4. Substitute these values into the equation $y = -x$. Since $-4 = -4$, the point is on the line. For a point to be in Quadrant IV, it must have an x-coordinate that is positive and a y-coordinate that is negative. The point $(4, -4)$ is in Quadrant IV.



D Incorrect. For a point to be on the line $y = -x$, its coordinates must satisfy the equation. The x-coordinate of point $(1, -2)$ is 1. The y-coordinate of point $(1, -2)$ is -2. Substitute these values into the equation $y = -x$. Since $-2 \neq -1$, the point is not on the line.

Objective 4

Question 31 (page 185)

C Correct. The surface area of a solid figure is the sum of the areas of its surfaces. The surfaces of the prism are all rectangles.
To find the area of each rectangle, multiply its length by its width.
There are two rectangles with dimensions of 2 by 9 in. Each has an area of 18 in.^{2} Their combined area is 36 in.^{2}
There are two rectangles with dimensions of 2 by 10 in. Each has an area of 20 in.^{2} Their combined area is 40 in.^{2}
There are two rectangles with dimensions of 9 by 10 in. Each has an area of 90 in.^{2} Their combined area is 180 in.^{2}

$$S = 36 \text{ in.}^2 + 40 \text{ in.}^2 + 180 \text{ in.}^2 = 256 \text{ in.}^2$$

The surface area of the prism is 256 square inches.

Question 32 (page 185)

C Correct. The area of the label is the lateral surface area of the can.
The formula for the lateral surface area of a cylinder is $S = 2\pi rh$. Measure the height and radius of the cylinder in centimeters.
Radius = 1 cm
Height = 6 cm
Substitute 1 for r, 6 for h, and 3.14 as an approximate value for π.

$$S = 2\pi rh$$
$$S \approx 2(3.14)(1)(6)$$
$$S \approx 37.68$$

The lateral surface area of the cylinder is approximately 38 square centimeters.

Question 33 (page 185)

A Correct. The amount of cement needed is equal to the volume of the slab, which will be a rectangular prism.
The formula for the volume of a rectangular prism is $V = Bh$.
First convert 4 inches to feet, since the question asks for the number of cubic feet of cement needed.

$$\frac{4 \text{ inches}}{x \text{ feet}} = \frac{12 \text{ inches}}{1 \text{ foot}}$$
$$12x = 4$$
$$x = \frac{1}{3}$$

Calculate B, the area of the base.

$$12.5 \cdot 9 = 112.5 \text{ ft}^2$$

Find the volume of the slab.

$$V = Bh$$
$$V = 112.5 \cdot \frac{1}{3}$$
$$V = 37.5 \text{ ft}^3$$

It will take 37.5 cubic feet of cement to build the slab.

Question 34 (page 186)

C Correct. Since the ladder and the wall of the building form a right triangle, use the Pythagorean Theorem to find the missing side.

Let b represent the distance from the bottom of the ladder to the base of the building, which is the missing leg of the right triangle. The length of the ladder is the hypotenuse of the right triangle. Replace c with 12. The known leg of the triangle, a, is 10.

$$a^2 + b^2 = c^2$$
$$10^2 + b^2 = 12^2$$
$$100 + b^2 = 144$$
$$b^2 = 144 - 100$$
$$b^2 = 44$$
$$\sqrt{b^2} = \sqrt{44}$$
$$b = \sqrt{44}$$

Since $6^2 = 36$ and $7^2 = 49$, $\sqrt{44}$ is between 6 and 7. Only answer choice C is a value between 6 and 7 feet.

Question 35 (page 186)

D Correct. The triangle formed by home plate, first base, and third base is a right triangle. The segment between first base and third base forms the hypotenuse.

Use the Pythagorean Theorem to find the length of the missing side.

Replace a with 90 and b with 90.

$$c^2 = a^2 + b^2$$
$$c^2 = 90^2 + 90^2$$
$$c^2 = 8{,}100 + 8{,}100$$
$$c^2 = 16{,}200$$
$$\sqrt{c^2} = \sqrt{16{,}200}$$
$$c = \sqrt{16{,}200}$$

Since $120^2 = 14{,}400$ and $130^2 = 16{,}900$, $\sqrt{16{,}200}$ is between 120 and 130. Only answer choice D is a value between 120 and 130 feet.

Question 36 (page 187)

B Correct. The model and the house can be represented with two rectangles. The rectangles are similar figures, so the measurements of their corresponding sides are proportional.

Let x represent the width of the house.

The ratio of the length of the house to the length of the model is $\frac{64 \text{ ft}}{12.5 \text{ in.}}$.

The ratio of the width of the house to the width of the model is $\frac{x \text{ ft}}{7.75 \text{ in.}}$.

Write a proportion.

$$\frac{x}{7.75} = \frac{64}{12.5}$$
$$12.5x = 496$$
$$x = 39.68$$

To the nearest foot, the width of the actual house will be 40 feet.

Question 37 (page 187)

A Correct. The perimeter of the new rectangle will decrease by the same scale factor.

The scale factor is $\frac{1}{2}$. To find the perimeter of the new rectangle, multiply the scale factor by the perimeter of the original rectangle. The perimeter of the new rectangle is $39 \cdot \frac{1}{2} = 19.5$ centimeters.

Question 38 (page 187)

C Correct. The area of the new triangle will increase by the square of the scale factor.

The scale factor is 5. The square of the scale factor is $5^2 = 25$.

To find the area of the new triangle, multiply the area of the original triangle by the square of the scale factor.

The area of the new triangle is $6 \cdot 25 = 150 \text{ ft}^2$.

Question 39 (page 187)

D Correct. The volume of the new cylinder will decrease by the cube of the scale factor.

The scale factor is $\frac{1}{4}$. The cube of the scale factor is $\left(\frac{1}{4}\right)^3$.

$$\left(\frac{1}{4}\right)^3 = \frac{1^3}{4^3} = \frac{1}{64}$$

To find the volume of the new cylinder, multiply the volume of the original cylinder by the cube of the scale factor.

The volume of the new cylinder is $3{,}600 \cdot \frac{1}{64} = 56.25$ cubic centimeters.

Question 40 (page 188)

B Correct. The dimensions of the smaller tent are $\frac{3}{4}$ the dimensions of the larger tent. The smaller tent is a dilation of the larger tent, with a scale factor of $\frac{3}{4}$.

The cube of the scale factor is $\left(\frac{3}{4}\right)^3$.

$$\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$$

The ratio of the volumes of the tents will be proportional to the cube of the scale factor.

Let x represent the volume of the larger tent.

The ratio of the volume of the smaller tent to the volume of the larger tent is $\frac{1{,}000}{x}$.

Write a proportion.

$$\frac{27}{64} = \frac{1{,}000}{x}$$
$$27x = 64{,}000$$
$$x = 2{,}370.37$$
$$x \approx 2{,}370 \text{ ft}^3$$

The volume of the larger tent is approximately 2,370 cubic feet.

Objective 5

Question 41 (page 216)

B Correct. The outcome of the first spin has no effect on the outcome of the second spin. Therefore, this is a compound event made up of two independent events. There are 3 red sections on the spinner, out of a total of 8 sections. Calculate the probability that the spinner will land on a red section on the first spin.

$$P(\text{red}) = \frac{3}{8}$$

There are 2 green sections on the spinner. Calculate the probability that the spinner will land on a green section on the second spin.

$$P(\text{green}) = \frac{2}{8}$$

To find the probability of spinning red on the first spin and green on the second spin, multiply the probabilities of the two events.

$$P(\text{red and green}) = P(\text{red}) \cdot P(\text{green}) =$$
$$\frac{3}{8} \cdot \frac{2}{8} = \frac{6}{64} = \frac{3}{32}$$

Question 42 (page 216)

D Correct. The number of tiles in the bag will be different for the second draw than for the first draw because Erica does not replace the tile after the first draw. Therefore, this is a compound event made up of two dependent events.

For the first draw, there are 7 vowels out of a total of 15 tiles in the bag.

$$P(\text{vowel}_{\text{first}}) = \frac{7}{15}$$

If a vowel is drawn on the first draw, there will be only 6 vowels and a total of 14 tiles in the bag for the second draw.

$$P(\text{vowel}_{\text{second}}) = \frac{6}{14}$$

To find the probability that Erica will draw two vowels, multiply the probabilities of the two events.

$$P(\text{vowel}_{\text{first}} \text{ and vowel}_{\text{second}}) =$$
$$P(\text{vowel}_{\text{first}}) \cdot P(\text{vowel}_{\text{second}}) =$$
$$\frac{7}{15} \cdot \frac{6}{14} = \frac{42}{210} = \frac{1}{5}$$

Question 43 (page 216)

C Correct. Use a proportion to solve the problem. The ratio of students voting for Nancy in the survey should equal the ratio of students voting for Nancy in the election.

$$\frac{5}{30} = \frac{x}{332}$$
$$30x = 1{,}660$$
$$x \approx 55.33$$

Nancy should get about 55 votes in the election.

Question 44 (page 216)

B Correct. Use Mark's experimental probability of running the 440-meter race in less than 65 seconds to predict his future performance. Mark has run 8 races. Of those, he ran 3 in less than 65 seconds. His experimental probability of running the 440-meter race in less than 65 seconds is $\frac{3}{8}$. Running his next two races in less than 65 seconds is a compound event. Running the first race in less than 65 seconds is the first event, and running the second race in less than 65 seconds is the second event. To find the probability of Mark's running the first and second races in less than 65 seconds each, multiply the probabilities of the two events.

$P(\text{1st under 65 and 2nd under 65}) =$
$P(\text{1st under 65}) \cdot P(\text{2nd under 65}) =$
$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$

The theoretical probability that Mark will run both of his next two races in less than 65 seconds each is $\frac{9}{64}$.

Question 45 (page 217)

D Correct. The range of a set of numbers tells how large the spread is between the greatest and the smallest values in the set. Use the range to find the greatest value when you are given the smallest value.

Question 46 (page 217)

A Correct. As you move from left to right on the coordinate grid, the years increase from 1991 to 1998. As the years increase, the cost of the vehicles neither increases nor decreases in any pattern. This means that there is no trend in the data.

Question 47 (page 218)

D Correct. A total of 180 sandwiches were served. Of those, 40 were tuna salad sandwiches. The fraction of the total number of sandwiches served that were tuna salad is $\frac{40}{180}$, or $\frac{2}{9}$. The sum of all the angles in a circle graph is 360°. The angle that should form the section for tuna salad is $\frac{2}{9} \cdot 360° = 80°$. An 80° angle should form the section for tuna salad.

Question 48 (page 218)

A Correct. Janis surveyed only people who were shopping at Food Superstore. The people shopping at Food Superstore are more likely to choose Food Superstore as their favorite grocery store. This was a biased sample, not representative of all the people in Janis's town.

Question 49 (page 219)

D Correct. For each year, the height of the left bar should be equal to the number of seventh-grade participants that year, and the height of the right bar should be equal to the number of eighth-grade participants that year. Only the heights of the bars in the graph in choice D match the data in the table.

Question 50 (page 220)

A Correct. This circle graph best represents the data. The total number of pounds of litter collected was 507 pounds. Divide each weight by 507 to find the percent each represents.

Barryville Litter Drive

Type of Litter	Pounds	Percent
Paper	305	≈60%
Aluminum	25	≈5%
Wood	51	≈10%
Misc.	126	≈25%
Total	507	100%

Only the graph in choice A appears to match these percentages.

Objective 6

Question 51 (page 235)

A Incorrect. To determine whether the package can be mailed, she needs to add the three dimensions together, not find the area of the package.

B Correct. To find which shape would require the greater number of tiles, she needs to compare the area of the circle to the area of the square.

C Incorrect. To determine the amount of chalk powder the machine will use, he needs to calculate the perimeter of the soccer field, not its area.

D Incorrect. To find which container will hold more sugar, he needs to find the volumes of the containers, not their areas.

Question 52 (page 235)

A Correct. Work backwards to find a solution. Celia is 15 years old. Tom is twice Celia's age, so he is $2 \cdot 15 = 30$ years old. Mary is 6 years younger than Tom, so she is $30 - 6 = 24$ years old. Jack is two-thirds of Mary's age, so he is $\frac{2}{3} \cdot 24 = 16$ years old. If Jack is 16 years old and Celia is 15 years old, then Jack is 1 year older than Celia.

Question 53 (page 235)

D Correct. To find the sale price, subtract the amount of the discount from the original price.

Find 25% of $34.80.

$$25\% = 0.25$$

$$34.80 \cdot 0.25 = 8.70$$

The discount is $8.70.

The sale price is

$$\$34.80 - \$8.70 = \$26.10.$$

To find the total price, add the amount of sales tax to the sale price.

Find 6% of $26.10.

$$6\% = 0.06$$

$$26.10 \cdot 0.06 = 1.566$$

The amount of tax Phil will pay is $1.57.

The total cost of the shoes is

$$\$26.10 + \$1.57 = \$27.67.$$

If Phil has only $25, then he needs $\$27.67 - \$25.00 = \$2.67$ more to buy the shoes.

Question 54 (page 235)

C Correct. Find the area of the patio.

$$A = lw$$

$$A = 15 \cdot 6 = 90 \text{ ft}^2$$

Find the area of a tile in square feet, not square inches.

The tiles are 9 inches, or $\frac{9}{12} = \frac{3}{4} = 0.75$ feet on a side.

$$A = 0.75 \cdot 0.75 = 0.5625 \text{ ft}^2$$

Write a proportion to find the number of tiles needed.

$$\frac{1}{0.5625} = \frac{x}{90}$$

$$0.5625x = 90$$

$$x = 160$$

Mr. Wilson will need to buy 160 tiles. They are sold in boxes of 12. Divide to find the number of boxes of tiles he will need to buy.

$$160 \div 12 \approx 13.33$$

Mr. Wilson will need to buy 14 boxes of tiles.

Question 55 (page 236)

B Correct. The formula for the area of a square is $A = s^2$. Find the area of the square in square inches. Since 1 foot = 12 inches, the area is 12^2 square inches.

The circle has a diameter of 8 inches. The radius is half of the diameter, or 4 inches. The formula for the area of a circle is $A = \pi r^2$. Substitute 4 for r, the radius of the circle, to find the area of the circle.

The area of the circle is $4^2\pi$ square inches.

Subtract the area of the circle from the area of the square to find the area of the shaded part. $12^2 - 4^2\pi$.

Question 56 (page 236)

D Correct. The ladder forms a right triangle with the building and the ground. In a right triangle, $a^2 + b^2 = c^2$, where a and b are the legs of the triangle and c is its hypotenuse. Choice D states the Pythagorean Theorem in words.

Question 57 (page 236)

C Correct. To solve this problem, you do not need to know the surface areas of the two boxes, just the ratio of their areas. If the dimensions of a figure are dilated by a scale factor, then the surface area of the dilated figure will change by the square of that scale factor. If the dimensions of the box are doubled (times 2), then the surface area of the larger box will be 2^2, or 4 times the surface area of the smaller box. Their areas are in the ratio of $\frac{1}{4}$. Use this ratio to compare the cost of covering the boxes.

$$\frac{\text{total cost to cover small box}}{\text{total cost to cover large box}} = \frac{1}{4} \cdot \frac{\$0.12}{\$0.04}$$

$$= \frac{\$0.12}{\$0.16}$$

If the smaller box had an area of 1 ft^2, then it would cost $0.12 to cover. If the smaller box had an area of 1 ft^2, then the larger box would have an area of 4 ft^2. It would cost 4($0.04), or $0.16, to cover.

If the smaller box had an area of 2 ft^2, then it would cost 2($0.12) = $0.24 to cover. If the smaller box had an area of 2 ft^2, then the larger box would have an area of 8 ft^2. It would cost 8($0.04), or $0.32, to cover.

Therefore, it will cost more to cover the larger box.

Question 58 (page 236)

C Correct. The pattern in the sequence of numbers depends on the term's position in the sequence. The first term in the sequence has a numerator of 1 and a denominator of 1^2, or 1. The second term has a numerator of 2 and a denominator of 2^2, or 4. The third term has a numerator of 3 and a denominator of 3^2, or 9. The fourth term has a numerator of 4 and a denominator of 4^2, or 16.

The fifth term has a numerator of 5 and a denominator of 5^2, or 25. The numerator equals the term's position in the sequence, while the denominator equals the square of the term's position in the sequence.

Question 59 (page 237)

B Correct. All the triangles except triangle S are right triangles. The lengths of their sides satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The lengths of the sides of triangle S do not satisfy the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$
$$10^2 + 30^2 \stackrel{?}{=} 32^2$$
$$100 + 900 \stackrel{?}{=} 1{,}024$$
$$1{,}000 \neq 1{,}024$$

Triangle S does not belong in this group.

Question 60 (page 237)

D Correct. To find the amount Kevin spent on the pizzas, subtract the cost of the soda from the total amount spent.

$$70 - 2.50 = 67.50$$

Kevin spent $67.50 on pizza. One way to solve this problem would be to make a table that shows the cost of the different combinations of medium and large pizzas that total 5 pizzas all together.

Number of Pizzas			Cost
Medium	Large	Total	
0	5	5	0 • 12.50 + 5 • 15 = $75
1	4	5	1 • 12.50 + 4 • 15 = $72.50
2	3	5	2 • 12.50 + 3 • 15 = $70.00
3	2	5	3 • 12.50 + 2 • 15 = $67.50
4	1	5	4 • 12.50 + 1 • 15 = $65.00
5	0	5	5 • 12.50 + 0 • 15 = $62.50

The only combination of five pizzas that totals $67.50 is 3 medium and 2 large pizzas.

Social Studies

Overview of the Texas Assessment of Knowledge and Skills (TAKS) Tests

The TAKS tests are administered throughout the state of Texas to measure academic achievement. Satisfactory performance on the Grade 11 exit level TAKS tests is a requirement for receiving a high school diploma. The Grade 8 social studies TAKS test includes some topics that are tested again in Grade 10 and Grade 11. All questions on the social studies TAKS tests use the multiple-choice format.

Purpose of This Book

This study guide is designed to help strengthen your understanding of the knowledge and skills tested on the Grade 8 TAKS tests. **It does not discuss every topic eligible for testing.** Example test questions and practice test questions provide insight into the kinds of questions you may be required to answer on the actual test. This study guide is organized into three sections: reading, mathematics, and social studies. This is the social studies section.

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Organization of the Social Studies Section

The social studies section of this study guide is organized according to the five objectives that are covered on the TAKS social studies assessments.

- The first part covers Objective 1, which requires you to demonstrate an understanding of issues and events in U.S. history. This objective includes topics such as important dates in early American history and the roles that George Washington and Thomas Jefferson played in the American Revolution.
- The second part covers Objective 2, which requires you to demonstrate an understanding of geographic influences on historical issues and events. This objective includes topics such as how maps, charts, and graphs communicate geographic information and how geographic factors influenced major events in U.S. history.
- The third part covers Objective 3, which requires you to demonstrate an understanding of economic and social influences on historical issues and events. This objective includes topics such as the development of the free-enterprise system and the importance of technological innovations in U.S. history.
- The fourth part covers Objective 4, which requires you to demonstrate an understanding of political influences on historical issues and events. This objective includes topics such as the development of representative government in colonial America and fundamental principles of the U.S. Constitution and the Bill of Rights.
- The fifth part covers Objective 5, which requires you to use critical-thinking skills to analyze social studies information.

This objective includes topics such as how to interpret written and visual sources of social studies information.

In addition, throughout this guide you will find activities and sample questions that will help you strengthen your knowledge and skills in social studies.

Finding More Information About Social Studies

To refresh and add to your knowledge of social studies, you may want to review your social studies textbooks and class notes. You can also find interesting and helpful information in books in your school or local library and on websites on the World Wide Web. Additionally, the Texas Education Agency has published a *TAKS Information Booklet for Social Studies*, which contains useful information about the tests and more sample questions. You can find a copy of the information booklet at your school or on the TEA website (www.tea.state.tx.us).

Objective 1

The student will demonstrate an understanding of issues and events in U.S. history.

What Major Events Led to the Founding and Growth of the United States?

Objective 1 covers major issues and events of the early history of the United States. It includes the period from the founding in 1607 of Jamestown, the first permanent British settlement in the New World, to the end of Reconstruction in 1877. You will need to be familiar with some major topics, including the American Revolution, westward expansion, and the Civil War. The following paragraphs go into more detail about the information you need to know to answer questions based on Objective 1.

Important Dates

Objective 1 requires you to answer questions about the significance of the following dates in American history: 1607, 1776, 1787, 1803, and 1861–1865. While many events occurred during these years, you will be tested on only the most important ones. Let's review the significance of the dates:

1607

- The first permanent English settlement in North America is founded at Jamestown, Virginia.

1776

- The Declaration of Independence, written mainly by Thomas Jefferson, is signed on July 4.
  The Declaration of Independence marks the colonies' formal separation from Great Britain and the establishment of the United States.

1787

- The Constitutional Convention meets in Philadelphia, Pennsylvania, and drafts the U.S. Constitution. The Constitution creates the basic structure of the federal government.

1803

- President Thomas Jefferson approves the purchase of the Louisiana Territory from France. The Louisiana Purchase doubles the size of the United States and encourages westward expansion.

1861–1865

- **1861**: Southern states establish the Confederate States of America. Abraham Lincoln is inaugurated as president of the United States. The Civil War begins a short time later when Confederate forces bombard Fort Sumter in South Carolina.
- **1863**: President Lincoln issues the Emancipation Proclamation, which frees slaves in areas controlled by the Confederacy. Also, in one of the war's major turning points, Union forces defeat Confederate forces at the Battle of Gettysburg in Pennsylvania.
- **1865**: Confederate general Robert E. Lee surrenders to Union general Ulysses S. Grant at Appomattox Court House, Virginia. President Lincoln is assassinated in Washington, D.C., a few days later.

Colonization

Objective 1 also requires you to know why Europeans established colonies in North America. Some important factors in the founding of the American colonies include the quest for religious freedom, the pursuit of economic gain, and rivalries between European countries.

- **Religious Freedom:** Groups in search of religious freedom founded several colonies. One of these colonies was Massachusetts, which was founded by a group called the Puritans. The Puritans sought to separate from the Church of England. Religious freedom was also an important factor in the founding of Pennsylvania. William Penn, a Quaker, founded Pennsylvania on the principle of religious toleration, or acceptance of most religions.
- **Economic Gain:** Some colonies were founded by groups of businesspeople who wanted an opportunity to make money. Virginia, the first English colony in North America, is the most famous example of an American colony that was formed as a business venture. Tobacco became Virginia's main source of wealth. South Carolina also was founded for business purposes. Rice farming became South Carolina's main source of wealth.
- **European Rivalries:** The most powerful countries of Europe saw colonies as a way to extend their economic and political power. Competition between European countries sometimes spilled over into the Americas. For example, England captured the Dutch colony of New Netherland in 1664 and renamed it New York.

The American Revolution

The next subject that Objective 1 requires you to know about is the American Revolution. In 1776 American colonists declared their independence from Great Britain. Two of the main reasons they took this action were British economic restrictions and a lack of political representation.

- **Economic Restrictions:** During the colonial period, European leaders followed an economic theory called **mercantilism**. According to the idea of mercantilism, the key to a country's power was its wealth, and a country could best maintain its wealth by strictly controlling the trade of its colonies. In the 1760s Great Britain began to enforce a set of mercantilist trade laws. These laws taxed colonial trade and prohibited the colonies from trading directly with any country except Great Britain. These restrictions angered many colonists.
- **Lack of Political Representation:** The colonists did not have the right to elect representatives to Parliament, the British legislature. When Parliament placed a series of taxes on the colonies, including the Sugar Act (1764), the Stamp Act (1765), and the Townshend Acts (1767), colonists became increasingly angry. They argued that it was unfair for them to be taxed by the British government without their consent. This argument was summed up by the phrase "No taxation without representation!"

The Boston Massacre took place in 1770, when British troops fired on a group of colonists, killing five. The incident added to American resentment of British rule.

The colonists' dissatisfaction with Great Britain eventually led to armed conflict. In 1775 the Revolutionary War began with the Battles of Lexington and Concord in Massachusetts. The last major battle of the war was in 1781, when the British surrendered to American forces at Yorktown, Virginia. The war officially ended in 1783 with the signing of the Treaty of Paris. Some of the main reasons for the American victory in the Revolutionary War were America's physical distance from Great Britain, George Washington's leadership of the Continental army, and the consequences of the American victory at the Battle of Saratoga.

- **Distance from Great Britain**: The United States and Great Britain were separated by thousands of miles of ocean. It took time to send news, orders, supplies, and troops across the Atlantic. This complicated the British war effort.
- **The Continental Army**: The Continental army, commanded by George Washington, was the main American military force. Washington's main strategy was to preserve his army and keep it from being defeated decisively. He chose to engage British forces when the Continental army had a clear advantage.
- **The Battle of Saratoga**: The Battle of Saratoga, which took place in New York in 1777, resulted in a major American victory. This victory convinced the French government that the Americans could win the war. France, eager to weaken the British, began to send supplies, money, and troops to help the Continental army.

Now let's look at a practice question that asks about some of the information you just read.

The Battles of Lexington and Concord were significant because they —

A halted all British attacks against the colonists

B were the first battles of the American Revolution

C proved to the British that the Americans were unbeatable

D demonstrated the superiority of the British army

Explanation of answer choices

A Incorrect. The Battles of Lexington and Concord, which took place in Massachusetts in April 1775, marked the beginning of the American Revolution.

B Correct. The Battles of Lexington and Concord were the first battles of the American Revolution. These battles showed Great Britain that additional military force would be necessary if it hoped to put down the colonial uprising.

C Incorrect. The Americans defeated the British in the Battles of Lexington and Concord largely by using hit-and-run tactics. However, the British were better equipped and better trained than their American opponents, and British military leaders remained confident they could put down the uprising.

D Incorrect. Despite their military advantages over the Americans, the British were defeated in the Battles of Lexington and Concord.

During the American Revolution the United States formed its first government under a document called the Articles of Confederation. An important accomplishment of this government was the passage of the **Northwest Ordinance** in 1787. The Northwest Ordinance set up the procedure for organizing new states from the land northwest of the Ohio River. This encouraged Americans to move westward in the decades that followed.

Despite this accomplishment, the Articles of Confederation did not give much power to the national government. In 1787 the states sent delegates to Philadelphia to discuss amending, or changing, the Articles. This meeting, which George Washington presided over, became known as the Constitutional Convention because the delegates decided to replace the Articles with a new document, the U.S. Constitution.

The Creation of the U.S. Constitution

Objective 1 requires you to understand the issues surrounding the creation of the U.S. Constitution. Three of the most important of these issues were the organization and power of the national government, the representation of states in Congress, and slavery.

- **Organization and Power of the National Government:** Most delegates to the Constitutional Convention agreed on the need for a stronger national government. However, there was disagreement on how this government should be organized. The delegates also disagreed on how power should be distributed between the national government and the state governments.

  Eventually the delegates agreed to have a strong national government with three branches: executive, legislative, and judicial. However, a number of important powers were left to the states. (You will learn more about this arrangement in the section on Objective 4.)

- **Representation:** Delegates also disagreed about how states should be represented in the legislative branch of the national government, or Congress. Delegates from states with small populations wanted each state to have the same number of legislators. States with large populations wanted representation to be based on population. This plan would have given some states more legislators than others.

  The representation issue was solved by an agreement known as the **Great Compromise**. The Great Compromise specified that Congress would be composed of two separate bodies: the Senate and the House of Representatives. In the Senate, each state would have two representatives. In the House of Representatives, representation would be based on population.

- **Slavery:** Although some delegates from northern states hoped to abolish slavery, they knew that trying to do so would have prompted many southern delegates to walk out of the convention. Instead, a debate arose over how slaves should be counted as part of a state's population for purposes of representation in the House of Representatives. Northern delegates did not want slaves to be counted at all. Southern delegates wanted slaves to be counted the same as free people so that the South would have as many representatives as possible.

  This issue was solved by an agreement known as the **Three-Fifths Compromise**. The Three-Fifths Compromise specified that three-fifths of a state's slaves would be counted in the state's population for purposes of representation. The delegates also agreed to forbid Congress from outlawing the importation of slaves to the United States until 1808.

The Constitution was sent to the states for ratification, or approval, in late 1787. Supporters of the Constitution were known as **Federalists**, while opponents were known as **Anti-Federalists**. Federalists believed that the country needed a national government that was more powerful than the state governments. Anti-Federalists argued that the Constitution gave too much power to the national government. They believed the states should hold more power, as they had under the Articles of Confederation. Anti-Federalists also called for more specific protections of individual rights.

The most famous writings to come out of this debate were *The Federalist Papers*, a series of essays written by Alexander Hamilton, James Madison, and John Jay. These essays explained and defended the Constitution and encouraged Americans to support the new form of government. The Constitution was ratified in 1788 after an agreement was reached to add more protections of individual rights. Ten constitutional amendments known as the Bill of Rights provided these protections. With the new Constitution in place, George Washington was sworn in as the first president of the United States in 1789. You will learn more about the U.S. Constitution and the Bill of Rights in the section on Objective 4.

Now let's look at a practice question that asks about some of the information you just read.

Which of the following statements about the Constitutional Convention of 1787 is true?

A A record of each day's discussions and debates from the convention was printed in the following day's newspaper.

B The original purpose of the convention was to sign the treaty ending the American Revolution.

C Thomas Jefferson and John Adams presided over the convention.

D The original purpose of the convention was to revise the Articles of Confederation.

Explanation of answer choices

A Incorrect. The discussions held by the Constitutional Convention were kept secret to encourage extensive and open discussion among the delegates.

B Incorrect. The Treaty of Paris, which officially ended the American Revolution, was signed by officials of the United States and Great Britain in 1783.

C Incorrect. Neither Thomas Jefferson nor John Adams attended the Constitutional Convention. Jefferson was serving as a diplomat in France. Adams was serving as a diplomat in Great Britain.

D Correct. The Constitutional Convention was assigned the task of revising the Articles of Confederation. Instead, the delegates created a whole new governmental structure.

Political Issues Faced by the New Nation

Objective 1 also requires you to know about major political developments that occurred during the late 1700s and early 1800s. Two of these developments were the formation of political parties and tensions with European powers.

- **Formation of Political Parties**: In the mid-1790s two political parties, the Federalists and the Democratic-Republicans, formed. The Federalists, led by Secretary of the Treasury Alexander Hamilton, wanted a strong national government. Federalists wanted the government to support the development of trade and industry and the creation of a national bank. The Democratic-Republicans, led by Thomas Jefferson, wanted a weaker national government. They opposed the Federalists' plan to increase the responsibilities of the national government.
- **Tensions with European Powers**: Foreign affairs presented several challenges to the new country. During the 1790s, conflict between Great Britain and France led both countries to seize American cargo ships. This eventually contributed to the outbreak of war between the United States and Great Britain in 1812. The war ended in 1815 without an obvious winner. A few years later, in 1823, President James Monroe issued the **Monroe Doctrine**, which declared the Western Hemisphere off-limits to further colonization by European powers.

After the War of 1812, the United States entered an era in which more and more Americans participated in politics. The rivalry between the Federalists and Democratic-Republicans died down, and in 1828 a new political party, the Democratic Party, formed. The Democrats were led by President Andrew Jackson. Politics became more democratic during this time as the Democratic Party encouraged more Americans to vote.

Westward Expansion

The period after the War of 1812 was also a time when an increasing number of Americans moved westward. Objective 1 requires you to know about the factors that encouraged westward expansion. Three of the main factors were economic growth, territorial expansion, and the concept of Manifest Destiny.

- **Economic Growth:** The Industrial Revolution, the spread of cotton farming, and advances in transportation all contributed to westward expansion. New England textile factories were the earliest industries in America. These factories made cloth from cotton grown in the South. The increasing demand for cotton helped make cotton farming more profitable. This encouraged more and more southern farmers to move west to grow cotton. The building of roads and canals and the development of the railroad locomotive and the steamboat made it faster and cheaper to move people and goods. These advances—sometimes called the Transportation Revolution—helped connect the economies of different regions of the United States. This, in turn, encouraged westward expansion.

- **Territorial Expansion:** Westward expansion was also promoted by the acquisition of new territories. (The map on page 272 shows these territories.) The most important territorial acquisitions were the Louisiana Purchase (1803), Texas (1845), the Oregon Territory (1846), and the Mexican Cession (1848).
- **Manifest Destiny:** Finally, westward expansion was encouraged by the idea of Manifest Destiny. Manifest Destiny was the belief that it was the mission of the United States to expand across the entire North American continent.

As well as requiring you to know about the causes of westward expansion, Objective 1 requires you to know about the consequences of westward expansion. Two major consequences were Indian Removal and the Mexican War.

- **Indian Removal:** In 1830 the U.S. government passed the Indian Removal Act. This act called for Native Americans living in the southeastern United States to be resettled west of the Mississippi River. Many Native Americans died during the relocation process. The most well-known example of removal was the forced march of the Cherokee Indians in 1838 and 1839, which became known as the **Trail of Tears**. This journey resulted in the death of thousands of Cherokee. The removal of Native Americans allowed white settlers to take over land that Native Americans had occupied.
- **The Mexican War:** The Mexican War started in 1846 and was related to the U.S. annexation of Texas, which had been part of Mexico. The war ended in 1848 when the United States and Mexico signed the Treaty of Guadalupe Hidalgo. This treaty granted the United States a large portion of northern Mexico, a region that now makes up much of the western United States.

Image courtesy of Woolaroc Museum, Bartlesville, Oklahoma

The Trail of Tears took place in the fall and winter of 1838–1839. The Cherokee did not have enough food or warm clothing, and thousands died during the journey.

Now let's look at a practice question that asks about some of the information you just read.

In 1838 and 1839 the Cherokee were forced to relocate to Indian Territory. This forced journey is known as —

A the Nullification Crisis

B the Chisholm Trail

C the Trail of Tears

D the Oregon Trail

Explanation of answer choices

A Incorrect. The Nullification Crisis took place in 1832 and 1833 when South Carolina attempted to ignore a federal tariff and threatened to secede.

B Incorrect. The Chisholm Trail stretched from Texas to Abilene, Kansas, and was a route used for cattle drives.

C Correct. Forced to travel in winter weather with inadequate food supplies, nearly 4,000 Cherokee died on the Trail of Tears.

D Incorrect. The Oregon Trail was an overland route to the western United States.

Sectionalism

Another subject that Objective 1 requires you to know about is the growth of **sectionalism**, or regional conflict between the North and South. Two of the main causes of sectionalism were disagreements over states' rights and the issue of slavery in the western territories.

The dispute over states' rights was based on differing interpretations of how the U.S. Constitution defined federalism. Federalism is the sharing of power between the national government and state governments. Americans disagreed over what that exact balance of power should be. Many northerners argued that the federal government had broad powers under the Constitution. On the other hand, many southerners argued that the federal government's powers under the Constitution were limited. One example of the conflict over states' rights was the **Nullification Crisis**.

The Nullification Crisis began in 1832 after the U.S. Congress placed a tariff, or tax, on imported manufactured goods. The tariff boosted profits for U.S. manufacturers, most of whom were located in the Northeast. At the same time, it raised prices for consumers throughout the country. Because there was very little manufacturing in the South, many southerners felt that the tariff unfairly benefited northerners. (For more information about tariffs, see the section on Objective 3.)

South Carolina responded to the tariff by nullifying it, or declaring it void in South Carolina, and threatening to **secede** from, or leave, the Union. In response, President Andrew Jackson threatened to send troops to South Carolina to enforce the law. In the end, a compromise was reached; Congress lowered the tariff, and South Carolina backed down. However, many southerners continued to distrust the federal government and resent the political power of northerners, and the question of states' rights was left unresolved.

The second major factor in the growth of sectionalism was the issue of slavery in the western territories. Many northerners opposed the westward expansion of slavery. This angered many southerners, who wanted the West to be open to slavery. However, a series of measures passed by Congress kept sectional conflict over slavery under control for several decades. The two most important of these measures were the Missouri Compromise and the Compromise of 1850.

- **The Missouri Compromise (1820):** To maintain a balance between the number of free and slave states, the Missouri Compromise admitted Missouri to the Union as a slave state and Maine as a free state. The Missouri Compromise also banned slavery in territories north of 36º 30´ latitude. Territories south of this line were open to slavery.
- **The Compromise of 1850:** In 1846 northern Democrats endorsed a proposal to ban slavery from all territories that would be gained from the Mexican War. Southerners were outraged by this proposal, and it failed to pass in Congress. However, Kentucky senator Henry Clay proposed a compromise that Congress did approve. The Compromise of 1850 contained a number of different provisions, including the admission of California to the Union as a free state and the passage of the Fugitive Slave Law. This law required the capture and return of runaway slaves, even in the North.

Compromises like the ones just described kept the slavery issue from exploding for a number of years. However, in 1854 Congress passed the Kansas-Nebraska Act, which allowed the territories of Kansas and Nebraska to vote on whether to allow slavery. The act outraged many northerners because these territories should have been free territories under the terms of the Missouri Compromise.

The Kansas-Nebraska Act worsened sectional tensions and helped lead to the Civil War. Three other events that continued this trend were the *Dred Scott* decision, John Brown's Raid, and the election of Abraham Lincoln as president in 1860.

- ***Dred Scott* Decision:** The U.S. Supreme Court decided in 1857 that slaves were property and could not become free by moving to a free state or territory. This decision pleased southerners and angered northerners.

- **John Brown's Raid:** In 1859 John Brown tried to start a slave rebellion in Virginia. Brown was unsuccessful. He and many of his followers were captured and hanged for treason. Brown's actions alarmed many southerners, while abolitionists saw Brown as a hero.
- **Election of Abraham Lincoln:** Abraham Lincoln was elected president in 1860. Lincoln was a member of the Republican Party, which had been formed in 1854 with the main goal of stopping the westward spread of slavery. Although Lincoln had promised not to interfere with slavery where it already existed, many southerners believed that Lincoln planned to abolish slavery.

Image courtesy of the Library of Congress

In 1859 John Brown and his followers attacked a federal arsenal in Harpers Ferry, Virginia. Their plan was to capture rifles to arm slaves for a mass rebellion. Government troops killed or captured all the raiders.

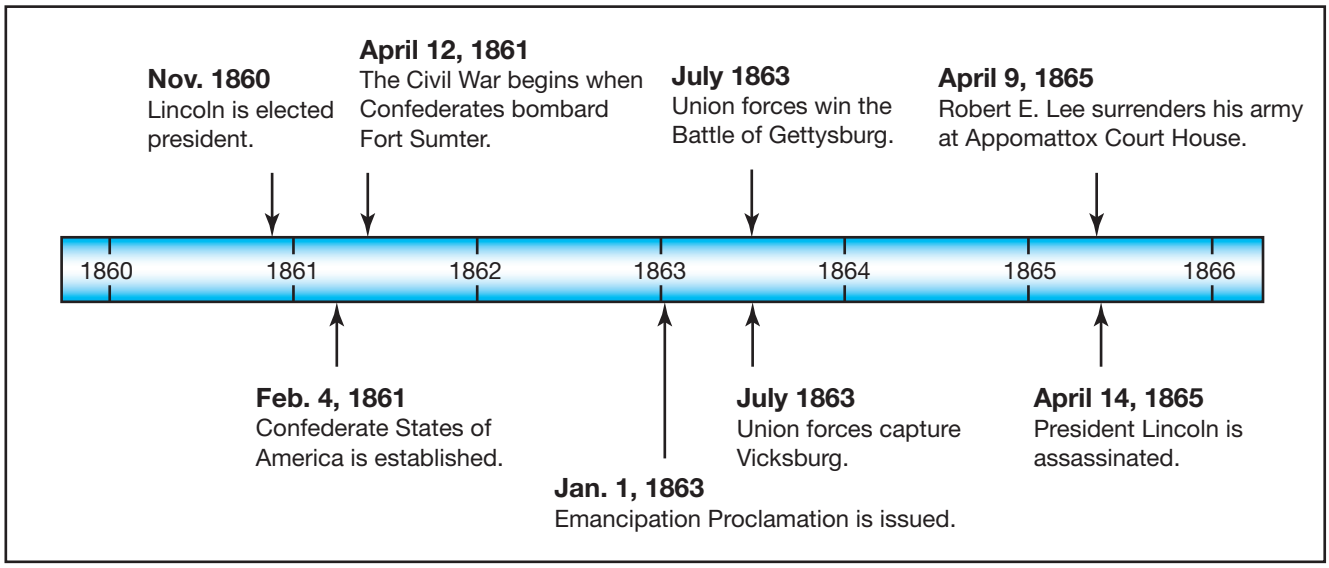
The Civil War

Following Lincoln's election, southern states seceded from the United States and formed the Confederate States of America. The Civil War began on April 12, 1861, when Confederate forces bombarded Fort Sumter in South Carolina. After four years of warfare, the Union finally defeated the Confederacy. Objective 1 requires you to know about factors that contributed to this outcome.

Both the North and the South had skilled political and military leaders. The Union's most important leaders were President Abraham Lincoln and General Ulysses S. Grant. The Confederacy's most important leaders were President Jefferson Davis and General Robert E. Lee. However, a number of factors gave the advantage to the North: a larger population, more economic resources, and the superior leadership of Abraham Lincoln.

- **Population and Economic Resources**: The North's larger population and industrial economy allowed it to field a larger army and produce more guns and other military supplies than the South. In addition, the North had more factories and controlled more miles of railroad.
- **Abraham Lincoln**: Lincoln helped lead the Union to victory through his careful management of the North's economy and military. He also gave northerners a sense of purpose through his moving speeches. Some examples are his first and second inaugural addresses and the Gettysburg Address. Lincoln also began the process of abolishing slavery with the release of the Emancipation Proclamation in 1863. Lincoln's words and actions defined the Civil War as a war to preserve the Union and promote liberty and equality.

Civil War Time Line



Now It's Your Turn

Question 1

Who served as president of the United States during the Civil War?

A Thomas Jefferson

B Abraham Lincoln

C John Adams

D James Polk

Answer Key: page 322

Question 2

Which list shows the events in the order they occurred?

A Declaration of Independence
Constitutional Convention
Missouri Compromise

B Missouri Compromise
Declaration of Independence
Constitutional Convention

C Missouri Compromise
Constitutional Convention
Declaration of Independence

D Declaration of Independence
Missouri Compromise
Constitutional Convention

Answer Key: page 322

Question 3

How did the U.S. Constitution address the issue of how slaves would be counted for purposes of representation in the House of Representatives?

A It stated that slaves would not be counted in a state's population.

B It stated that slaves would be counted only in southern states.

C It stated that three-fifths of a state's slaves would be counted in the state's population.

D It stated that slaves would be counted the same as free people.

Answer Key: page 322

Question 4

Thomas Paine's purpose for writing the political pamphlet *Common Sense* was to —

A show the British public that the American colonists had a powerful army

B point out weaknesses in the Articles of Confederation

C lift American spirits after a series of military defeats

D persuade colonists to support the cause of American independence

Answer Key: page 322

Question 5

The Treaty of Paris signed in 1783 —

A awarded Florida to Spain

B ended the Revolutionary War

C established the terms of the Louisiana Purchase

D prohibited settlement west of the Appalachian Mountains

Answer Key: page 322

Question 6

Which of the following contributed to the westward expansion of the United States?

A European warships attacked the eastern seaboard, forcing citizens to move westward for safety.

B Most of the land east of the Mississippi River was not suitable for farming.

C Many Americans felt it was their right and duty to move westward.

D The standard of living on the Pacific Coast was very high, so many people wanted to move there.

Answer Key: page 322

Question 7

Who served as president of the Confederate States of America?

A Jefferson Davis

B Ulysses S. Grant

C Robert E. Lee

D Abraham Lincoln

Answer Key: page 322

Question 8

Which of the following statements best explains the significance of the Northwest Ordinance of 1787?

A It created a process for U.S. territories to become states.

B It limited settlements in the Northwest Territory.

C It established a system for the Northwest Territory to be controlled by Great Britain.

D It confirmed Native American land rights north of the Ohio River.

Answer Key: page 323

Question 9

Which battle took place in 1781 and was the last major battle of the Revolutionary War?

A Battle of Lexington

B Battle of Trenton

C Battle of Quebec

D Battle of Yorktown

Answer Key: page 323

Question 10

Anti-Federalists agreed to ratify the U.S. Constitution on the condition that which of the following be added?

A A second house of the legislature

B A bill of rights

C The right of citizens to vote

D A ban on slavery

Answer Key: page 323

Objective 2

The student will demonstrate an understanding of geographic influences on historical issues and events.

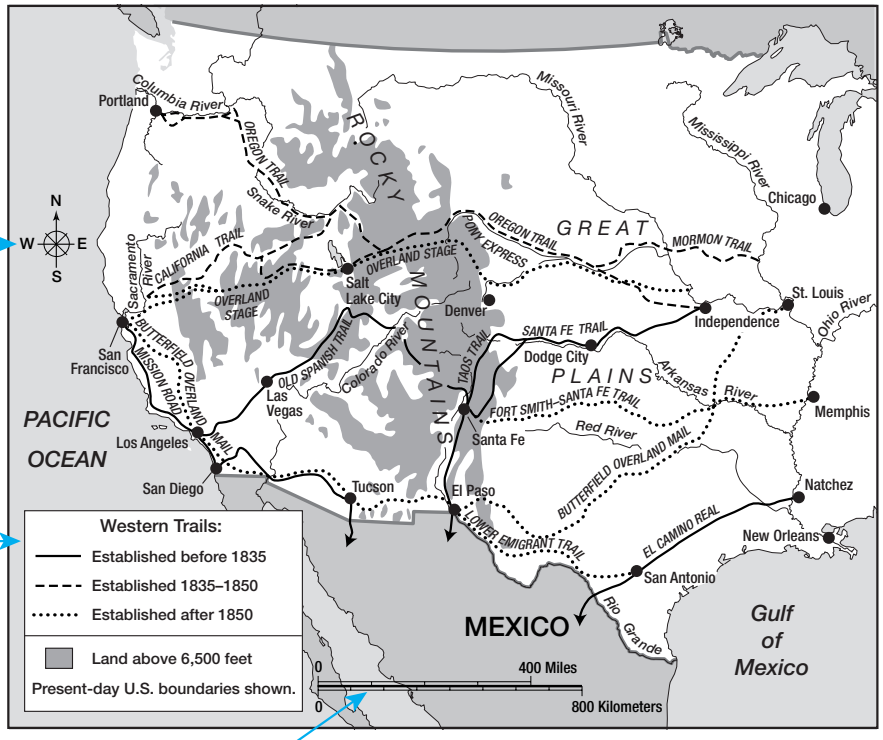
How Did Geography Influence the Development of the United States?

Objective 2 focuses on geographic influences on early American history. Geography is the study of the earth, its features, and the ways humans interact with their environment. To answer questions based on Objective 2, you must be able to read and interpret maps, tables, and graphs. You must also know about the territories that were acquired in the 1700s and 1800s to form the United States, the location and the characteristics of U.S. regions, and the impact of geography on major events in early American history.

Understanding Maps, Tables, and Graphs

The first skill that Objective 2 asks you to demonstrate is how to read maps, tables, and graphs. Let's begin by discussing maps. A map is a visual representation of a given area. Maps provide information about the land's physical features, such as mountains, rivers, and coastlines.

Title → **Selected Western Trails, 1800s**



Key → Western Trails

Scale → 0 400 Miles / 0 800 Kilometers

Labels → Rocky Mountains, Great Plains, Pacific Ocean, Gulf of Mexico, Mexico, rivers, trails, cities

Source: Facts on File *Atlas of American History*, 1995

Maps can also provide information about human activities, such as the construction of railroads or the types of crops that are grown in a region. The map on the previous page shows the location of the main overland trails that early American settlers used to migrate west.

The first step in reading a map is looking at the title. The title will tell you the theme or purpose of the map. Next look for a compass rose and scale. The **compass rose** tells you which way is north, south, east, and west on the map. The **scale** tells you the relationship between distance on the map and actual distance on Earth.

Next look at the map's key, or legend. The **key** explains the meaning of the different symbols used on the map. The key to this map uses different types of lines to show when different trails were established. For example, it shows you that the Oregon Trail, which runs from Independence, Missouri, to Portland, Oregon, was established between 1835 and 1850. The Lower Emigrant Trail, which runs between San Antonio and El Paso, was established after 1850. Using the key, you can also identify areas that are more than 6,500 feet above sea level, such as the Rocky Mountains.

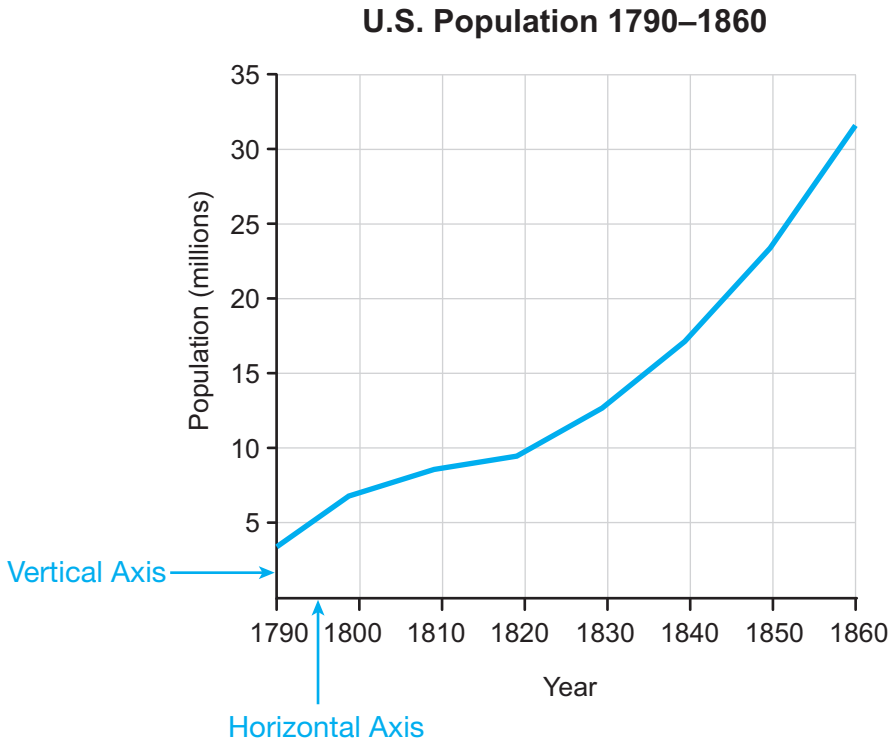
Finally, remember that not all the information on a map is in the key. You should also look for labels on the map itself. Common examples of things that may be labeled are the names of countries, cities, rivers, and mountains.

Objective 2 also requires you to interpret information from tables and graphs. Tables are used to organize and summarize information by sorting it into columns and rows. Graphs present statistical data using points, lines, bars, or other visual forms.

Although tables and graphs are different from maps, all three serve the same basic purpose: to communicate information visually.

Let's practice interpreting graphs by using the graph below. It shows population growth in the United States from 1790 through 1860.

The lines labeled "Year" and "Population" are called the axes of the graph. The line labeled "Year" is the horizontal axis. The line labeled "Population" is the vertical axis. The population for every tenth year is plotted as a point on the graph, and the points are connected to form the sloping line. When the statistics are presented this way, you can easily see a trend: U.S. population grew over time. You can also use the graph to find the population for a specific year. For example, in 1810 the population of the United States was a little over 7 million. In 1840 the U.S. population had grown to just over 17 million. Thus, the population of the United States grew by about 10 million in the 30 years between 1810 and 1840. You will learn more about maps and tables in the section on Objective 5.



Source: *Encyclopedia of American History*, 1996

Now let's look at a practice question that asks you to read a table.

Use the table and your knowledge of social studies to answer the following question.

Approximate Population of Selected U.S. Cities, 1800–1830

	1800	1810	1820	1830
New York	61,000	96,000	124,000	203,000
Philadelphia	41,000	54,000	64,000	80,000
Baltimore	27,000	47,000	63,000	81,000
Boston	25,000	34,000	43,000	61,000
Charleston	19,000	25,000	25,000	30,000

Source: U.S. Census Bureau

According to the table, the approximate population of Philadelphia in 1820 was —

A 124,000

B 80,000

C 64,000

D 25,000

Explanation of answer choices

A Incorrect. The table shows that the population of New York, not Philadelphia, was approximately 124,000 in 1820.

B Incorrect. The table shows that Philadelphia's population was approximately 80,000 in 1830, not in 1820.

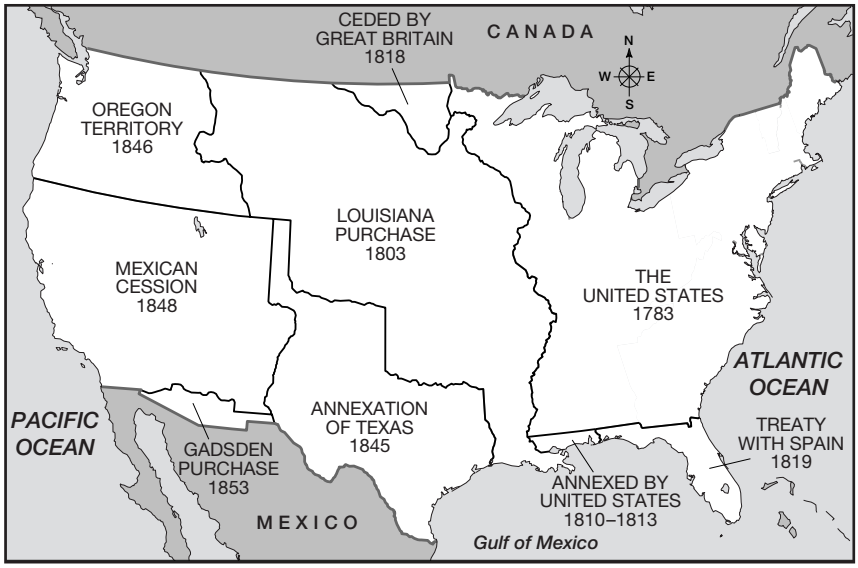
C Correct. The table shows that Philadelphia's population in 1820 was approximately 64,000.

D Incorrect. The table shows that the population of Charleston, not Philadelphia, was approximately 25,000 in 1820.

Territorial Acquisitions

By the 1850s the political boundaries of the United States had expanded across North America to the Pacific Ocean. Objective 2 requires you to identify areas that the United States acquired during this expansion. The map below shows these areas and the dates when they were acquired.

Territorial Acquisitions of the United States



You will be responsible for knowing about the most important territorial acquisitions, including land acquired through the Treaty of Paris, the Louisiana Purchase, Florida, Texas, the Oregon Territory, the Mexican Cession, and the Gadsden Purchase.

- **Treaty of Paris (1783):** Great Britain recognized U.S. land rights west of the original 13 colonies and east of the Mississippi River as part of the treaty that ended the Revolutionary War.
- **Louisiana Purchase (1803):** The United States bought the territory of Louisiana from France in 1803. The Louisiana Purchase more than doubled the size of the United States.
- **Florida (1819):** Florida was acquired by the United States through a treaty with Spain in 1819.
- **Texas (1845):** In 1836 the Republic of Texas gained its independence from Mexico. In 1845 the United States annexed Texas, making it the 28th state in the Union.
- **Oregon Territory (1846):** The Oregon Territory became part of the United States after Great Britain gave up its claim to the area in 1846.
- **Mexican Cession (1848):** The Mexican Cession was acquired from Mexico through the Treaty of Guadalupe Hidalgo, which ended the Mexican War. It includes much of what is now the southwestern United States.
- **Gadsden Purchase (1853):** The United States bought what is now part of New Mexico and Arizona from Mexico in 1853.

Regions of the United States

The United States is composed of regions with different characteristics. These regional characteristics have played a major role in the nation's historical development. For example, cotton farming developed in the South in part because of the region's warm, sunny climate and rich soil.

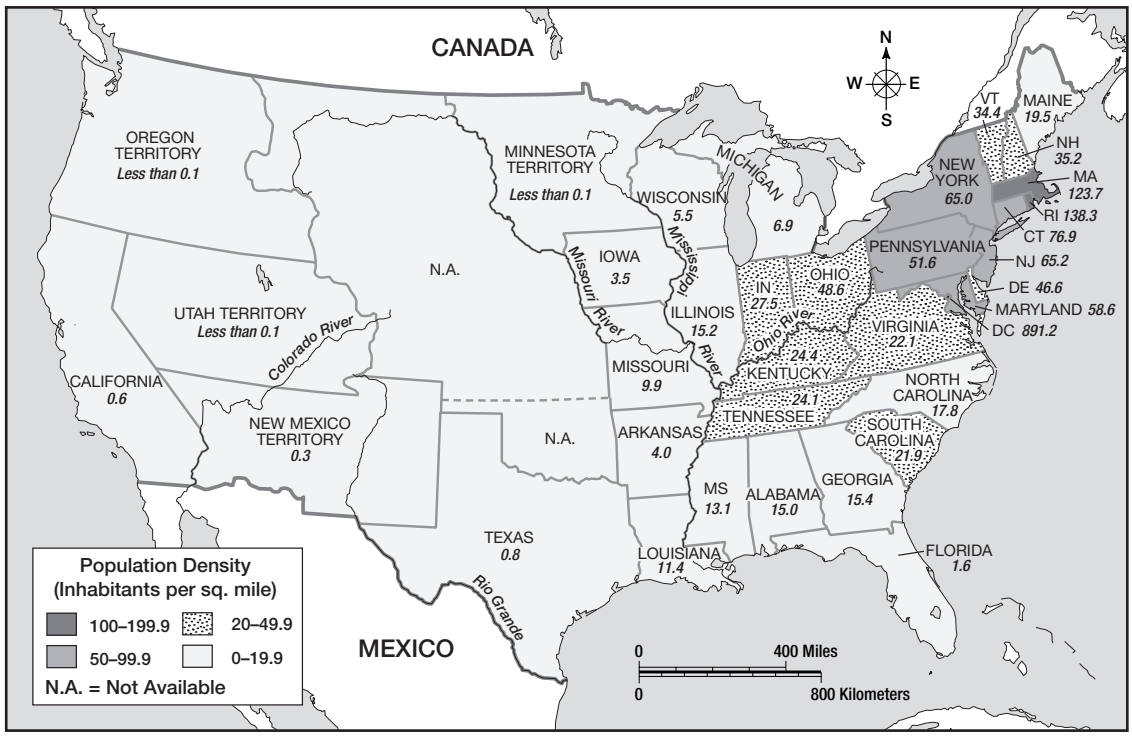
Objective 2 requires you to know the location and characteristics of the different regions of the United States. At the time of the Civil War, the three main regions of the United States were the North, South, and West. Each of these main regions was made up of smaller regions—New England, for instance, or the Deep South. The following paragraphs describe some of the characteristics of the North, South, and West and mention smaller regions contained in each.

- **The North:** The North included the area north of the Ohio River and east of the Mississippi River. Its climate was generally cooler than that of the South. The North was the most heavily populated region of the United States at the time of the Civil War. It also had the most cities and towns. In some parts of the North, such as New England, the cool climate and rocky soil were not well suited for agriculture. In other areas, such as the Mid-Atlantic states of New York and Pennsylvania, farming was an important activity—the main crops were corn and wheat. The North led the country in manufacturing. Other important economic activities included fishing and shipbuilding. The North became the home of most European immigrants to the United States.
- **The South:** At the time of the Civil War, the South included the area south of the Ohio River, as well as Arkansas, Louisiana, and Texas. The South had a smaller population than the North. It also had fewer cities and towns. The South's climate was warmer than the North's. This, along with rich soil, made the South well suited for growing a variety of crops. Tobacco was the main crop in the Upper South, for example, while cotton was grown throughout the Deep South. Some crops, especially cotton, were grown on large plantations with African American slaves doing most of the work. There were also many smaller family farms in the South.
- **The West:** At the time of the American Revolution, all the land west of the Appalachian Mountains was considered the West. As the United States expanded, however, definitions of the West changed. For example, states in the Midwest, such as Illinois, Indiana, and Ohio, gradually came to be considered part of the North. Texas was considered part of both the West and the South. At the time of the Civil War, most Americans defined the West as the lightly settled land west of the Mississippi River. It included the Great Plains, the Rocky Mountains, the Southwest, and the Pacific Coast. Because its population was small, the West contained few towns and no large cities. The landforms and climate of the West varied greatly. Important economic activities

included farming, fur trapping, and mining. People from many different ethnic groups, including Native Americans, interacted in the West as more and more settlers moved into the area.

The map below shows the population density of states and territories of the United States in 1850. It shows how the U.S. population was unevenly distributed. Most of the states with the highest population densities were in the North. Southern states generally had lower densities, and states and territories in the West had the lowest densities.

Population Density of U.S. States and Territories, 1850



Source: Facts on File *Atlas of American History*, 1995

Now let's look at a practice question that asks about some of the information you just read.

Many colonists in New England turned from farming to industrial activities in part because —

A they wanted to preserve the old-growth forests

B the winters were cold and the soil was rocky and hard

C they preferred to purchase their food from Native Americans

D farmers in the Mid-Atlantic colonies controlled the sale of cash crops

Explanation of answer choices

A Incorrect. New England colonists made extensive use of the area's forests for building materials and fuel. They had little interest in preserving them.

B Correct. Many colonists in New England turned to industrial activities as a way to make a living in part because the long, cold winters and poor soil made farming very difficult.

C Incorrect. While New England colonists did trade with Native Americans for a small amount of their food, they depended mostly on other food sources.

D Incorrect. The sale of crops from the colonies was controlled by the British government, not by colonists.

Geography's Role in Early American History

Finally, Objective 2 also requires that you understand how geographic factors influenced important events in early American history. One way to understand how geography affected an event is to first identify the geographic characteristics of the place where the event occurred. Then consider how those characteristics might have affected the actions of the people involved in the event.

There are many examples of how geography played a role in early American history. For instance, one of the main reasons for the federal government's removal of Native Americans from the Southeast to the West in the 1830s was that they occupied an area with gold deposits and fertile land. White settlers who wanted the land for mining and farming pressured the government to force the Native Americans living there to leave. Geography also played a role in the outcome of the American Revolution. The distance between the United States and Great Britain made it difficult for the British to communicate with and supply their troops, which weakened the British war effort. Geography also influenced the establishment and growth of cities and towns in early America. Most cities and towns were located on coastlines or near rivers largely because waterways were used to transport people and goods. For example, New Orleans became one of the South's largest cities because its location near the mouth of the Mississippi River made it a major port and business center. Similarly, Boston, New York, and Philadelphia became major cities in the Northeast largely because they were located near the Atlantic Ocean.

Review Activity

This map shows the boundaries of the territories that the United States acquired before the Civil War. Fill in the appropriate blanks with the name of the territory, the country that the United States acquired the territory from, and the date of the territory's acquisition. Use the information on page 272 to check your answers.

The Continental United States

Now It's Your Turn

Question 11

Which port city lies near the mouth of the Mississippi River, making it important for commerce?

A New York City

B Philadelphia

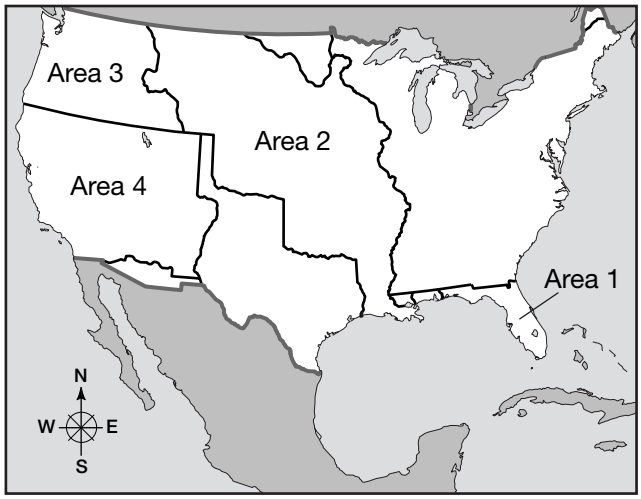
C Charleston

D New Orleans

Answer Key: page 323

Use the map and your knowledge of social studies to answer the following question.

U.S. Territorial Acquisitions



Question 12

Which area did the United States purchase from France in 1803?

A Area 1

B Area 2

C Area 3

D Area 4

Answer Key: page 323

Use the map and your knowledge of social studies to answer the following question.

Selected Revolutionary War Battles

Question 13

The British army was defeated at Yorktown in 1781. This was the last battle of the Revolutionary War. Which location on the map indicates where the battle was fought?

A I

B II

C III

D IV

Answer Key: page 323

Question 14

Most towns and cities in the British colonies in North America were located —

A in mountainous areas where there were deposits of gold and silver

B far from the coast in order to avoid attacks by French and Spanish ships

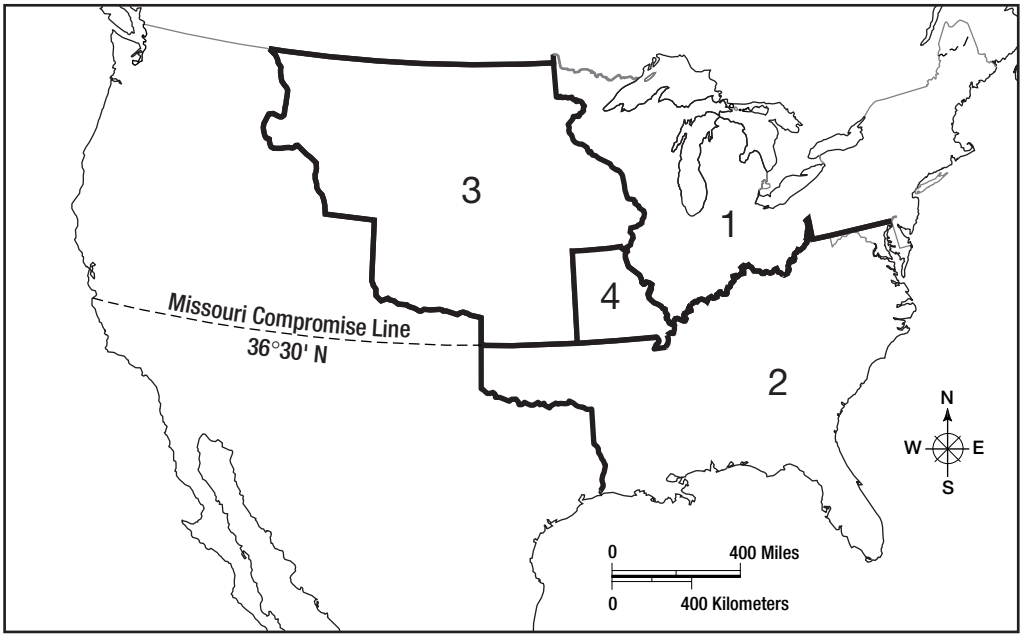
C in deep forests where there were large supplies of timber

D on the coast or along rivers to provide access to transportation

Answer Key: page 324

Use the map and your knowledge of social studies to answer the following question.

The Missouri Compromise, 1820



Question 15

Which area was closed to slavery by the Missouri Compromise of 1820?

A 1

B 2

C 3

D 4

Answer Key: page 324

Objective 3

The student will demonstrate an understanding of economic and social influences on historical issues and events.

How Did Economic and Social Issues Influence the Development of the United States?

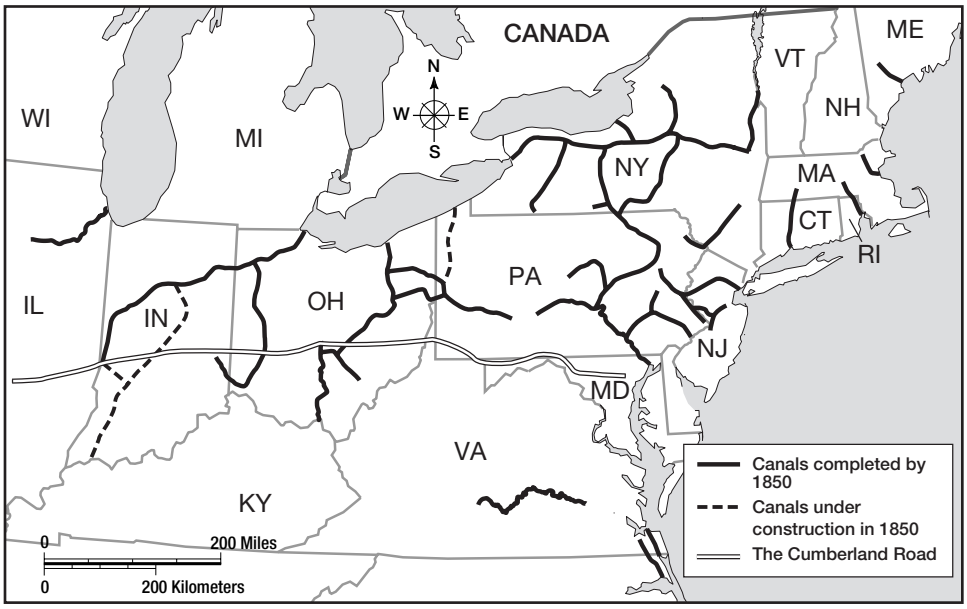
Objective 3 focuses on the influence of economic and social factors on early American history. In the period between the ratification of the U.S. Constitution and the beginning of the Civil War, the United States developed a large and complex national economy. In order to answer questions based on Objective 3, you need to know about the main factors that contributed to this development. You must also be familiar with the various social reform movements that emerged as the national economy took shape. The following paragraphs go into more detail about these topics.

Economic Growth

Objective 3 requires you to know about the factors that led to the rapid economic growth of the United States. Five of the most important factors were the growth of the free-enterprise system, the development of a national economy, industrialization, the expansion of the plantation system in the South, and the introduction of various technological innovations.

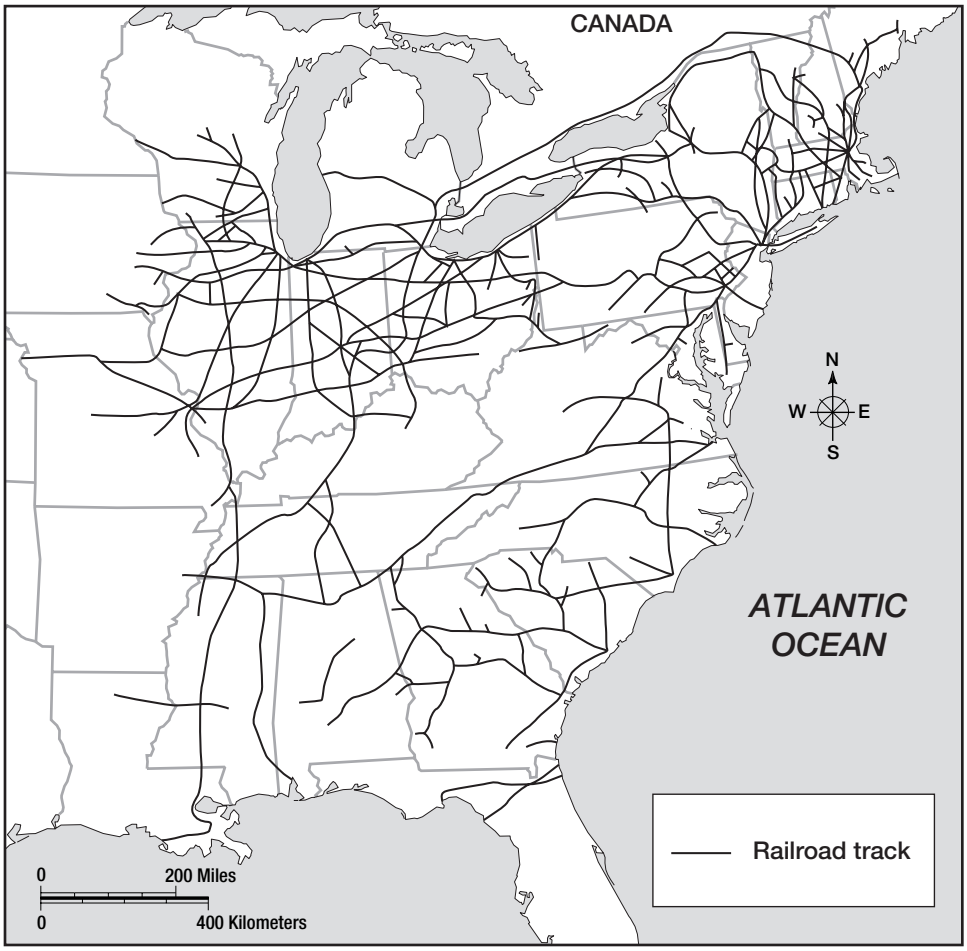
- **Free Enterprise:** One of the most important developments in the early history of the United States was the growth of the **free-enterprise** economic system. This system allowed individuals to participate in business activities with little government interference. The growth of the free-enterprise system encouraged many Americans to start and invest in new businesses.
- **National Economy:** At the end of the American Revolution, most economic activity in the United States took place at the local level or involved trade with Europe. Trade between different regions of the country was just beginning to develop. By 1860, however, economic activities in different regions were connected through an extensive web of trade and commerce. One of the main factors in the creation of this national economy was the construction of roads, canals, and railroads, as well as the development of new forms of transportation such as the steamboat. These advances significantly reduced the amount of time it took to move goods over long distances. Collectively, these advances are sometimes called the Transportation Revolution. The maps on the next page show how advances in transportation connected different regions of the country by the time of the Civil War.

U.S. Canals and the Cumberland Road, 1850



Source: *Atlas of American History*, 1995

U.S. Railroads, 1860



Source: *Atlas of American History*, 1995

- **Industrialization**: Another factor that contributed to the rapid economic growth of the United States was **industrialization**, or the widespread use of machines to produce goods. Industrialization first began in England in the 1700s with the development of water- and steam-powered machinery and the factory system. These innovations were then introduced to the United States by people such as Samuel Slater, a British immigrant who designed the first textile, or cloth production, factory in the United States in 1790. A few years later inventor Eli Whitney contributed to the industrialization process by developing a system of manufacturing that used interchangeable parts. Industrialization made the mass production of goods possible in the United States for the first time.

  Industrialization had a number of important effects on U.S. society. The factory system gradually became the main system for manufacturing goods, especially in the North. Cottage industry, an older system of manufacturing, had involved people working in their homes to create products. Their employers then paid them for what they produced. As the factory system became more common, cottage industry declined in the United States.

  As more factories were established, people's ideas about work began to change. Factory workers had to arrive at their jobs on time and work a precise number of hours. Farmers and craftspeople had always worked hard, but they had enjoyed the freedom to set their own hours. In addition, factory workers usually worked for a fixed wage, paid every day, week, or month. Their situation was different from that of farmers, who received money only when they sold their crops, or craftspeople, whose pay varied according to the number of items they sold.

  Unlike craftspeople, factory workers usually did not have many skills or much experience. Factory owners often hired women and children, who worked for lower wages than men did.

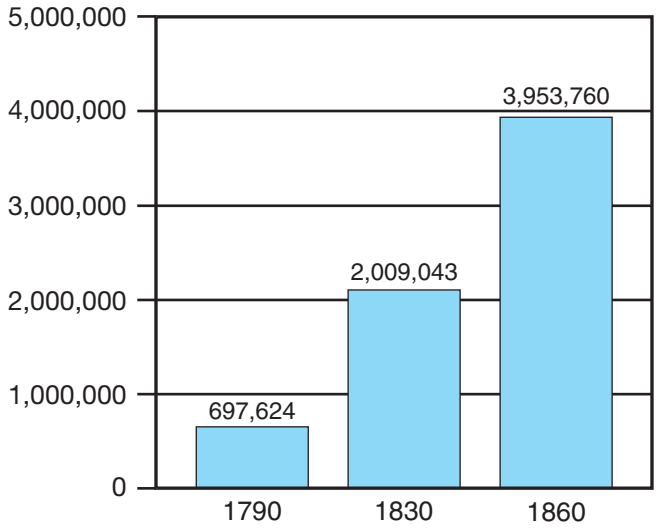
  Another major effect of industrialization was to speed up the growth of cities. Since most factories were located in cities, large numbers of people moved from rural to urban areas, hoping to find jobs. This process is called **urbanization**. As you learned in the section on Objective 2, the North was the most urbanized region of the United States.

  Finally, industrialization contributed to immigration. During the late 1700s and early 1800s, most immigrants had come to the United States hoping to own farmland. As the nation became more industrialized, however, factory jobs attracted more and more immigrants. This steady stream of immigrants contributed to urbanization and helped make cities culturally diverse places.

- **The Plantation Economy**: A fourth factor that led to the rapid economic growth of the United States was the expansion of the plantation system in the South. This expansion began soon after the invention of the cotton gin by Eli Whitney in 1793. The cotton gin reduced the time and cost of removing seeds from cotton, which increased production and made cotton a much more profitable crop. The increased supply of cotton was used by textile manufacturers in New England and Europe.

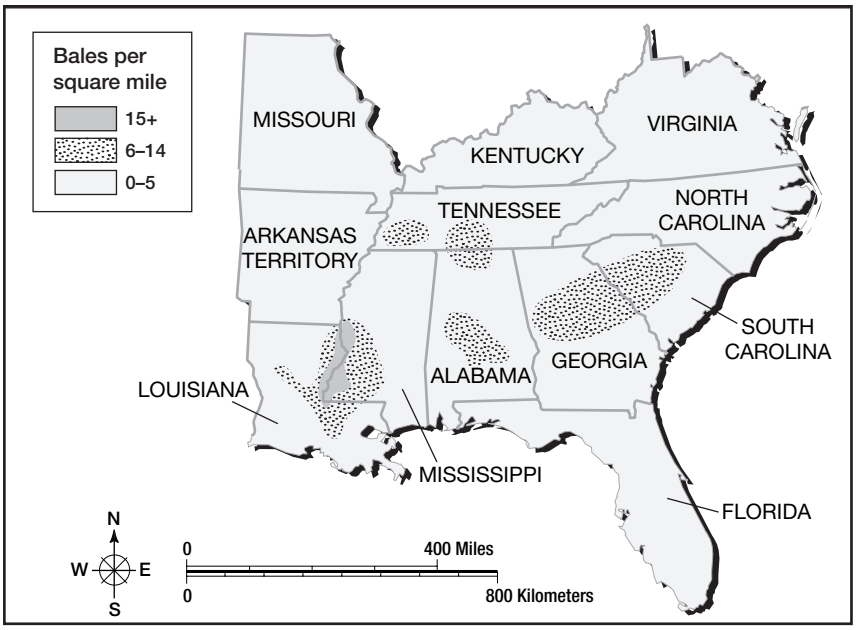
  The invention of the cotton gin and the strong demand for cotton by textile manufacturers contributed to the spread of cotton **plantations**. These were large farms that depended on slave labor. This, in turn, contributed to an increased demand for slaves in the South. In 1790 there were about 700,000 slaves in the United States. By 1860 there were almost four million slaves. The growth of the southern cotton industry and the increase in the South's slave population are illustrated in the following graph and maps.

U.S. Slave Population



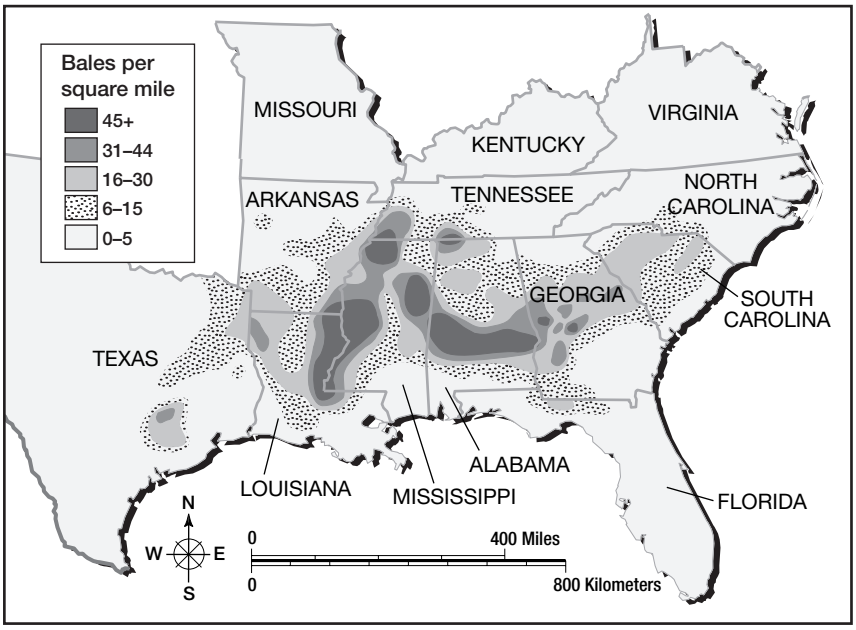
Source: *Reader's Companion to American History*

U.S. Cotton Production, 1830



Source: *Routledge Historical Atlas of the American South*, 1999

U.S. Cotton Production, 1860



Source: *Routledge Historical Atlas of the American South*, 1999

- **Technological Innovations**: As the information on the last few pages has shown, the rapid growth of the U.S. economy was encouraged by a number of important technological innovations. The following table provides short descriptions of some of these innovations.

Technological Innovation	Description
Cotton gin	The cotton gin removed seeds from cotton, a task that had previously been done by hand. Eli Whitney invented the cotton gin in the 1790s.
Textile factories	Textile factories used machines to produce cloth in large quantities. Samuel Slater established the first successful textile factory in the United States in 1793.
Interchangeable parts	Interchangeable parts were parts that could be substituted for one another in the manufacturing process. This reduced the amount of time it took to produce goods. Eli Whitney developed interchangeable parts in the early 1800s.
Steamboats	Steamboats, which were powered by steam engines, increased the speed of river travel. Robert Fulton developed the first practical steamboat in 1807.
Trains	Trains, which were powered by steam locomotives, increased the speed of overland travel. Railroad construction began in the United States in the 1830s. By 1860 there were more than 30,000 miles of railroad track in the United States.
Telegraph	The telegraph, an early form of electronic communication, allowed people to communicate rapidly across long distances. Samuel Morse built the first U.S. telegraph system in the 1840s.

Now let's look at a practice question that asks about some of the information you just read.

Beginning in the early 1800s, advances in transportation in the United States helped lead to —

A a decrease in the size of cities

B the creation of a national economy

C the beginning of trade with foreign countries

D a decline in agricultural production

Explanation of answer choices

A Incorrect. Advances in transportation made it cheaper and faster to move people and goods. This encouraged, rather than discouraged, the growth of cities.

B Correct. Advances in transportation reduced the cost and increased the speed of transporting goods. This allowed businesspeople to sell their goods on a national scale.

C Incorrect. Americans had been involved in international trade since the colonial era.

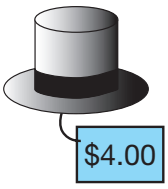
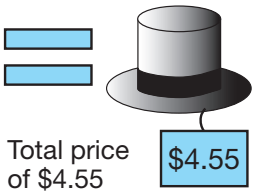
D Incorrect. Advances in transportation helped create new markets for agricultural goods. This encouraged an increase in agricultural production.

The Tariff System

Objective 3 also requires you to understand the impact of protective tariffs on the U.S. economy during the 1800s. You should review the discussion of protective tariffs and their role in the Nullification Crisis on pages 259 and 260.

A **tariff** is a tax, or fee, charged on imports—goods brought in from foreign countries. There are two kinds of tariffs, and they serve different purposes. Revenue tariffs are designed to raise revenue, or money, to pay for the costs of government. Protective tariffs are designed to protect a country's industries from foreign competition. The chart below will help you understand the impact of protective tariffs on the price of goods.

Effect of a Protective Tariff

United States	 It costs $3.00 to produce a hat in the United States	 Add $1.00 to the selling price for profit	 Total price of $4.00 for American consumers to buy the hat	
Great Britain	 It costs $2.50 to produce a hat in Great Britain	 Add $1.00 to the selling price for profit	 Add $1.05 (30%) to the selling price for the tariff Great Britain must pay the U.S. government to bring the hat into the United States to sell	 Total price of $4.55 for American consumers to buy the hat

Protective tariffs benefited U.S. manufacturers by making American-made goods less expensive than similar goods imported from other countries. However, the economic benefits of protective tariffs were not evenly distributed throughout the nation. Since most U.S.-manufactured goods were produced in the North, the northern economy benefited most from protective tariffs. Consumers in both the North and the South had to pay more for American-made goods than they would have paid for similar imported goods if a tariff were not in place. Southerners, however, did not profit as manufacturers in the North did from increased domestic sales. The protective tariff issue was one of the causes of sectional tension that eventually led to the Civil War.

Reform Movements

As the United States developed a national economy, many Americans began to take note of social problems that were becoming increasingly severe. This prompted the development of social movements that sought to reform, or improve, American society. The most important of these reform movements were abolitionism, temperance, public education, and women's rights. The **abolitionist movement** called for the elimination of slavery. The **temperance movement** urged Americans to quit drinking alcohol. The **public education movement** pressed for government-funded schools to be available to all students. The **women's rights movement** supported the expansion of the rights of women, including the rights to own property and vote.

A number of Americans became well known for their involvement with these reform movements. William Lloyd Garrison, for example, published an abolitionist newspaper called *The Liberator*. Garrison's paper called for the immediate emancipation, or freeing, of all slaves. Other famous abolitionists included writer and public speaker Frederick Douglass, a former slave, and novelist Harriet Beecher Stowe. Stowe's book *Uncle Tom's Cabin* caused many Americans, especially northerners, to oppose slavery.

Famous leaders of the women's rights movement included Lucretia Mott and Elizabeth Cady Stanton. In 1848 Mott and Stanton helped organize the Seneca Falls Convention, the first women's rights convention in the United States. Delegates to the convention issued a document known as the Declaration of Sentiments. This document called for American women to have the same rights as American men.

Advertisement for *Uncle Tom's Cabin*

Now let's look at a practice question that asks about some of the information you just read.

The main goal of the abolitionist movement was to —

A secure for women the right to vote

B eliminate slavery

C establish public schools

D prohibit the consumption of alcohol

Explanation of answer choices

A Incorrect. People who supported women's right to vote were known as suffragists.

B Correct. Abolitionists wanted to abolish, or eliminate, slavery.

C Incorrect. Although many abolitionists thought it was important to educate former slaves, the establishment of public schools was not the main goal of the abolitionist movement.

D Incorrect. People who opposed the consumption of alcohol were part of the temperance movement.

Image courtesy of the Library of Congress

Supporters of temperance argued that alcohol destroyed families. This illustration shows the "fruits of temperance"—a sober husband greeting his loving family.

Review Activity

On the line next to each statement, write the name of the region described (North, South, or both) for the period before the Civil War.

1. _____ Cotton production was a major economic activity.
2. _____ Rivers were important for the transportation of goods.
3. _____ The free-enterprise economic system helped business activities grow.
4. _____ Factories were a major part of the economy.
5. _____ Plantations were a major part of the economy.
6. _____ Cotton was imported to this region to make textiles.
7. _____ The demand for agricultural laborers resulted in a growing slave population.
8. _____ Technological innovations encouraged economic growth.
9. _____ The economy included trade with foreign countries.
10. _____ Protective tariffs on manufactured goods meant that consumers had to pay more.
11. _____ Protective tariffs on manufactured goods were most beneficial.
12. _____ Railroads made transportation faster and cheaper.

Answers: South=1, 5, 7; North=4, 6, 11; both=2, 3, 8, 9, 10, 12

Now It's Your Turn

Use the map and your knowledge of social studies to answer the following question.

Question 16

In which of these areas was cotton farming the main economic activity during the mid-1800s?

A I

B II

C III

D IV

Answer Key: page 324

Use the table and your knowledge of social studies to answer the following question.

Population of Selected U.S. Cities, 1820 and 1860

1820		1860	
City	Population	City	Population
New York	123,700	New York	1,080,330
Philadelphia	112,800	Philadelphia	565,529
Baltimore	62,700	Baltimore	212,418
Boston	43,300	Boston	177,840

Source: *Encyclopedia of American History*, 1999

Question 17

Which factor contributed most to the population trend shown in the table?

A Migration of African Americans from the South

B A decrease in the number of U.S. farms

C An increase in the number of cottage industries

D Immigration from Europe

Answer Key: page 324

Question 18

Which of these statements best describes the effect of the Industrial Revolution on the types of jobs held by Americans?

A Most Americans became factory managers.

B Few Americans started new businesses.

C The number of factory jobs increased.

D The number of skilled craftspeople increased.

Answer Key: page 324

Question 19

Which of the following technological innovations contributed most to increased trade and settlement along the Mississippi River during the early 1800s?

A Clipper ships

B Steamboats

C Interchangeable parts

D Railroads

Answer Key: page 324

Question 20

Which of the following best describes the economic significance of slavery in the South before the Civil War?

A Slavery lifted almost all white southerners out of poverty.

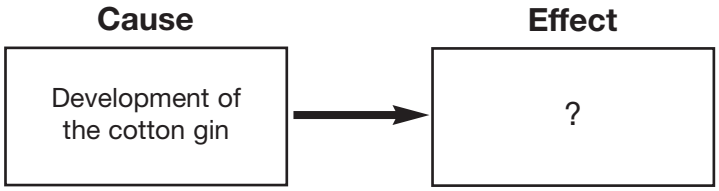
B Slavery enabled most small farmers to use slaves to do all their work.

C Slavery forced most small farms to go out of business.

D Slave labor was a key component of the plantation system.

Answer Key: page 325

Use the diagram and your knowledge of social studies to answer the following question.



Question 21

Which of the following best completes the diagram above?

A Decrease in cotton profits

B Decreased use of slave labor

C Loss of industrial jobs

D Increase in cotton production

Answer Key: page 325

Question 22

By 1832 many southerners opposed protective tariffs because these tariffs —

A increased the cost of owning slaves

B lowered the price of cotton

C increased the price of foreign manufactured goods

D discouraged the construction of new factories

Answer Key: page 325

Use the information in the box and your knowledge of social studies to answer the following question.

- Lucretia Mott
- Elizabeth Cady Stanton
- Sarah and Angelina Grimké

Question 23

These women gained national attention by encouraging women to —

A become politically involved and economically independent

B supervise family-oriented activities

C participate in church revivals and the temperance movement

D join labor unions

Answer Key: page 325

Objective 4

The student will demonstrate an understanding of political influences on historical issues and events.

How Did the U.S. System of Representative Government Develop?

Objective 4 focuses on the early political history of the United States. It includes topics such as the development of representative government in the American colonies and major principles of the Declaration of Independence, U.S. Constitution, and Bill of Rights. It also deals with some of the issues that U.S. political leaders faced regarding federalism, or the division of political power between the national and state governments. The following paragraphs go into more detail about the information you need to know to answer questions based on Objective 4.

The Growth of Representative Government

Objective 4 requires you to know about the reasons for the growth of representative government in the American colonies. A **representative government** is a government where the people choose individuals to make and enforce laws on their behalf. Representative governments developed in the colonies for several reasons. First, many settlers had come to North America in search of greater freedom, especially religious freedom. Second, a philosophical movement known as the Enlightenment was an important influence on the political leaders of the colonial period. The Enlightenment encouraged the use of reason to improve society and government. Third, most colonists were familiar with the British system of government. Under this system, British citizens were represented in Parliament by an elected legislature called the House of Commons. The colonists expected to have some form of representative government in the colonies as well. Most colonies established their own elected legislative assemblies. These assemblies played an important role governing the colonies. Fourth, the British government realized it would be impossible to govern the day-to-day affairs of the colonies from the other side of the Atlantic Ocean. The colonies needed local governments to help them run their local affairs. As a result, most colonies had a royal governor who served as the representative of the king. These governors shared power with the colonial legislatures, combining democracy with monarchy much as Parliament and the king shared power in Great Britain.

Two early examples of representative government in the colonies were the Virginia House of Burgesses and the Mayflower Compact.

- **Virginia House of Burgesses (1619):** The Virginia House of Burgesses was the first elected legislature, or lawmaking body, in colonial America.
- **Mayflower Compact (1620):** The Mayflower Compact was an agreement signed by the Pilgrims prior to the founding of the Plymouth colony in Massachusetts. It created a government for the new colony that would rule according to the wishes of the majority.

Patrick Henry Before the Virginia House of Burgesses (1851) by Peter Rothermel. Red Hill, Patrick Henry Memorial, Brookneal, Virginia.

Patrick Henry, an important leader of the American Revolution, addresses the Virginia House of Burgesses in 1765. Henry later became an Anti-Federalist.

The Declaration of Independence

The Declaration of Independence (1776) announced the American colonies' independence from Great Britain and established the United States of America as a nation. You will need to know the main political principles discussed in the Declaration of Independence. These include equality, unalienable rights, and consent of the governed.

- **Equality:** The Declaration of Independence states, "all men are created equal." This means that all Americans possess the same rights and are entitled to equal treatment from the government.
- **Unalienable Rights:** Unalienable rights are the basic rights of all human beings; these rights cannot be taken away by governments without due process of law. The Declaration of Independence states that "all men" possess unalienable rights and that "among these are life, liberty, and the pursuit of happiness."
- **Consent of the Governed:** Consent of the governed means that a government should be based on the agreement of the people being governed. The Declaration of Independence argues that British rule of the colonies was not based on consent of the governed; therefore, the colonists had the right to form a new government.

From the Articles of Confederation to the U.S. Constitution

As you read in the section on Objective 1, the United States adopted the Articles of Confederation in 1781 as its first plan of government. The Articles of Confederation created a national government to manage affairs that could not be handled by the separate states. For instance, the national government was responsible for making treaties and alliances with foreign governments, establishing a national army and navy, and settling disputes between states. The national government was also responsible for regulating westward expansion under the Articles of Confederation, as shown by the passage of the Northwest Ordinance in 1787.

Although it had these responsibilities, the national government under the Articles of Confederation was designed to have less power than the state governments. Because of their experience as British colonies, the states did not want to be controlled by a powerful central government. Objective 4 requires you to be familiar with the main weaknesses of the Articles of Confederation. Three of the most important weaknesses involved taxation, regulation of commerce, and the process for amending the Articles.

- **Taxation:** Under the Articles of Confederation, the national government did not have the power to impose taxes. This made it difficult for the national government to raise enough money to perform its duties.
- **Regulation of Commerce:** The national government could not regulate commerce, or trade, between the states. This caused economic problems, since the states often refused to cooperate with one another. For instance, some states passed laws taxing goods produced in other states.
- **Amending the Articles:** All 13 states had to agree to any amendments, or changes, to the Articles of Confederation. This made it especially hard to make improvements to the structure of the national government.

In 1787 delegates from the states met in Philadelphia to try to revise the Articles of Confederation. Instead, the convention decided to create a new plan of government. The document they created was the Constitution of the United States. The Constitution created a much stronger national government with three separate branches—executive, legislative, and judicial. Each branch was given specific powers and responsibilities. The **legislative branch**—Congress—makes laws. The **executive branch**—the president and the agencies under the president's control—enforces laws. The **judicial branch**—the federal courts headed by the Supreme Court—interprets laws.

Branch	Includes	Function
Legislative	Congress	Makes laws
Executive	President and various executive agencies	Enforces laws
Judicial	Supreme Court and lower federal courts	Interprets laws

The table below summarizes the main differences between the Articles of Confederation and the U.S. Constitution.

Differences Between the Articles of Confederation and the U.S. Constitution

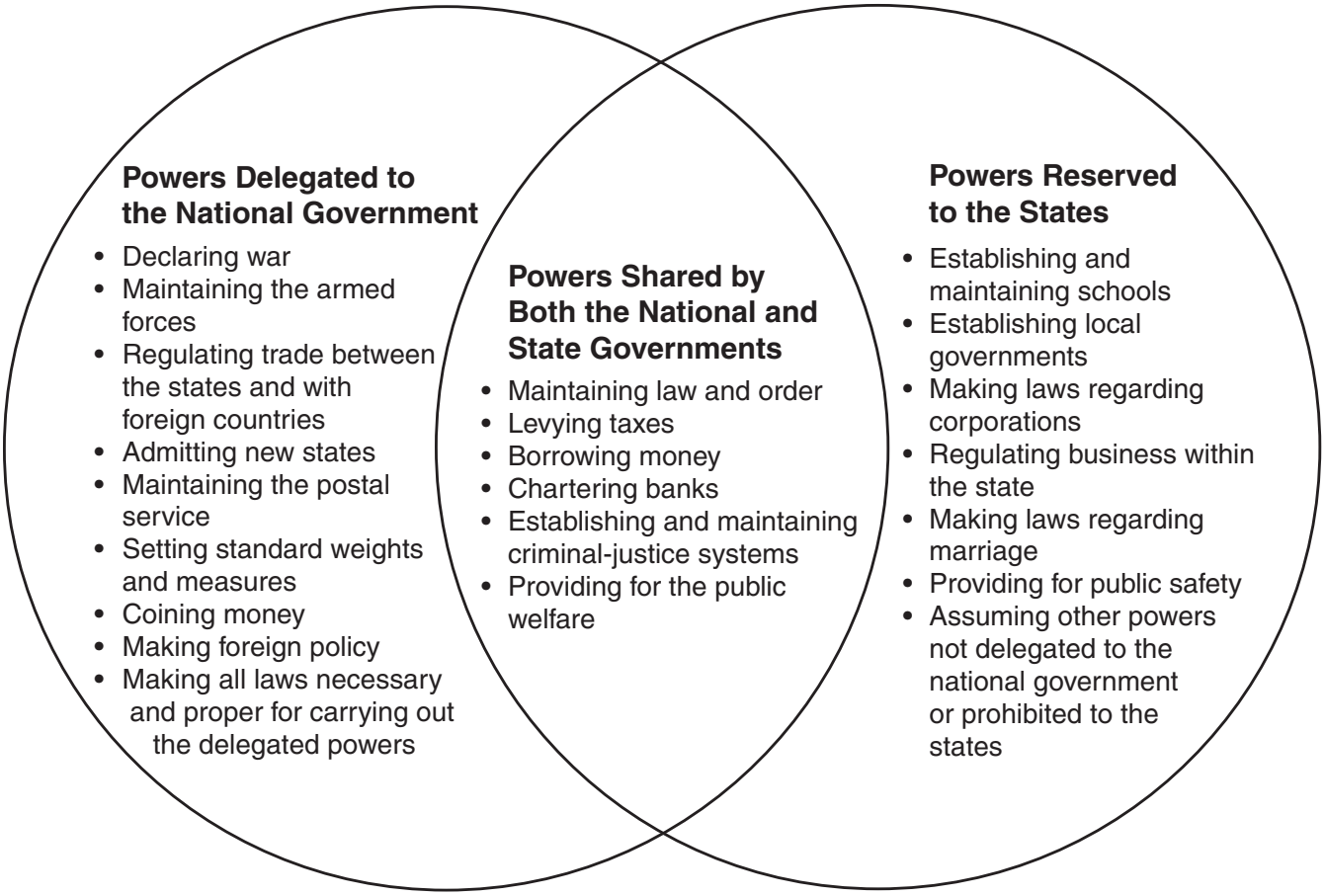
The Articles of Confederation	The U.S. Constitution
1. Legislative branch: Congress is made up of one house	1. Legislative branch: Congress is made up of two houses—the Senate and the House of Representatives
2. Each state has one vote in Congress	2. Each state has two votes in the Senate; each state has one or more votes in the House of Representatives, depending on the state's population
3. At least 9 of the 13 states must approve a law	3. A majority of each house of Congress must approve a law
4. No executive branch	4. The executive branch, headed by the president, carries out laws
5. No judicial branch	5. The judicial branch, headed by the Supreme Court, interprets laws
6. Only states can impose taxes	6. Congress can impose taxes
7. Each state can issue its own money	7. Only Congress can issue money
8. Each state can regulate its trade with other states	8. Only Congress can regulate trade between states

Objective 4 requires you to be familiar with the U.S. Constitution's main political principles. The table below provides brief definitions of these principles.

Main Political Principles of the Constitution of the United States

Principle	Definition	Example
Popular Sovereignty	Popular sovereignty means that a government receives its authority from the people. A government cannot rule without the people's consent.	The will of the population is expressed through the people elected to serve in local, state, and federal government.
Limited Government	A limited government is not all-powerful. Its powers are limited to those given to it by the people.	The U.S. government can have no powers or responsibilities beyond those assigned in the U.S. Constitution.
Republicanism	Republicanism is the concept of representative government. A republic is a form of government where the people rule through elected representatives.	U.S. citizens elect legislators to represent them in Congress.
Federalism	Federalism is the division of power between the state governments and the national, or federal, government.	The U.S. Constitution grants certain powers to the national government and others to the states.
Separation of Powers	Separation of powers is the division of the national government into three branches—the executive, legislative, and judicial—with each branch having different powers and responsibilities.	The legislative branch makes laws. The executive branch enforces laws. The judicial branch interprets laws.
Checks and Balances	Each branch of the national government has the ability to check, or control, the power of the other two branches. Checks and balances keep any one branch of the government from becoming too powerful.	The president can veto a bill passed by Congress.
Individual Rights	Individual rights are rights that the government is obligated to protect. These protections are provided by various amendments to the Constitution.	The right to free speech is protected by the First Amendment.

The Federal System Under the U.S. Constitution



Now let's look at a practice question that asks about some of the information you just read.

Use the information in the box and your knowledge of social studies to answer the following question.

- The House of Representatives has the power to impeach the president.
- The president has the power to veto laws passed by Congress.
- The Senate must approve judicial appointments made by the president.

The information in the box illustrates which of the following constitutional principles?

A Individual rights

B Federalism

C Checks and balances

D Republicanism

Explanation of answer choices

A Incorrect. The information in the box lists ways in which one branch of the national government can limit the power of another. It does not give examples of individual rights.

B Incorrect. Federalism is the division of power between the national and state governments. The information in the box relates only to the national government.

C Correct. The information in the box lists ways in which one branch of the national government can limit the power of another branch. These are examples of checks and balances.

D Incorrect. Republicanism is the concept of representative government. The examples in the box illustrate the principle of checks and balances.

The Ratification Debate

Following the Constitutional Convention, the U.S. Constitution was sent to the states for ratification, or approval. People who were opposed to the ratification of the Constitution were called **Anti-Federalists**. Patrick Henry and George Mason were two of the leading Anti-Federalists. There were a number of important reasons Anti-Federalists did not want the Constitution to be approved. First, the Constitution was written in a convention that had been called to revise the Articles of Confederation, not to create an entirely new government. Second, by increasing the power of the national government, the Constitution reduced the power and independence of the states. Third, the Constitution created a strong executive branch, which Anti-Federalists feared might lead to an abuse of power by the president. Finally, the Constitution did not include specific protections of individual rights.

Supporters of the Constitution were called **Federalists**. Their leaders included Alexander Hamilton and James Madison. Federalists argued that the United States needed a stronger national government to better protect the interests of all the states, especially the smaller, less powerful ones. They also argued that a strong national government with the power to regulate commerce would be good for U.S. businesses. In response to the Anti-Federalists' concerns about individual rights, Federalists offered to amend the Constitution by adding a bill of rights. Hamilton, Madison, and John Jay wrote *The Federalist Papers* to explain the Constitution and convince the states to approve it.

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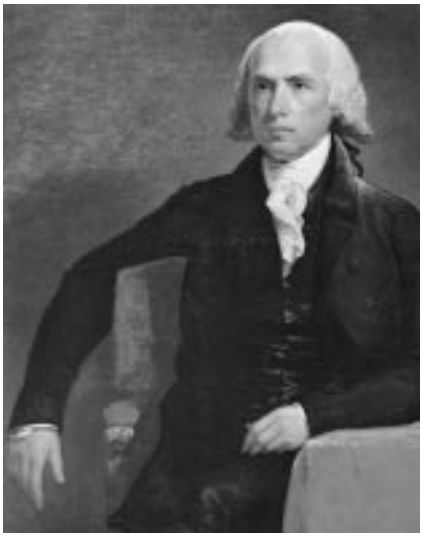
Alexander Hamilton



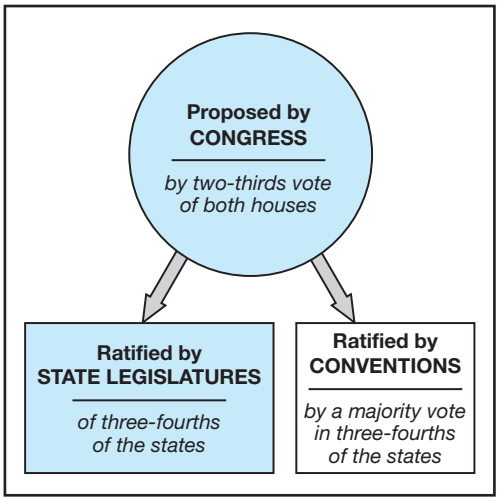
Library of Congress

James Madison

After much debate the Constitution was ratified in 1789. In order to allow the national government to adapt to changing circumstances, the Constitution provided ways for amendments, or changes, to be made to the document. In the most frequently used process, an amendment is proposed by a two-thirds vote of both houses of Congress and then must be ratified by three-quarters of the states.

The diagram below will help you understand the amendment process.

Amending the U.S. Constitution



In 1791, shortly after the U.S. Constitution was ratified, 10 amendments were added to the document. These amendments are known as the Bill of Rights. The list below summarizes the basic principles of the Bill of Rights.

Basic Principles of the Bill of Rights

1. Freedom of speech, press, religion, assembly, and the right to petition the government
2. The right to bear arms
3. The guarantee that civilians will not be forced to house soldiers
4. Protection against unreasonable searches by law-enforcement officers
5. The right of a person under arrest to know why he or she has been arrested and to refuse to testify against himself or herself in a court of law
6. The right to a speedy trial by a jury of one's peers in criminal cases
7. The right to a trial by jury in civil cases involving substantial amounts of money
8. Protection against excessive bail and cruel and unusual punishment
9. The guarantee that rights not specifically listed in the Constitution are not automatically denied to the people
10. The guarantee that the people and the states are to keep powers not specifically granted to the federal government

The Bill of Rights was influenced by earlier political documents, such as the English Bill of Rights and the Declaration of Independence. Objective 4 asks you to explain how the Bill of Rights addressed important issues that were raised in the Declaration of Independence. There are several examples. For instance, one of the complaints about the British government that colonists listed in the Declaration of Independence involved the quartering, or housing, of troops in citizens' homes. The Third Amendment prohibits the U.S. government from quartering troops in citizens' homes. A second example concerns the right to a trial by jury. A jury is a group of citizens that decides the verdict of a court trial. The Declaration of Independence criticized Great Britain for depriving colonists of the right to a jury trial. The Fourth Amendment protects this right.

Now let's look at a practice question that asks about some of the information you just read.

Which of the following individuals was an author of *The Federalist Papers*, which supported ratification of the U.S. Constitution?

A Patrick Henry

B James Madison

C Abraham Lincoln

D George Washington

Explanation of answer choices

A Incorrect. Patrick Henry, a leading political figure from the American Revolution, was suspicious of a strong central government. He argued against ratification of the Constitution.

B Correct. James Madison supported the creation of a strong central government. He, Alexander Hamilton, and John Jay wrote *The Federalist Papers* to try to win support for the Constitution.

C Incorrect. Abraham Lincoln, the 16th president of the United States, was not born until 1809. *The Federalist Papers* were written in 1787 and 1788.

D Incorrect. Although George Washington presided over the Constitutional Convention, he was not an author of *The Federalist Papers*.

Interpreting the Constitution

Objective 4 also requires you to know the significance of two important cases decided by the U.S. Supreme Court: *Marbury* v. *Madison* and *Dred Scott* v. *Sandford*.

- ***Marbury* v. *Madison* (1803):** This case arose after President John Adams appointed several new judges in 1800, near the end of his term. In 1801 newly elected president Thomas Jefferson tried to deny the appointments. One of the appointees, William Marbury, filed a lawsuit against James Madison, Jefferson's secretary of state, who was responsible for judicial commissions.

  The case turned into a power struggle between the judicial and executive branches. The most important outcome of the case was that it established the power of judicial review. Judicial review allows federal courts to strike down laws that violate the U.S. Constitution.

- ***Dred Scott* v. *Sandford* (1857):** This case was filed by Dred Scott, a slave whose owner had taken him to a western territory where slavery was prohibited. After the owner died, Scott sued for his freedom and managed to get the case into federal court.

  The Supreme Court ruled that slaves were property and could never become citizens, even if they moved to areas where slavery was prohibited. This meant that Scott was not a citizen and therefore did not have the right to sue in federal court. The Court went on to rule that it was unconstitutional for the U.S. government to prohibit slavery in territories. *Dred Scott* had far-reaching effects and helped set the stage for the Civil War.

Some of the most important political controversies in early U.S. history involved the concept of federalism, or the division of political power between the national and state governments. As we have already seen, arguments about federalism were an important part of the debate surrounding ratification of the U.S. Constitution. Anti-Federalists, who favored states' rights, opposed ratification largely because they believed the Constitution gave too much power to the national government. Federalists supported ratification largely because they thought the United States needed a strong central government.

As mentioned in the section on Objective 1, federalism was also an important factor in the formation of the first political parties. The Federalist Party supported a strong national government, while the Democratic-Republican Party favored states' rights and a limited national government.

Federalism also influenced how these two political parties interpreted the Constitution. Democratic-Republicans argued that the Constitution should be interpreted as narrowly as possible. By sticking to the exact wording of the Constitution, they hoped to limit the power of the national government. Federalists, who wanted a strong national

government, supported a broad reading of the Constitution. The U.S. Supreme Court's decision in *Marbury* v. *Madison* is an example of this type of broad interpretation. In the *Marbury* case the U.S. Supreme Court established the principle of judicial review even though this principle was not specifically mentioned in the Constitution.

The Civil War and the Reconstruction Amendments

Controversy about the division of power between the federal and state governments resulted in a major crisis in 1860. Abraham Lincoln, who was running for president under the banner of the Republican Party, stated his intention to prevent slavery from spreading to any new territories. Southerners suspected that Lincoln would try to eliminate slavery in the South as well. Lincoln won the election, and even before he was sworn in, seven southern states announced their secession from the Union. These states, which were soon joined by four more, declared themselves a new country. They formed the Confederate States of America, often called the Confederacy. The constitution for this new country asserted that states would possess greater power than the national government and that slavery would remain lawful. This time there was no room for compromise with the U.S. government, and it took years of war to finally settle the issue. The remaining states of the United States, known as the Union, eventually defeated the Confederacy. The Union's victory in the Civil War made it clear that the federal government would assert its power over state governments. It also ensured that no state could secede from the Union in the future.

After the Civil War ended, there was still the question of how the southern states should be brought back into the Union. Over time they were allowed to rejoin, but first they were required to ratify the 13th, 14th, and 15th Amendments to the Constitution. These amendments officially ended the institution of slavery, guaranteed equal protection of the laws to African Americans, and gave African American men the right to vote.

The Reconstruction Amendments

13th Amendment (1865)	14th Amendment (1868)	15th Amendment (1870)
• Abolishes slavery in the United States	• Declares that all persons born in the United States are citizens • States that all citizens are guaranteed equal protection of the laws	• States that citizens cannot be denied the right to vote because of "race, color, or previous condition of servitude"

Now It's Your Turn

Question 24

The Declaration of Independence states that all men possess unalienable rights. What are these rights?

A Protection from cruel and unusual punishment

B Freedom of religion, speech, and the press

C The guarantee of a speedy and public trial

D Life, liberty, and the pursuit of happiness

Answer Key: page 325

Question 25

The significance of the Mayflower Compact was that it —

A served as a model for the English Bill of Rights

B served as an example of self-rule in the colonies

C declared colonial independence from England

D established the ideal of political equality

Answer Key: page 325

Question 26

Which of the following was one of the main weaknesses of the Articles of Confederation?

A The Articles were easy to amend.

B The Articles gave the federal government more power than they gave the states.

C The Articles did not allow the federal government to regulate commerce between states.

D The Articles did not allow the federal government to regulate westward expansion.

Answer Key: page 326

Question 27

Which of the following best summarizes the U.S. Supreme Court's decision in *Marbury* v. *Madison* (1803)?

A The federal government does not have the power to ban slavery in the territories.

B Federal courts have the power to declare acts of Congress unconstitutional.

C The federal government has the power to establish a national bank.

D Federal courts have broad powers to regulate commerce.

Answer Key: page 326

Use the information in the box and your knowledge of social studies to answer the following question.

- The federal government must protect citizens by creating a Bill of Rights.
- The president will have too much power and will become a tyrant.
- The federal government will take away the rights of the state governments.

Question 28

A member of which of the following groups might have made these statements in 1787?

A Tories

B Anti-Federalists

C Patriots

D Federalists

Answer Key: page 326

Objective 5

The student will use critical-thinking skills to analyze social studies information.

What Skills Do You Need to Study History?

Historians are like detectives. They use clues to learn about what happened in the past. Objective 5 tests your ability to use some of the same critical-thinking skills that historians use. Objective 5 focuses on three main areas. First, you are required to understand the difference between primary and secondary sources. Second, you are required to recognize point of view and bias in primary sources. Third, you are required to interpret historical information from maps, tables, and graphs. The following paragraphs go into more detail about the information you need to know to answer questions based on Objective 5.

Understanding Sources

The first thing you need to know to answer questions based on Objective 5 is the difference between primary and secondary sources. Historians use sources to provide evidence about the past. There are many types of sources, including letters, court records, statistics, maps, and the work of other historians. Historians classify sources into two categories: primary and secondary.

A **primary source** is something produced at or near the time of a historical event by someone close to the event. Primary sources are usually some kind of written record, but they can also be other types of evidence, such as political cartoons, photographs, songs, posters, or objects, like a gun used during the American Revolution. A newspaper from 1789 describing the debate over ratification of the U.S. Constitution is one example of a primary source. Primary sources on the social-reform movements of the early 1800s might include a political cartoon supporting the abolition of slavery or the text of a temperance speech on the harm caused by drinking alcohol. Remember, primary sources are created during or around the time of the event being studied by someone who observed the event firsthand.

Historians also use **secondary sources** to understand the past. Secondary sources are usually created a long time after an event and are based mainly on evidence found in primary sources. Examples of secondary sources include a book on the American Revolution, a documentary film on the Civil War, and the maps and charts in this study guide. It may help to remember that all history books, including your textbook, are secondary sources.

The chart below summarizes the differences between primary and secondary sources.

Differences Between Primary and Secondary Sources

I. Primary Sources	II. Secondary Sources
A. Definition: an item made at or near the time of a historical event by someone who observed the event firsthand 1. Usually a written record 2. Can also include visual evidence and physical objects	A. Definition: an item that provides a secondhand interpretation of an event after some time has passed 1. Often based on primary sources 2. May also use other secondary sources
B. Examples: newspaper articles, census records, letters, diaries, photographs, maps, and sound recordings	B. Examples: history books, reference books, maps, and tables based on other sources

Identifying Point of View and Bias

Once you know how to tell whether a source is primary or secondary, you will need to use critical-thinking skills to analyze your source. First, you will need to be able to recognize the point of view expressed in a primary or secondary source. Participants in historical events often have different points of view or opinions on the causes and meanings of the event. For instance, suppose you were trying to determine the reasons the Union won the Civil War. One source, such as an editorial from a southern newspaper, might state that the Union won because it had a large advantage in economic and military resources. Another source, a diary written by a Union soldier, might state that the Union won because Abraham Lincoln was a better leader than Confederate president Jefferson Davis.

Objective 5 also requires that you be able to recognize bias in a primary or secondary source. **Bias** refers to prejudices, stereotypes, or distortions of fact expressed in a source. Bias can be used to portray an event or person in either positive or negative ways. It is important to recognize the words, symbols, and images that express bias because they help determine the reliability of a source.

There are several examples of negative bias in this political cartoon from the early 1830s depicting President Andrew Jackson.

King Andrew the First

Library of Congress

The cartoon shows Jackson as a king, although he was actually the president of a democracy. It also shows him stepping on the U.S. Constitution, although he probably never physically put his foot on this document. Both of these distortions of fact, which are meant to express the opinion that Jackson was abusing the power of the presidency, demonstrate bias against the president. Can you find any examples of bias in your everyday life? You may want to practice recognizing bias by looking for it in television advertisements, newspaper editorials, and political cartoons.

Now let's look at a practice question that asks you to identify point of view in a written source.

Use the excerpt and your knowledge of social studies to answer the following question.

> The blessings in which you, this day, rejoice are not enjoyed in common. The rich inheritance of justice, liberty, prosperity, and independence bequeathed [given] by your fathers is shared by you, not by me. The sunlight that brought light and healing to you has brought stripes [beatings] and death to me.
>
> — *Frederick Douglass, Independence Day speech, 1841*

In this excerpt, Frederick Douglass is expressing —

A opposition to the consumption of alcohol

B support for southern secession

C opposition to the institution of slavery

D support for the ratification of the U.S. Constitution

Explanation of answer choices

A Incorrect. The excerpt mentions nothing about the temperance movement, which opposed the consumption of alcohol.

B Incorrect. The excerpt makes no reference to southern secession.

C Correct. Douglass was himself a former slave and leader of the abolition movement. The excerpt shows that he considered the institution of slavery a brutal contradiction to the freedom enjoyed by white Americans.

D Incorrect. The date of the speech shows that it is not related to the debate over ratification of the U.S. Constitution.

Interpreting Maps, Tables, and Graphs

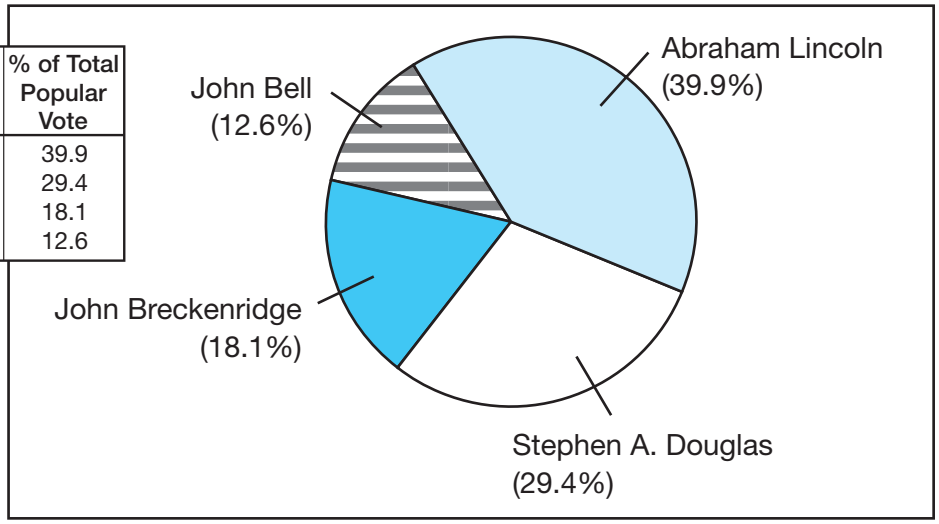
Objective 5 also requires you to interpret historical information from maps, tables, and graphs. You may want to review the section on Objective 2, which discusses some of the ways maps and graphs can help you learn about history and geography.

In addition to the types of graphs you learned about in Objective 2, you need to be familiar with circle graphs. A circle graph is another way to show data. A circle graph looks like a pie cut into triangular pieces. Each slice of the pie represents a percentage of the whole. The circle graph below shows the results of the election of 1860, when Abraham Lincoln was elected president. According to the graph, what percentage of the popular vote did Abraham Lincoln win?

Answer: 39.9%

Popular Vote in the Presidential Election of 1860

Candidate	Electoral Votes	Popular Votes	% of Total Popular Vote
Lincoln	180	1,866,452	39.9
Douglas	12	1,375,157	29.4
Breckenridge	72	847,953	18.1
Bell	39	590,631	12.6

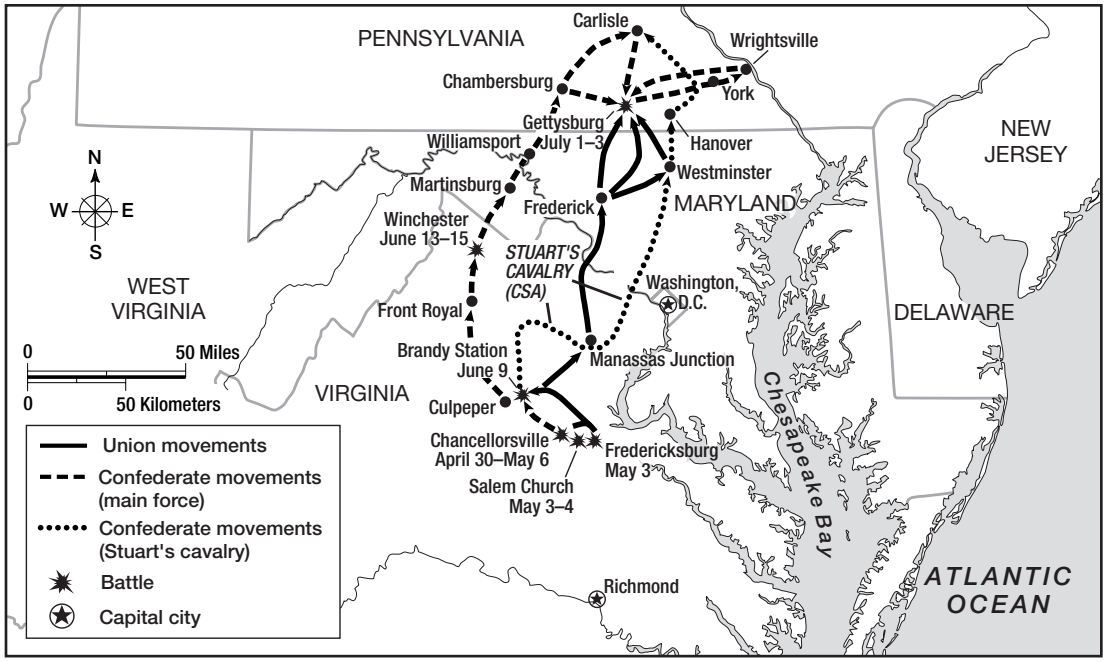


Source: *Images of American Political History*

In addition to graphs, you will need to be familiar with maps. Historians use maps to interpret data about historical events. Maps come in a variety of sizes, colors, and patterns. Sometimes a map might seem complicated or confusing at first glance. When looking at an unfamiliar map, you should ask yourself what the map is trying to show you—what the purpose of the map is. The map's title, labels, and key should help you.

Now let's go through the steps of reading a historical map. First look at the title and key of the map on the next page. From its title, "The War in the East, 1863: From Chancellorsville to Gettysburg," you should be able to tell that it refers to the Civil War. The key should make it clear that the map shows troop movements and battles. The purpose of this map is to show how Union and Confederate armies moved and fought during a specific span of time in 1863.

The War in the East, 1863: From Chancellorsville to Gettysburg



Source: National Park Service

Now follow the arrows and the dates on the map to get an idea of what took place. The title of the map and the dates on the map indicate that Chancellorsville, Virginia, is the starting point of the action. Finding it on the map, you will see that a battle was fought there. You may notice that two other battles were fought at about the same time, at Salem Church and Fredericksburg. The map does not indicate which side won these battles, although from your study of the Civil War, you may remember that Chancellorsville was a Confederate victory.

After the Battle of Chancellorsville, both armies began a series of movements. The arrows show that both armies moved west and then north. Trace the Confederate force shown by the dashed line. Notice that the Confederate force divided into two parts. One part, Stuart's cavalry, fought a battle at Brandy Station, Virginia, and then rode east, moving around the main Union army and then north into Maryland, passing not far from Washington, D.C. The rest of the Confederate army fought a battle at Winchester, Virginia, marched northeast into Pennsylvania, and then split again. Finally, all the forces of both armies converged for a battle at Gettysburg, Pennsylvania. This was one of the most decisive battles of the war and resulted in a Union victory.

You should now have a good grasp of the information conveyed on the map. Of course, you may find that you now have more questions than you did before! For example, why did Lee decide to lead the Confederate army in an invasion of Union territory in the first place? Why did both sides divide their forces as they marched cross-country? What were the results of Stuart's cavalry raid behind the Union army? And why did the small town of Gettysburg become the site of this key battle? You may want to review a history textbook or look for other sources for information on the Civil War and the Battle of Gettysburg.

Review Activity

Look at the data in this table:

Approximate Population of Selected Counties in Alabama, 1840–1870

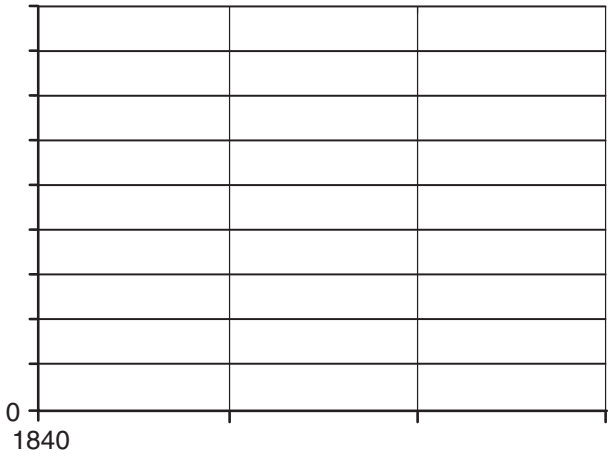
County	1840	1850	1860	1870
Barbour	12,000	23,500	31,000	29,500
Chambers	17,000	24,000	23,000	17,500
Montgomery	24,500	30,000	36,000	43,500
Tuscaloosa	16,500	18,000	23,000	20,000

Source: Inter-University Consortium for Political and Social Research

The data in this table could be presented in different ways to make it easier to understand or to emphasize different aspects. For example, a line graph is a good way to show change over time. Using the axes provided below, go through the following steps.

- Label the vertical axis "Population (thousands)" and label the horizontal lines with 0, 5, 10, 15, 20, 25, 30, 35, 40, and 45.
- Label the horizontal axis "Year" and label the vertical lines on the graph with 1840, 1850, 1860, and 1870.
- Place a dot to show Montgomery County's population in 1840, 1850, 1860, and 1870. Then connect all four dots to show the trend over time. You should see the line moving upward as it goes from left to right, showing population growing steadily. Label this line "Montgomery County."
- Now plot the data for each of the other counties in the table. Be sure to label each line with the county it represents. You may want to use colored pencils or pens to make it easier to tell the different lines apart. Do you see the same trend in these counties?

Approximate Population of Selected Counties in Alabama, 1840–1870



Answer Key: page 326

Now It's Your Turn

Use the graph and your knowledge of social studies to answer the following question.

Source: *Historical Statistics of the United States*

Question 29

What can be concluded from this graph?

A The production of cotton had no impact on the slave population.

B The production of cotton increased as the slave population increased.

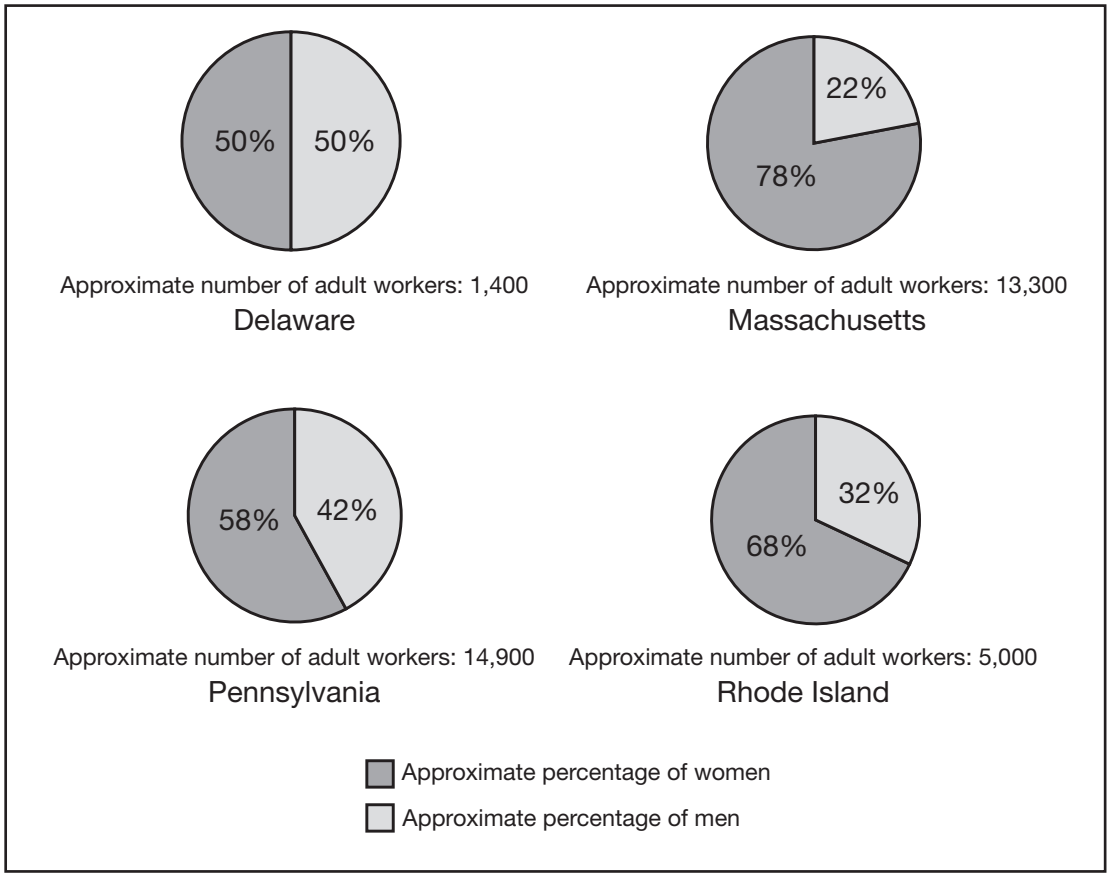
C The slave population was unnecessary for the production of cotton.

D The slave population decreased as the production of cotton increased.

Answer Key: page 326

Use the graphs and your knowledge of social studies to answer the following questions.

Textile-Mill Employment in Selected States, 1831



Source: *Routledge Historical Atlas of Women in America*, 2000

Question 30

According to the graphs, which state had the largest percentage of female textile-mill workers in 1831?

A Delaware
B Pennsylvania
C Rhode Island
D Massachusetts

Answer Key: page 326

Question 31

Acording to the graphs, which state had the smallest number of female textile-mill workers in 1831?

A Delaware
B Pennsylvania
C Rhode Island
D Massachusetts

Answer Key: page 327

Use the excerpt and your knowledge of social studies to answer the following question.

> I have heard something said about allegiance to the South. I know no South, no North, no East, no West, to which I owe any allegiance.
>
> —*Henry Clay, 1848*

Question 32

In this excerpt, Henry Clay opposes the growth of —

A republicanism

B sectionalism

C mercantilism

D nationalism

Answer Key: page 327

Use the map and your knowledge of social studies to answer the following question.

Northwest Territory, 1787

Question 33

According to the map, what geographical feature provided a northern boundary for most of the Northwest Territory?

A The Great Lakes

B The Ohio River

C The Mississippi River

D Lake Winnipeg

Answer Key: page 327

Use the excerpt and your knowledge of social studies to answer the following question.

> Because . . . the power to protect manufacture[r]s is nowhere expressly granted to Congress, nor can [it] be considered as necessary and proper to carry into effect any specified power, it seems to be expressly reserved to the state.
>
> —*The South Carolina Protest Against the Tariff of 1828*

Question 34

This excerpt is from a protest issued by the South Carolina legislature and addressed to the U.S. Congress. Which of these political issues was it related to?

A The Bank War

B *Marbury* v. *Madison*

C The Nullification Crisis

D The Alien and Sedition Acts

Answer Key: page 327

Social Studies Answer Key

Objective 1

Question 1 (page 263)

A Incorrect. Thomas Jefferson served as president from 1801 to 1809, before the Civil War.

B Correct. Abraham Lincoln took office in 1861, and his presidency lasted until his assassination, which occurred just a few days after Confederate general Robert E. Lee surrendered his army in 1865.

C Incorrect. John Adams served as president from 1797 to 1801, before the Civil War.

D Incorrect. James Polk served as president from 1845 to 1849, before the Civil War.

Question 2 (page 263)

A Correct. The Declaration of Independence was written in 1776, the Constitutional Convention occurred in 1787, and the Missouri Compromise was negotiated in 1820 and 1821.

B Incorrect. The events are out of order.

C Incorrect. The events are out of order.

D Incorrect. The events are out of order.

Question 3 (page 264)

A Incorrect. Southern delegates to the Constitutional Convention argued that slaves had to be counted in some way to give southerners a fair voice in government.

B Incorrect. The Constitution stated that three-fifths of any state's slaves would be counted in its population.

C Correct. There was a compromise at the convention between delegates who did not want slaves to be counted for purposes of representation and delegates who wanted slaves to be counted the same as the rest of the population. The final decision, known as the Three-Fifths Compromise, was to count three-fifths of a state's slave population.

D Incorrect. Slaves were not counted in the same way as free people were.

Question 4 (page 264)

A Incorrect. *Common Sense* was aimed at an American audience, not a British one.

B Incorrect. The Articles of Confederation had not yet been written in 1776, when *Common Sense* was published.

C Incorrect. After he wrote *Common Sense*, Thomas Paine wrote a series of pamphlets called *The American Crisis*. These pamphlets were intended to improve American morale during difficult military times.

D Correct. *Common Sense* urged American colonists to support the colonies' separation from Great Britain. The pamphlet presented several reasons why the colonies should declare independence.

Question 5 (page 265)

A Incorrect. The treaty was between the United States and Britain.

B Correct. The Treaty of Paris was signed by representatives of the United States and Great Britain on September 3, 1783. It formally ended the Revolutionary War, recognized U.S. independence, established U.S. boundaries, and settled other issues.

C Incorrect. The United States purchased the Louisiana Territory from France in 1803.

D Incorrect. The Proclamation of 1763, which was issued by Great Britain, prohibited colonial settlement west of the Appalachian Mountains.

Question 6 (page 265)

A Incorrect. Naval attacks did not drive Americans westward.

B Incorrect. While many settlers moved west to obtain farmland, farming remained a major activity east of the Mississippi River.

C Correct. Many people held a firm belief that the United States had the right and duty to expand westward. This belief was known as Manifest Destiny.

D Incorrect. In the early years of settlement, pioneers found life on the Pacific Coast to be very difficult.

Question 7 (page 266)

A Correct. Jefferson Davis served as president of the Confederate States of America during the Civil War.

B Incorrect. General Ulysses S. Grant commanded the Union army during the Civil War. He served as president of the United States from 1869 to 1877.

C Incorrect. General Robert E. Lee commanded the most important Confederate army, the Army of Northern Virginia, during the Civil War.

D Incorrect. Abraham Lincoln served as president of the United States during the Civil War.

Question 8 (page 266)

A Correct. The Northwest Ordinance established guidelines for settlement and a process for the formation of new states.

B Incorrect. The Northwest Ordinance was designed to encourage westward expansion while providing a system for territories to obtain statehood.

C Incorrect. The Northwest Territory was acquired from Great Britain at the end of the Revolutionary War. The Northwest Ordinance did not grant control of this area to Great Britain.

D Incorrect. The Northwest Ordinance did not address Native American land rights.

Question 9 (page 267)

A Incorrect. The Battle of Lexington, along with the Battle of Concord, marked the beginning of the Revolutionary War in 1775.

B Incorrect. The Battle of Trenton occurred in New Jersey in 1776. George Washington led his troops across the Delaware River on Christmas Day and launched a surprise attack on British and Hessian troops in Trenton.

C Incorrect. On December 31, 1775, a small American army attacked British forces in Quebec and was decisively defeated. The Americans soon gave up on conquering British Canada.

D Correct. The Battle of Yorktown was fought in 1781. American and French troops cooperated to trap more than 7,000 British troops in Virginia. After several weeks of fighting, the British surrendered. The Battle of Yorktown was the last major battle of the American Revolution.

Question 10 (page 267)

A Incorrect. The Constitution already included two houses of the legislature when it was submitted to the states for ratification.

B Correct. Several states ratified the Constitution only on the condition that a bill of rights be added to specifically address individual rights.

C Incorrect. The right to vote was already part of the Constitution when it was submitted to the states for ratification.

D Incorrect. Slavery was not banned in the United States until the 13th Amendment was ratified after the Civil War.

Objective 2

Question 11 (page 278)

A Incorrect. New York City is a major port city located at the mouth of the Hudson River, near the Atlantic Ocean.

B Incorrect. Philadelphia, Pennsylvania, is a major port city on the Delaware River, near the Atlantic Ocean.

C Incorrect. Charleston, South Carolina, is a port city located on the Atlantic coast.

D Correct. New Orleans, Louisiana, is located on the Mississippi River near the Gulf of Mexico and is an important commercial port.

Question 12 (page 278)

A Incorrect. This area was acquired by the United States from Spain in 1819.

B Correct. The Louisiana Purchase was acquired by the United States from France in 1803.

C Incorrect. This area, the Oregon Territory, was jointly occupied by the United States and Great Britain until 1846, when it was acquired by the United States.

D Incorrect. This area was part of Mexico before it was acquired by the United States in 1848.

Question 13 (page 279)

A Incorrect. This is the location of the Battles of Lexington and Concord in 1775.

B Incorrect. This is the location of the Battle of Saratoga in 1777.

C Incorrect. This is the location of the Battle of Trenton in 1776.

D Correct. This is the location of the Battle of Yorktown in 1781.

Question 14 (page 280)

A Incorrect. Most towns and cities were not located in mountainous areas, and gold and silver were not major products of the 13 colonies.

B Incorrect. Although France and Spain were hostile toward Great Britain during parts of the colonial period, naval attack was not a major fear of the colonists. Many towns and cities were located on the coast.

C Incorrect. Some towns and cities were located near timber supplies, but the vast majority were located on the coast or along rivers.

D Correct. Because boats were a major means of transporting people and goods, most towns and cities were located near water.

Question 15 (page 280)

A Incorrect. This area consisted of free states and territories.

B Incorrect. This area consisted of slave states and territories.

C Correct. This area was closed to slavery by the Missouri Compromise.

D Incorrect. By the terms of the Missouri Compromise, Missouri entered the Union as a slave state.

Objective 3

Question 16 (page 291)

A Incorrect. In this region, New England, manufacturing and trade were the main economic activities.

B Incorrect. In this region, the Midwest, grain farming was the main economic activity.

C Incorrect. In this region, the Upper South, tobacco farming was the main economic activity.

D Correct. In this region, the Lower South, cotton farming was the main economic activity.

Question 17 (page 292)

A Incorrect. The migration of African Americans from the South to northern cities was not a major factor in urban population growth until the early 1900s.

B Incorrect. The number of U.S. farms increased rather than decreased between 1820 and 1860.

C Incorrect. The growth of the factory system, which encouraged urban growth, led to the decline of cottage industries.

D Correct. Increased immigration from Europe between 1820 and 1860 was a major factor in the rapid growth of cities in the United States.

Question 18 (page 292)

A Incorrect. The Industrial Revolution led to the creation of factory management jobs. However, most Americans were farmers or industrial workers, not managers.

B Incorrect. The Industrial Revolution resulted in economic growth and an increase in business opportunities. As a result, many Americans started new businesses.

C Correct. The growth of the factory system was a key element of the Industrial Revolution. As more and more factories were built, the number of factory jobs increased.

D Incorrect. Factories produced most goods more cheaply than skilled craftspeople did. As a result, the number of skilled craftspeople decreased.

Question 19 (page 293)

A Incorrect. Clipper ships were extremely fast sailing ships that were used mainly for ocean travel. They were not important in opening up the Mississippi River system.

B Correct. Steamboats were well suited for moving cargo and passengers up and down the Mississippi River. They helped open this huge region to increased trade and settlement.

C Incorrect. Interchangeable parts made mass production possible. This development had far-reaching economic effects, but it was relatively insignificant to the opening of the Mississippi River system.

D Incorrect. Railroads were not important for trade and settlement along the Mississippi River during the early 1800s. Railroad lines did not reach this area until the 1850s.

Question 20 (page 293)

A Incorrect. There were many poor white southerners.

B Incorrect. Most southern whites owned no slaves. Some farmers with small farms owned one or two slaves and worked in the fields alongside them.

C Incorrect. There were many small farms in the South.

D Correct. Slave labor was used extensively on cotton and tobacco plantations in the South.

Question 21 (page 294)

A Incorrect. The development of the cotton gin allowed southern cotton growers to decrease production costs, which increased profits.

B Incorrect. The large increase in cotton production made possible by the cotton gin resulted in an increase in the use of slave labor.

C Incorrect. The increased amount of raw cotton made available to northern textile mills after the development of the cotton gin helped create more industrial jobs.

D Correct. The development of the cotton gin allowed the processing of raw cotton to become more efficient and profitable. This resulted in a great increase in cotton production.

Question 22 (page 294)

A Incorrect. Protective tariffs increased the price of imported manufactured goods. This had little direct effect on the cost of owning slaves.

B Incorrect. Protective tariffs raised the price of the machinery needed to process raw cotton, which made cotton production more expensive. This did not lead to a decrease in the price of cotton.

C Correct. Protective tariffs were a tax, or fee, that added to the price of foreign manufactured goods. The tariffs were protested by southerners because they felt that they were being unfairly burdened by these price increases.

D Incorrect. The intended purpose of the protective tariffs was to protect U.S. manufacturers. Ultimately, this encouraged new factory construction.

Question 23 (page 295)

A Correct. The women listed in the text box were all leaders of the women's rights movement. They worked to gain equal property rights and the right to vote for all women in the United States.

B Incorrect. While the women's rights movement was generally supportive of women's role in the traditional family, the women listed gained national attention from their political activism.

C Incorrect. Some of the women listed did support temperance. However, they were most active and visible in their support for economic rights and suffrage for women.

D Incorrect. The women's rights movement was primarily concerned with women's property rights and suffrage, not labor unions.

Objective 4

Question 24 (page 308)

A Incorrect. This right was first guaranteed by the English Bill of Rights, which placed limits on the powers of the English monarch over his or her subjects. This right was later guaranteed by the Eighth Amendment to the U.S. Constitution.

B Incorrect. These rights are guaranteed by the First Amendment to the U.S. Constitution.

C Incorrect. This guarantee is included in the Sixth Amendment to the U.S. Constitution.

D Correct. These are the unalienable rights listed in the Declaration of Independence.

Question 25 (page 308)

A Incorrect. The English Bill of Rights of 1689 established the principle of parliamentary supremacy in England. The Mayflower Compact was not a model for the English Bill of Rights.

B Correct. The Mayflower Compact was an agreement signed by the Pilgrims prior to the founding of the Plymouth colony in Massachusetts. It created a government for the new colony that would rule according to the wishes of the majority.

C Incorrect. The independence of the colonies from Great Britain was established by the Declaration of Independence in 1776.

D Incorrect. The Mayflower Compact did not deal with the issue of political equality. The most important document promoting the ideal of equality was the Declaration of Independence.

Question 26 (page 308)

A Incorrect. The Articles of Confederation were difficult to amend. All 13 states had to approve any changes to the document.

B Incorrect. The states had more power than the federal government under the Articles.

C Correct. The Articles did not allow the federal government to regulate interstate commerce. This caused economic problems since the states often refused to cooperate with one another.

D Incorrect. One of the national government's most important successes under the Articles was the Northwest Ordinance of 1787. This law established orderly procedures for westward expansion.

Question 27 (page 309)

A Incorrect. The U.S. Supreme Court case *Dred Scott* v. *Sandford* (1857) held that the federal government does not have the right to ban slavery in the territories.

B Correct. *Marbury* v. *Madison* held that federal courts have the power to declare acts of Congress unconstitutional.

C Incorrect. The U.S. Supreme Court case *McCulloch* v. *Maryland* (1819) held that the federal government has the power to establish a national bank.

D Incorrect. The U.S. Supreme Court case *Gibbons* v. *Ogden* (1824) held that Congress has broad powers to regulate commerce.

Question 28 (page 309)

A Incorrect. Tories, also known as Loyalists, were Americans who supported the British during the American Revolution. These people opposed the Declaration of Independence.

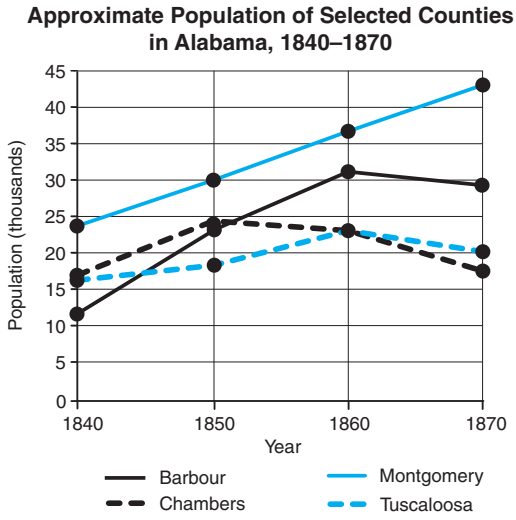
B Correct. Anti-Federalists opposed the ratification of the U.S. Constitution. They argued that the Constitution did not protect the rights of individuals and gave the federal government too much power over the state governments. They were also worried that the president might abuse the power given to him under the Constitution.

C Incorrect. Patriots were colonists who favored independence from British rule.

D Incorrect. Federalists believed that a strong national government was necessary to hold the United States together. They supported ratification of the Constitution.

Objective 5

Review Activity (page 316)



Question 29 (page 317)

A Incorrect. The graph shows that the slave population increased along with cotton production. This indicates that cotton production may have had an impact on the slave population. However, you need more information to prove there was a relationship.

B Correct. The graph shows that both the slave population and cotton production increased greatly from 1800 to 1860.

C Incorrect. The graph gives information on the increase in cotton production and the corresponding rise in slave population. It does not address the issue of whether slave labor was necessary or unnecessary to produce cotton.

D Incorrect. The graph clearly shows an increase in the slave population between 1800 and 1860, not a decrease.

Question 30 (page 318)

A Incorrect. The graph for Delaware shows that approximately 50% of that state's textile-mill workers were women.

B Incorrect. The graph for Pennsylvania shows that approximately 58% of that state's textile-mill workers were women.

C Incorrect. The graph for Rhode Island shows that approximately 68% of that state's textile-mill workers were women.

D Correct. The graph for Massachusetts shows that approximately 78% of that state's textile-mill workers were women.

Question 31 (page 318)

A Correct. The graph for Delaware shows that approximately 50% of that state's 1,400 textile-mill workers were women. This means that Delaware had approximately 700 female textile-mill workers.

B Incorrect. The graph for Pennsylvania shows that approximately 58% of that state's 14,900 textile-mill workers were women. This means that Pennsylvania had approximately 8,642 female textile-mill workers.

C Incorrect. The graph for Rhode Island shows that approximately 68% of that state's 5,000 textile-mill workers were women. This means that Rhode Island had approximately 3,400 female textile-mill workers.

D Incorrect. The graph for Massachusetts shows that approximately 78% of that state's 13,300 textile-mill workers were women. This means that Massachusetts had approximately 10,374 female textile-mill workers.

Question 32 (page 319)

A Incorrect. Republicanism is the concept of representative government. The excerpt does not address the issue of republicanism.

B Correct. In this excerpt Clay asserts that he does not owe "allegiance," or loyalty, to any particular region of the United States. This is a statement against sectionalism, or devotion to a single area of the United States instead of to the nation as a whole.

C Incorrect. Mercantilism was an economic theory that Great Britain used to guide its economic policies during much of the colonial period. The excerpt does not address the issue of mercantilism.

D Incorrect. Nationalism is the belief that loyalty to one's country is the most important responsibility an individual has. The excerpt shows Clay's opposition to sectionalism, not nationalism.

Question 33 (page 320)

A Correct. The map shows the Great Lakes as the northern boundary for the Northwest Territory.

B Incorrect. The Ohio River is shown as the southern boundary for the Northwest Territory.

C Incorrect. The Mississippi River is shown as the western boundary for the Northwest Territory.

D Incorrect. Lake Winnipeg is located northwest of the farthest reaches of the Northwest Territory and does not form a boundary for any part of it.

Question 34 (page 321)

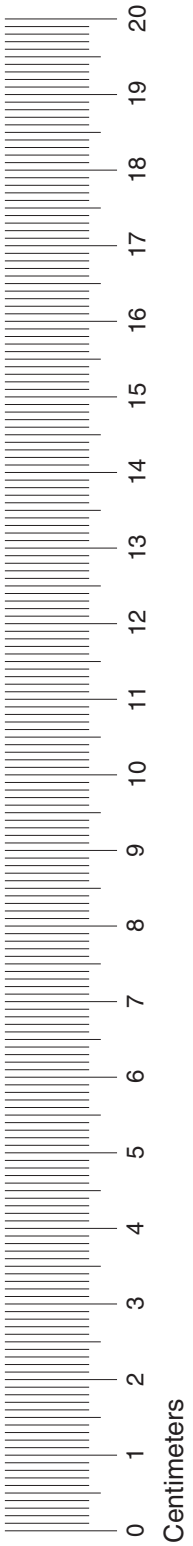
A Incorrect. The excerpt concerns the constitutionality of protective tariffs. The Bank War concerned the constitutionality of a national bank.

B Incorrect. The excerpt concerns the constitutionality of protective tariffs. *Marbury* v. *Madison* (1803) established the doctrine of judicial review.

C Correct. The excerpt concerns the constitutionality of protective tariffs. The Nullification Crisis began when South Carolina's leaders declared they would ignore a tariff they believed to be unconstitutional.

D Incorrect. The excerpt concerns the constitutionality of protective tariffs. The Alien and Sedition Acts were laws restricting free speech and the rights of immigrants in the late 1700s.

Grade 8
Mathematics Chart



LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days

1 year = 12 months

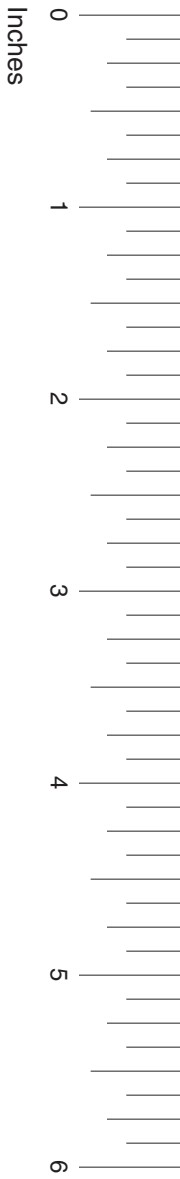
1 year = 52 weeks

1 week = 7 days

1 day = 24 hours

1 hour = 60 minutes

1 minute = 60 seconds



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Grade 8 Mathematics Chart

Perimeter	square	$P = 4s$
	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	square	$A = s^2$
	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	circle	$A = \pi r^2$
Surface Area	cube	$S = 6s^2$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism	$V = Bh^*$
	cylinder	$V = Bh^*$
	pyramid	$V = \frac{1}{3}Bh^*$
	cone	$V = \frac{1}{3}Bh^*$
	sphere	$V = \frac{4}{3}\pi r^3$
**B represents the area of the Base of a solid figure.*		
Pi	π	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Simple Interest Formula		$I = prt$

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