GRADE 10

Revised 2007

TAKS STUDY GUIDE

Texas Assessment of Knowledge and Skills

Mathematics

A Student and Family Guide

Revised Based on TEKS Refinements

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Texas Assessment of Knowledge and Skills

Grade 10

Mathematics

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A Letter from the Deputy Associate Commissioner for Student Assessment

Dear Student and Parent:

The Texas Assessment of Knowledge and Skills (TAKS) is a comprehensive testing program for public school students in grades 3–11. TAKS, including TAKS (Accommodated) and Linguistically Accommodated Testing (LAT), has replaced the Texas Assessment of Academic Skills (TAAS) and is designed to measure to what extent a student has learned, understood, and is able to apply the important concepts and skills expected at each tested grade level. In addition, the test can provide valuable feedback to students, parents, and schools about student progress from grade to grade.

Students are tested in mathematics in grades 3–11; reading in grades 3–9; writing in grades 4 and 7; English language arts in grades 10 and 11; science in grades 5, 8, 10, and 11; and social studies in grades 8, 10, and 11. Every TAKS test is directly linked to the Texas Essential Knowledge and Skills (TEKS) curriculum. The TEKS is the state-mandated curriculum for Texas public school students. Essential knowledge and skills taught at each grade build upon the material learned in previous grades. By developing the academic skills specified in the TEKS, students can build a strong foundation for future success.

The Texas Education Agency has developed this study guide to help students strengthen the TEKS-based skills that are taught in class and tested on TAKS. The guide is designed for students to use on their own or for students and families to work through together. Concepts are presented in a variety of ways that will help students review the information and skills they need to be successful on TAKS. Every guide includes explanations, practice questions, detailed answer keys, and student activities. At the end of this study guide is an evaluation form for you to complete and mail back when you have finished the guide. Your comments will help us improve future versions of this guide.

There are a number of resources available for students and families who would like more information about the TAKS testing program. Information booklets are available for every TAKS subject and grade. Brochures are also available that explain the Student Success Initiative promotion requirements and the new graduation requirements for eleventh-grade students. To obtain copies of these resources or to learn more about the testing program, please contact your school or visit the Texas Education Agency website at www.tea.state.tx.us/student.assessment.

Texas is proud of the progress our students have made as they strive to reach their academic goals. We hope the study guides will help foster student learning, growth, and success in all of the TAKS subject areas.

Sincerely,

Gloria Zyskowski

Gloria Zyskowski
Deputy Associate Commissioner for Student Assessment
Texas Education Agency

Contents

Mathematics

MATHEMATICS

What Is This Book?

This is a study guide to help you strengthen the skills tested on the Grade 10 Texas Assessment of Knowledge and Skills (TAKS). TAKS is a state-developed test administered with no time limit. It is designed to provide an accurate measure of learning in Texas schools.

By acquiring all the skills taught in tenth grade, you will be better prepared to succeed on the Grade 10 TAKS test and during the next school year.

What Are Objectives?

Objectives are goals for the knowledge and skills that you should achieve. The specific goals for instruction in Texas schools were provided by the Texas Essential Knowledge and Skills (TEKS). The objectives for TAKS were developed based on the TEKS.

How Is This Book Organized?

This study guide is divided into the ten objectives tested on TAKS. A statement at the beginning of each objective lists the mathematics skills you need to acquire. The study guide covers a large amount of material. You should not expect to complete it all at once. It may be best to work through one objective at a time.

Each objective is organized into review sections and a practice section. The review sections present examples and explanations of the mathematics skills for each objective. The practice sections feature mathematics problems that are similar to the ones used on the TAKS test.

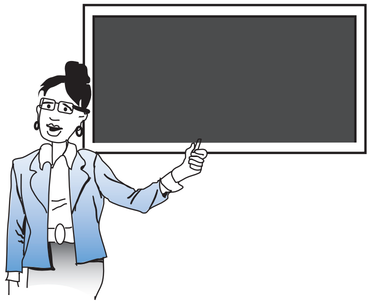
How Can I Use This Book?

First look at your Confidential Student Report. This is the report the school gave you that shows your TAKS scores. This report will tell you which TAKS subject-area test(s) you passed and which one(s) you did not pass. Use your report to determine which skills need improvement. Once you know which skills need to be improved, you can read through the instructions and examples that support those skills. You may also choose to work through all the sections. Pace yourself as you work through the study guide. Work in short sessions. If you become frustrated, stop and start again later.

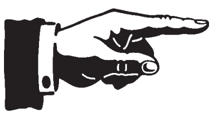
What Are the Helpful Features of This Study Guide?

- Look for the following features in the margin:

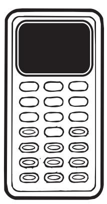
  Ms. Mathematics provides important instructional information for a topic.



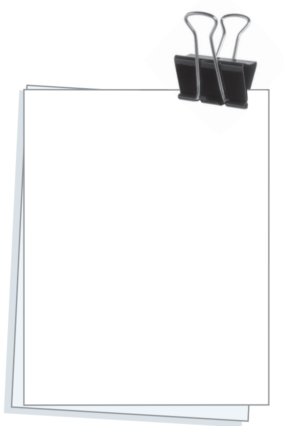
Do you see that . . . points to a significant sentence in the instruction.



Calculator suggests that using a graphing calculator might be helpful.



Memo provides page references in this study guide for additional information.



- There are several words in this study guide that are important for you to understand. These words are boldfaced in the text and are defined when they are introduced. Locate the boldfaced words and review the definitions.
- Examples are contained inside shaded boxes.
- Each objective has “Try It” problems based on the examples in the review sections.
- A Mathematics Chart for the Grade 10 TAKS test is included on pages 8–9 and also as a tear-out page in the back of the book. This chart includes useful mathematics information. The tear-out Mathematics Chart in the back of the book also provides both a metric and a customary ruler to help solve problems requiring measurement of length.

How Should the “Try It” Problems Be Used?

“Try It” problems are found throughout the review sections of the mathematics study guide. These problems provide an opportunity for you to practice skills that have just been covered in the instruction. Each “Try It” problem features lines for your responses. The answers to the “Try It” problems are found immediately following each problem.

While completing a “Try It” problem, cover up the answer portion with a sheet of paper. Then check the answer.

What Kinds of Practice Questions Are in the Study Guide?

The mathematics study guide contains questions similar to those found on the Grade 10 TAKS test. There are two types of questions in the mathematics study guide.

- Multiple-Choice Questions: Most of the practice questions are multiple choice with four answer choices. These questions present a mathematics problem using numbers, symbols, words, a table, a diagram, or a combination of these. Read each problem carefully. If there is a table or diagram, study it. You should read each answer choice carefully before choosing the best answer.
- Griddable Questions: Some practice questions use an eight-column answer grid like those used on the Grade 10 TAKS test.

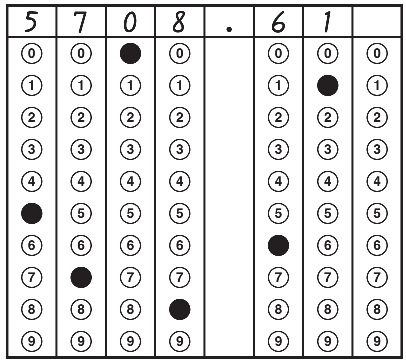
How Do You Use an Answer Grid?

The answer grid contains eight columns, which include three decimal places: tenths, hundredths, and thousandths.

Suppose 5708.61 is the answer to a problem. First write the number in the blank spaces. Be sure to use the correct place value. For example, 5 is in the thousands place, 7 is in the hundreds place, 0 is in the tens place, 8 is in the ones place, 6 is in the tenths place, and 1 is in the hundredths place.

Then fill in the correct bubble under each digit. Notice that if there is a zero in the answer, you need to fill in the bubble for the zero.

The grid shows 5708.61 correctly entered. The zero in the tens place is bubbled in because it is part of the answer. It is not necessary to bubble in the zero in the thousandths place, because this zero will not affect the value of the correct answer.



Where Can Correct Answers to the Practice Questions Be Found?

The answers to the practice questions are in the answer key at the back of this book (pages 235–254). Each question includes a reference to the page number in the answer key for the answer to the problem. The answer key explains the correct answer, and it also includes some explanations for incorrect answers. After you answer the practice questions, you can check your answers.

If you still do not understand the correct answer after reading the answer explanations, ask a friend, family member, or teacher for help. Even if you have chosen the correct answer, it is a good idea to read the answer explanation because it may help you better understand why the answer is correct.

Grades 9, 10, and Exit Level Mathematics Chart

TAKS Texas Assessment of Knowledge and Skills

LENGTH

Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces

TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Metric and customary rulers can be found on the tear-out Mathematics Chart in the back of this book.

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$
P represents the Perimeter of the Base of a three-dimensional figure.		
B represents the Area of the Base of a three-dimensional figure.		
Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$
	pyramid or cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	30°, 60°, 90°	$x,\ x\sqrt{3},\ 2x$
	45°, 45°, 90°	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$

Objective 1

The student will describe functional relationships in a variety of ways.

For this objective you should be able to recognize that a function represents a dependence of one quantity on another and can be described in a number of ways.

What Is a Function?

A **function** is a set of ordered pairs (x, y) in which each x-coordinate is paired with only one y-coordinate. In a list of ordered pairs belonging to a function, no x-coordinate is repeated.

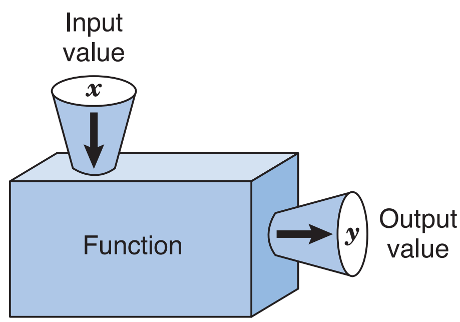
When people eat at a restaurant, they often use a tip chart to determine the amount of the tip to leave the server. A tip chart is a good example of a function.

Tip Chart

Cost of Meal	Recommended Tip
$5.00	$1.00
$6.00	$1.20
$7.00	$1.40
$8.00	$1.60
$9.00	$1.80
$10.00	$2.00

On the tip chart above, each meal cost listed has exactly one recommended tip listed. Since the amount of the tip depends on how much the meal costs, the recommended tip is a function of the cost of the meal.

In a functional relationship, for any given input there is a unique output.



Input an x-value and you get a y-value.

If you are given an x-value belonging to a function, you can find the corresponding y-value.

Do you see that . . .

If you input $5.00 into the above function, the output will be $1.00.

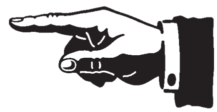
Objective 1

There are two ways to test a set of ordered pairs to see whether it is a function.

Examine the list of ordered pairs.

If a set of ordered pairs is a function, no x-coordinate in the set is repeated. No x-coordinate should be listed with two different y-coordinates.

> Is this set of ordered pairs a function?
>
> $$\{(1, 4), (5, 7), (-1, 7), (10, 12)\}$$
>
> Examine the set of ordered pairs.
>
> - None of the x-coordinates in the set are repeated.
> - Two ordered pairs, (5, 7) and $(-1, 7)$, have the same y-coordinate but different x-coordinates. This does not prevent this set of ordered pairs from being a functional relationship.
>
> This set of ordered pairs is a function.



Do you see that . . .

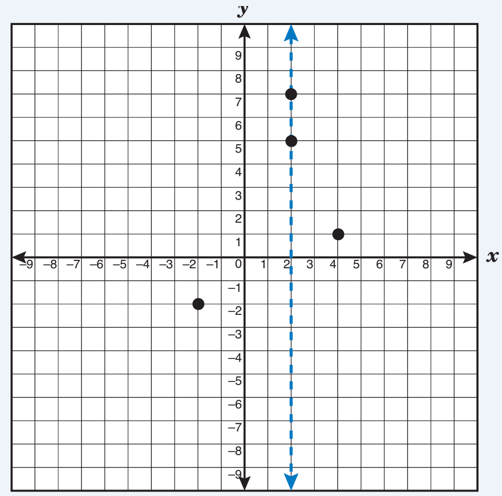
> Is this set of ordered pairs a function?
>
> $$\{(-2, 5), (0, 7), (1, 4), (-2, 6)\}$$
>
> - The number -2 is paired with 5; 0 is paired with 7; 1 is paired with 4; and -2 is paired with 6.
> - Two ordered pairs, $(-2, 5)$ and $(-2, 6)$, have the same x-coordinate. In a functional relationship, no x-coordinate should repeat.
>
> This set of ordered pairs is not a function.

Examine a graph of a set of ordered pairs.

Use a vertical line to determine whether two points have the same x-coordinate. If two points in the graph of a function lie on the same vertical line, then they have the same x-coordinate, and the set of ordered pairs is not a function.

Objective 1

Do the ordered pairs graphed below represent a function?



The ordered pairs (2, 5) and (2, 7) lie on a common vertical line.

They have the same *x*-coordinate, 2, but different *y*-coordinates, 5 and 7.

This graph does not represent a function because two points lie on the same vertical line.

In a function, the *y*-coordinate is described in terms of the *x*-coordinate. The value of the *y*-coordinate depends on the value of the *x*-coordinate.

Do you see that . . .

Suppose the number of miles you walk is equal to 4 times the number of hours you walk. Which is the dependent quantity in this function?

If you walked for 1 hour, you would have walked 4 miles.
If you walked for 3 hours, you would have walked 12 miles.

The distance you walk depends on, or is described in terms of, the number of hours you walk.

In this function, the number of hours you walk is the **independent** quantity. The distance you walk is the **dependent** quantity.

An equation that describes this function is $d = 4h$, where d represents the number of miles you walk and h represents the number of hours. In this equation, d, the distance you walk, depends on h, the number of hours you walk.

- The variable h is the independent variable.
- The variable d is the dependent variable.
- The number 4 is a **constant**, a quantity in an equation that does not change.

Suppose the equation $c = 0.07m + 0.25$ describes c, the cost of a phone call, in terms of m, the number of minutes the phone call lasts.

In this function, c is the dependent variable, m is the independent variable, and 0.07 and 0.25 are constants.

Jeremy works at an appliance store. He is paid $180.00 a week for his base salary plus a commission equal to 5% of his total sales. The equation $s = 180 + 0.05d$ represents Jeremy's weekly salary, s, in terms of d, his total weekly sales in dollars.

Which variable is the dependent variable in this equation? What are the constants?

- Jeremy's weekly salary, s, is the dependent quantity because it depends on d, his total sales.
- The constants are 180, Jeremy's base salary, and 0.05, his commission, because these numbers do not change.

Try It

Cara buys milk for her scout camp each morning. The function below shows the relationship between c, the total cost of the milk she buys, and n, the number of quarts she purchases.

$$c = 1.25n$$

In this functional relationship, which value is the dependent quantity?

The ______________________ quantity is the number of quarts of milk Cara purchases.

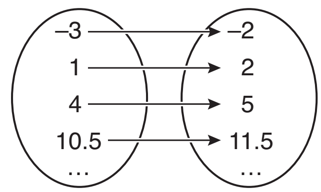
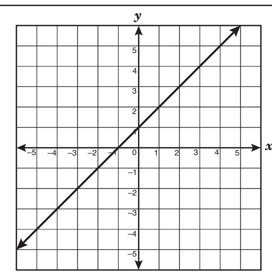
The ______________________ quantity is the total cost of the milk because the cost depends on the number of quarts of milk Cara buys.

The independent quantity is the number of quarts of milk Cara purchases. The dependent quantity is the total cost of the milk because the cost depends on the number of quarts of milk Cara buys.

Objective 1

How Can You Represent a Function?

Functional relationships can be represented in a variety of ways.

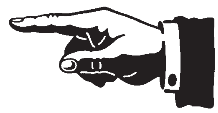
Method	Description	Example
List	List several ordered pairs.	{(−3, −2), (1, 2), (4, 5), (10.5, 11.5), …}
Table	Place the ordered pairs in a table.	x: −3, 1, 4, 10.5, … / y: −2, 2, 5, 11.5, …
Mapping	Draw a picture that shows how the ordered pairs are formed.	
Description	Use words to describe the functional relationship.	The *y*-values for a set of points are 1 more than the corresponding *x*-values.
Equation	Write an equation that describes the *y*-coordinate in terms of the *x*-coordinate.	$y = x + 1$
Function notation	Write a special type of equation that uses $f(x)$ to represent *y*.	$f(x) = x + 1$
Graph	Graph the ordered pairs.	

To use **function notation** to describe a function, give the function a name, typically a letter such as *f*, *g*, or *h*. Then use an algebraic expression to describe the *y*-coordinate of an ordered pair.

Suppose $f(x) = 3x - 1$.

- This function is read as "*f* of *x* equals 3 times *x* minus 1."
- If you input *x*, the output will be $3x - 1$.
- The *y*-coordinate of the ordered pair is $3x - 1$.

The function described by $f(x) = 3x - 1$ is the same as the function described by $y = 3x - 1$. In this function, an ordered pair looks like this: $(x, 3x - 1)$.



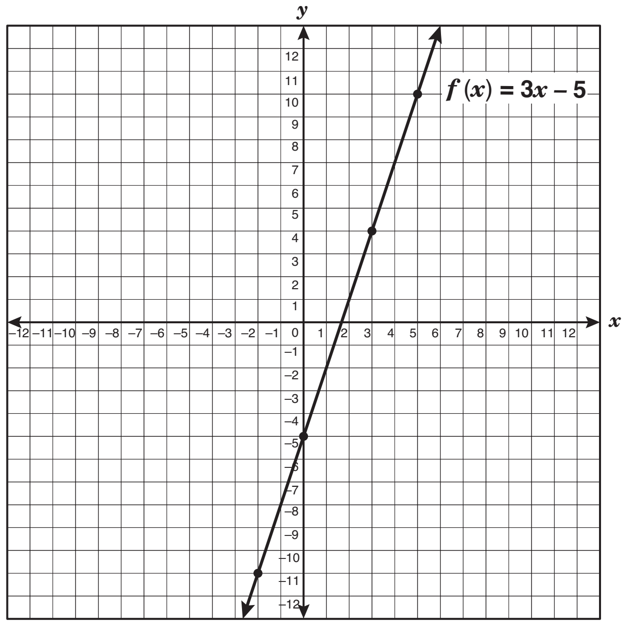
Do you see that . . .

Here are three methods you can use to determine whether two different representations of a function are equivalent.

Method	Action
Match a list or table of ordered pairs to a graph.	• Show that each ordered pair listed matches a point on the graph.
Match an equation to a graph.	• Determine whether they are both linear or quadratic functions. • Find points on the graph and show that their coordinates satisfy the equation. • Find points that satisfy the equation and show that they are on the graph.
Match a verbal description to a graph, an equation, or an expression written in function notation.	• Use the verbal description to find ordered pairs belonging to the function and then show that they satisfy the graph, equation, or function rule. • Find points on the graph or ordered pairs satisfying the equation or rule and show that they satisfy the verbal description.

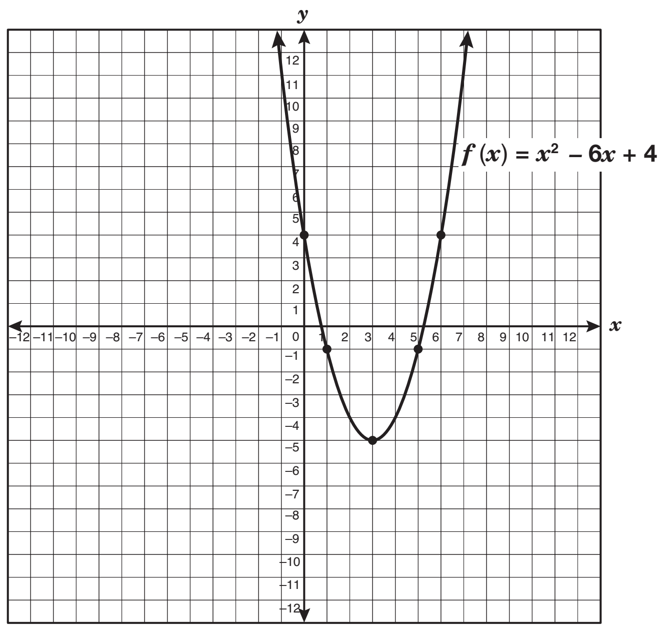
Any equation of the form $y = mx + b$ is a **linear function**. Its graph will be a line.

x	y
−2	−11
0	−5
3	4
5	10



Any equation of the form $y = ax^2 + bx + c$, where $a \neq 0$, is a **quadratic function**. Its graph will be a parabola.

x	y
0	4
1	−1
3	−5
5	−1
6	4



Which ordered pair, (7, 11) or (−4, −1), belongs to the function in which the *y*-coordinate is 3 more than the *x*-coordinate?

Match the ordered pairs to the verbal description.

- For the ordered pair (7, 11), the *y*-coordinate should be 3 more than 7.

$$7 + 3 = 10, \text{ not } 11$$

The ordered pair (7, 11) does not belong to this function.

- For the ordered pair (−4, −1), the *y*-coordinate should be 3 more than −4.

$$-4 + 3 = -1$$

The ordered pair (−4, −1) belongs to this function.

A variety of methods of representing a function are shown below. Which of these examples represents a function that is different from the other functions?

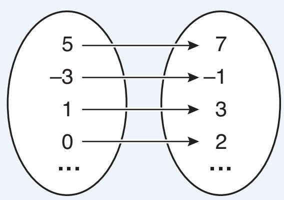
A. Verbal Description
The value of *y* is double the value of *x*.

B. List of Selected Values
{(3, 6), (−2, −4), (1, 2), (0, 0), (−5, −10), …}

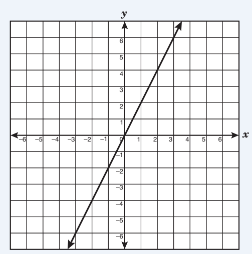
C. Table of Selected Values

x	*y*
−1	−2
2	4
0	0
5	10
…	…

D. Mapping of Selected Values



E. Graph



F. Equation
$y = 2x$

Look at the ordered pairs that make up each function.

- In Examples A, B, C, E, and F, each *x*-coordinate is paired with a *y*-coordinate that is its double, or two times as great.
- In Example D, for each ordered pair listed, the *x*-coordinate is paired with a *y*-coordinate that is two more rather than two times as great. So Example D includes ordered pairs that are not in the other examples.

Only Example D represents a function that is different from the other functions shown.

Objective 1

The table below presents selected values in a functional relationship.

x	-2	1	5	7
y	3	6	10	12

Write an equation that describes this functional relationship.

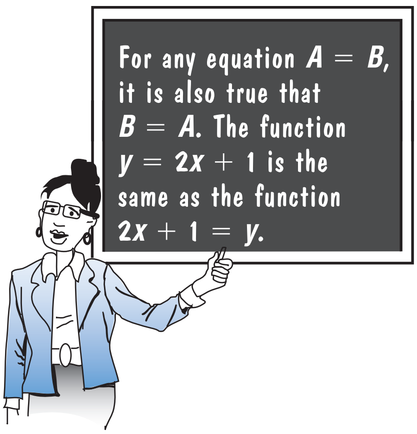
- Look for a pattern in the ordered pairs that belong to the function.

  The y-coordinate appears to be 5 more than the x-coordinate, so $x + 5 = y$ should represent the pattern.
- Check this equation for each pair.

	$\underline{x + 5 = y}$
For $x = -2$	$-2 + 5 = 3$
For $x = 1$	$1 + 5 = 6$
For $x = 5$	$5 + 5 = 10$
For $x = 7$	$7 + 5 = 12$

The y-coordinate in each ordered pair is 5 more than the x-coordinate.

The rule for this function can be represented by the equation $x + 5 = y$ or by the equation $y = x + 5$.



The table on the right presents selected values in a functional relationship between x and y.

x	y
-1	-3
1	7
3	17
8	42

Using function notation, write a rule that represents the relationship.

- Look for a pattern in the function's ordered pairs.

  The y-coordinate appears to be 5 times the x-coordinate plus 2.
- Check this pattern for each pair.

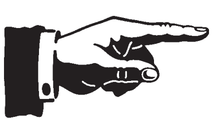
	$\underline{5x + 2 = y}$
$(-1, -3)$	$5 \cdot -1 + 2 = -3$
$(1, 7)$	$5 \cdot 1 + 2 = 7$
$(3, 17)$	$5 \cdot 3 + 2 = 17$
$(8, 42)$	$5 \cdot 8 + 2 = 42$

The y-coordinate is equal to 5 times the x-coordinate plus 2.

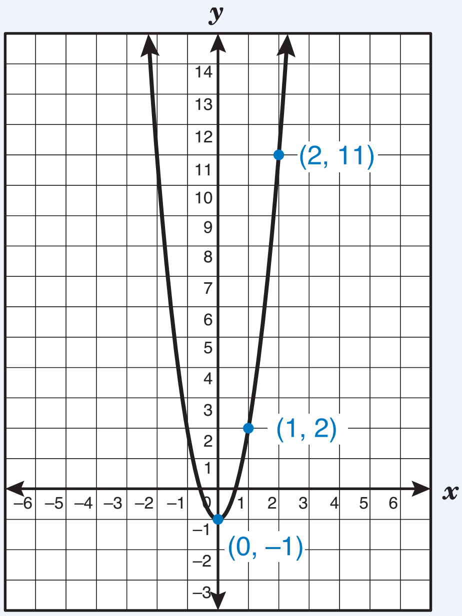
The rule for this function can be represented by the equation $y = 5x + 2$.

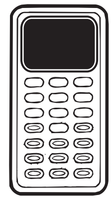
Replace y with $f(x)$ to express the rule in function notation: $f(x) = 5x + 2$.

Do you see that . . .



Does the graph below represent the same function as the equation $y = 3x^2 - 1$?





- The function $y = 3x^2 - 1$ is a quadratic function because it can be described by an equation in the form $y = ax^2 + bx + c$, where $a \neq 0$. Its graph should be a parabola. The above graph is a parabola, so the equation and the graph are the same type of function.
- Show that the equation and the graph represent the same quadratic function with the same set of ordered pairs.
- First check the coordinates of points on the graph to see whether they satisfy the equation. Pick points on the graph whose coordinates are easy to read. For example, you might choose (0, −1), (1, 2), and (2, 11).

  Substitute these values into $y = 3x^2 - 1$ and determine whether the equation is true.

Point (0, –1)	Point (1, 2)	Point (2, 11)
$x = 0$ and $y = -1$	$x = 1$ and $y = 2$	$x = 2$ and $y = 11$
$y = 3x^2 - 1$	$y = 3x^2 - 1$	$y = 3x^2 - 1$
$-1 \stackrel{?}{=} 3(0)^2 - 1$	$2 \stackrel{?}{=} 3(1)^2 - 1$	$11 \stackrel{?}{=} 3(2)^2 - 1$
$-1 \stackrel{?}{=} 3(0) - 1$	$2 \stackrel{?}{=} 3(1) - 1$	$11 \stackrel{?}{=} 3(4) - 1$
$-1 \stackrel{?}{=} 0 - 1$	$2 \stackrel{?}{=} 3 - 1$	$11 \stackrel{?}{=} 12 - 1$
$-1 = -1$	$2 = 2$	$11 = 11$

  The points (0, −1), (1, 2), and (2, 11) are points on the graph, and their coordinates satisfy the equation.

- Next find ordered pairs that satisfy the equation and confirm that the points are on the graph. Pick values that are easy to substitute, like $x = 1$, $x = 0$, or $x = -2$, and find the corresponding values for y.

Substitute these values into $y = 3x^2 - 1$ and determine the value for y.

$\underline{x = 1}$	$\underline{x = 0}$	$\underline{x = -2}$
$y = 3x^2 - 1$	$y = 3x^2 - 1$	$y = 3x^2 - 1$
$y = 3(1)^2 - 1$	$y = 3(0)^2 - 1$	$y = 3(-2)^2 - 1$
$y = 3(1) - 1$	$y = 3(0) - 1$	$y = 3(4) - 1$
$y = 3 - 1$	$y = 0 - 1$	$y = 12 - 1$
$y = 2$	$y = -1$	$y = 11$

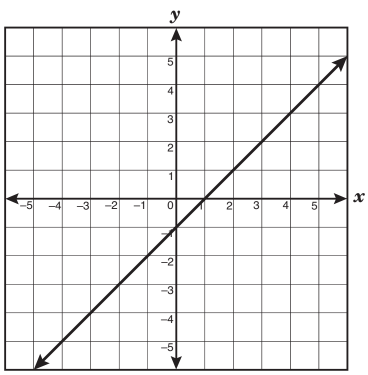
The ordered pairs (1, 2), (0, −1), and (−2, 11) satisfy the equation.

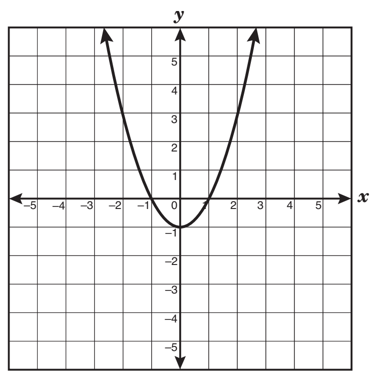
Confirm that these ordered pairs are points on the graph. Yes, all three points are on the graph.

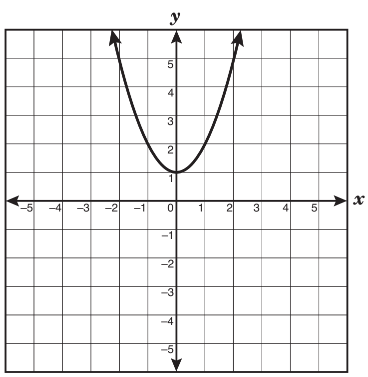
The graph does represent the relationship $y = 3x^2 - 1$.

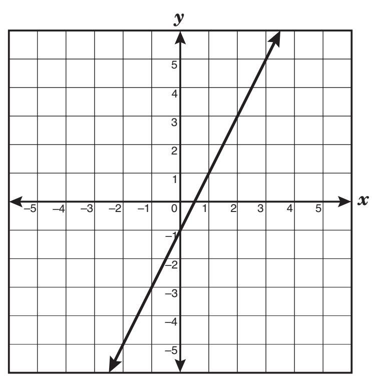
Try It

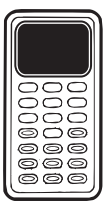
The equation $y = x^2 - 1$ represents a functional relationship. Which graph represents this function?

A


C


B


D




Determine whether the equation is a linear or quadratic function.

The equation is a ________________ function because it can be described by an equation in the form $y = ax^2 + bx + c$, where $a \neq 0$.

Its graph must be a ________________.

Answer choices ______ and ______ cannot be the graph of this function because they are _______________.

Determine which parabola is the correct graph.

See whether the point (0, 1) in answer choice B satisfies the equation $y = x^2 - 1$.

When $x = 0$ and $y =$ ______, is the equation $y = x^2 - 1$ true?

Does ______ $=$ ______ $-$ ______?

No, ______ $\neq$ ______.

Answer choice B is _______________.

See whether the point $(0, -1)$ in answer choice C satisfies the equation $y = x^2 - 1$.

When $x =$ ______ and $y =$ ______, is the equation $y = x^2 - 1$ true?

Does ______ $=$ ______ $-$ ______?

Yes, ______ $=$ ______.

Answer choice C is ___________________.

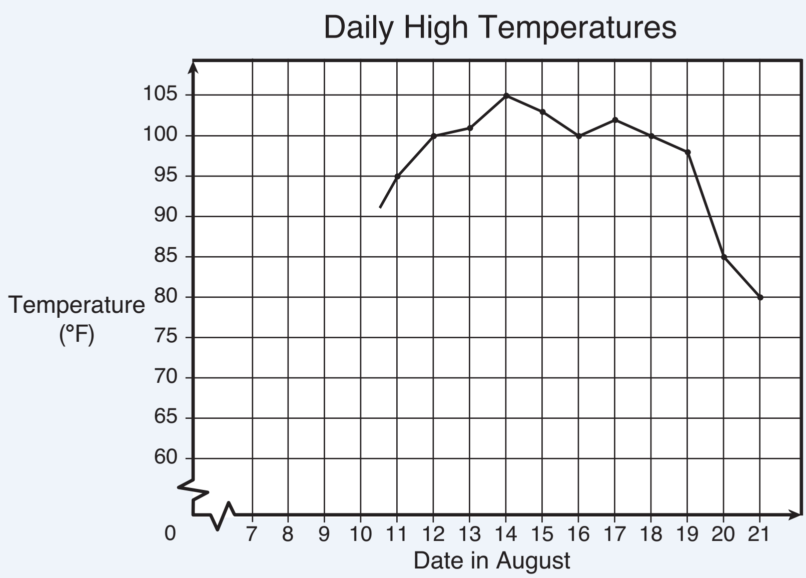
The equation is a quadratic function because it can be described by an equation in the form $y = ax^2 + bx + c$, where $a \neq 0$. Its graph must be a parabola. Answer choices A and D cannot be the graph of this function because they are lines. When $x = 0$ and $y = 1$, is the equation $y = x^2 - 1$ true? Does $1 = 0^2 - 1$? No, $1 \neq -1$. Answer choice B is not correct. When $x = 0$ and $y = -1$, is the equation $y = x^2 - 1$ true? Does $-1 = 0^2 - 1$? Yes, $-1 = -1$. Answer choice C is correct.

How Can You Draw Conclusions from a Functional Relationship?

Use these guidelines when interpreting functional relationships in a real-life problem.

- Understand the problem.
- Identify the quantities involved and any relationships between them.
- Determine what the variables in the problem represent.
- For graphs: Determine what quantity each axis on the graph represents. Look at the scale that is used on each axis.
- For tables: Determine what quantity each column in the table represents.
- Look for trends in the data. Look for maximum and minimum values in graphs.
- Look for any unusual data. For example, does a graph start at a nonzero value? Is one of the problem's variables negative at any point?
- Match the data to the equations or formulas in the problem.

The graph below shows the daily high temperatures in degrees Fahrenheit over a two-week period in August. During which three-day period did the temperature decrease by the greatest number of degrees?



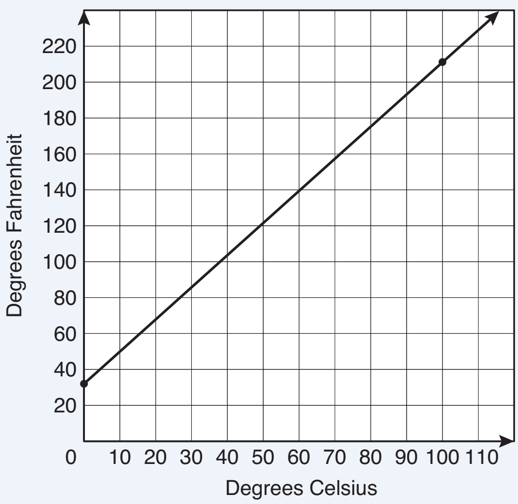
The high temperature decreased from August 14 to August 16 and then again from August 17 to August 21. To determine which three-day period it decreased by the greatest number of degrees, you need to find the coordinates of these points and calculate the total drop in the daily high temperature.

You could build a table to organize your work.

3-Day Period	Day 1 High Temperature	Day 2 High Temperature	Day 3 High Temperature	Decrease
Aug. 14, 15, 16	105°F	103°F	100°F	5°F
Aug. 17, 18, 19	102°F	100°F	98°F	4°F
Aug. 18, 19, 20	100°F	98°F	85°F	15°F
Aug. 19, 20, 21	98°F	85°F	80°F	18°F

The high temperature decreased by 18°F from August 19 to August 21. This was the greatest decrease in temperature for any three-day period on the graph.

The formula for converting degrees Celsius, C, to degrees Fahrenheit, F, is $F = \frac{9}{5}C + 32$.



Does the graph represent this function accurately?

- The points increase at a rate of 9°F for every 5°C, and the slope of the graph should be $\frac{9}{5}$, the coefficient of C.
- Two points on the graph are (0, 32) and (100, 212). When the coordinates are substituted into the equation $F = \frac{9}{5}C + 32$, the equation is true.

$F = \frac{9}{5}C + 32$	$F = \frac{9}{5}C + 32$
$212 = \frac{9}{5}(100) + 32$	$32 = \frac{9}{5}(0) + 32$
$212 = 180 + 32$	$32 = 0 + 32$
$212 = 212$	$32 = 32$

Yes, the graph accurately represents this function.

Now practice what you've learned.

Question 1

Rhonda works at a grocery store after school. She is paid $5.50 per hour. Her weekly salary, s, is described by the function $s = 5.5h$, where h is the number of hours she works in a week. What is the dependent quantity in this functional relationship?

A The number of hours she works in a week

B The number of dollars she is paid per hour

C The total salary for a week

D The number of days she works in a week

Answer Key: page 235

Question 2

The number of pretzels, p, that can be packaged in a box with a volume of V cubic units is given by the equation $p = 45V + 10$. In this relationship, which is the dependent variable?

A 10

B 45

C p

D V

Answer Key: page 235

Question 3

Which of the following tables does not represent a function?

A

x	y
1	2
4	1
−1	2
−4	1

C

x	y
1	1
−2	4
3	9
1	16

B

x	y
−1	−1
3	3
2	2
5	5

D

x	y
2	0
1	1
3	5
4	0

Answer Key: page 235

Question 4

The table shows the independent and dependent values in a functional relationship. Which function best represents this relationship?

Independent	Dependent
0	1
1	5
2	13
3	25

A $f(x) = 2x^2 - 2x + 1$

B $f(x) = 2x^2 + 2x + 1$

C $f(x) = x^2 + 1$

D $f(x) = 8x + 9$

Answer Key: page 235

Question 5

Jane started a weekend pet-care business. She bought the necessary supplies for $210. Jane charges $25 per weekend for each pet she cares for. Which function best represents her net profit in terms of x, the number of pets she cares for?

A $f(x) = x + 25 - 210$

B $f(x) = 210 - 25x$

C $f(x) = 210x - 25$

D $f(x) = 25x - 210$

Answer Key: page 235

Question 6

Jeb's stereo is playing at a volume of 75 decibels. If the decibel level reaches 120 decibels, the neighbors will complain. Which inequality models q, the number of decibels Jeb can increase the volume before the neighbors complain?

A $75 + q < 120$

B $75 - q > 120$

C $75 + q > 120$

D $75 - q < 120$

Answer Key: page 235

Question 7

Which table best represents the function $f(x) = \frac{2}{5}x - 3$?

A

x	y
0	−3
5	−1
10	−1
15	9

B

x	y
−10	−7
0	−3
10	1
20	5

C

x	y
−1	−5
10	1
30	9
−20	5

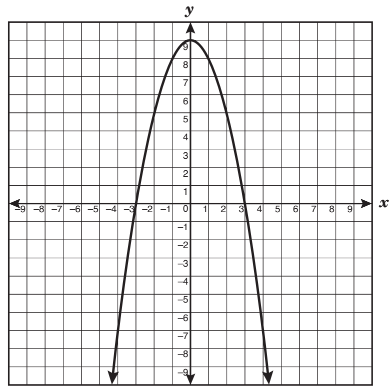
D

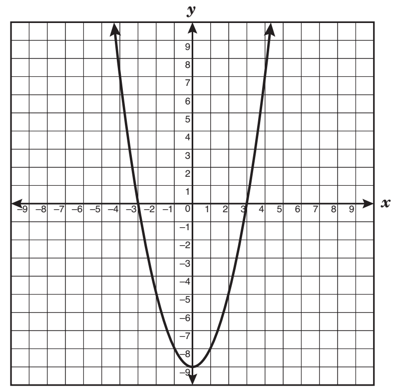
x	y
−10	−7
30	9
60	−21
−15	3

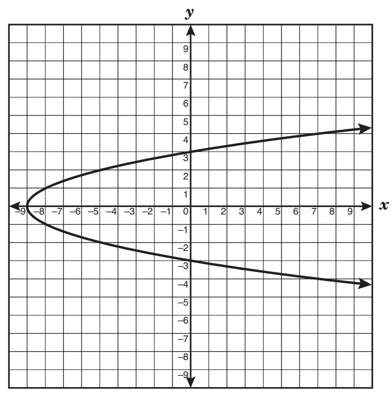
Answer Key: page 235

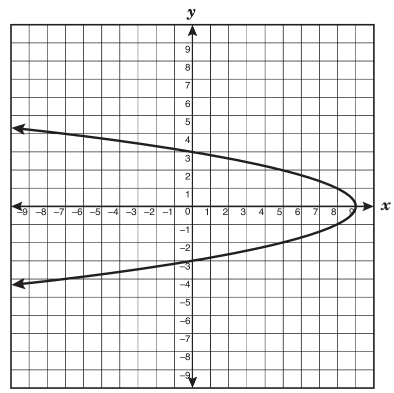
Question 8

Which graph best represents the function $f(x) = x^2 - 9$?

A


C


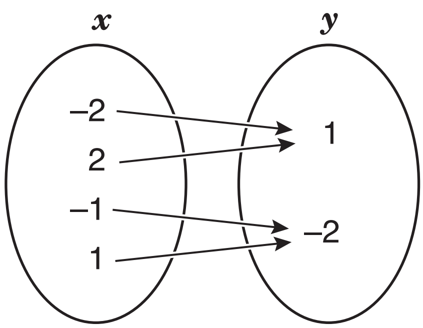
B


D


Answer Key: page 236

Question 9

Which equation best represents the mapping shown below?



A $y = -3x - 5$

B $y = x^2 - 3$

C $y = 3 - x^2$

D $y = 3x - 5$

Answer Key: page 236

Question 10

A fitness club charges a one-time registration fee, plus a monthly fee for a membership. The table below shows the total membership charges paid after different numbers of months.

Fitness-Club Membership Charges

Number of Months	Total Charges
1	$170
3	$230
5	$290
7	$350

Which conclusion can be drawn from the information presented in the table?

A The monthly fee is $50.

B The monthly fee is $60.

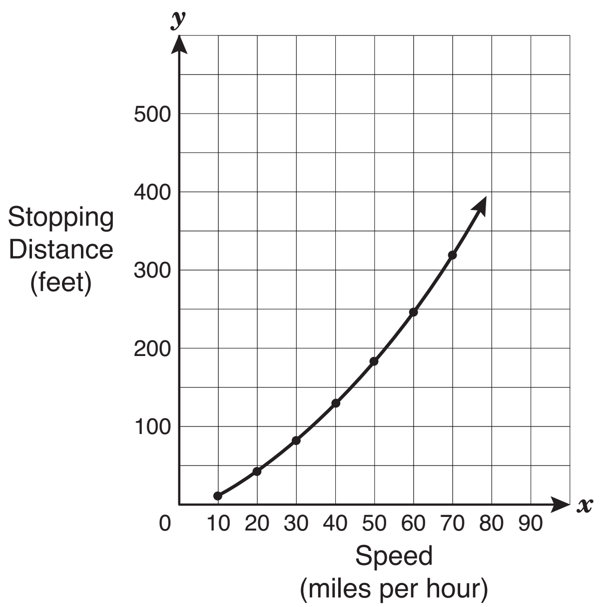
C The registration fee is $110.

D The registration fee is $140.

Answer Key: page 236

Question 11

A graph of the relationship between the speed of a car in miles per hour and the car's approximate stopping distance in feet is shown below.



What is the approximate stopping distance for a car traveling 70 miles per hour?

A 350 ft

B 325 ft

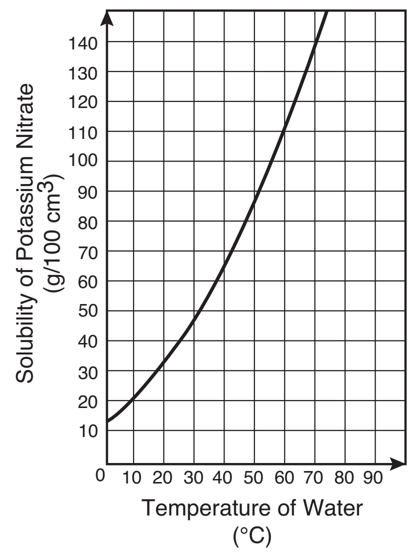
C 250 ft

D 300 ft

Answer Key: page 236

Question 12

Potassium nitrate dissolves in water to form a solution. The graph below shows the solubility of potassium nitrate in water as a function of the temperature of the water.



According to this graph, about how many grams of potassium nitrate will dissolve in 100 cm^3 of water at 42°C?

A 58 grams

B 82 grams

C 77 grams

D 68 grams

Answer Key: page 236

Question 13

A home-builders group recently published a study comparing the cost of building homes from 1,000 to 3,000 square feet in area in four different communities. The study found that the formulas below predicted the approximate cost, c, of building a new home in each of these communities in terms of f, the area of the home in square feet.

Community	Cost of Building a New Home
R	$c = 15{,}000 + 80f$
S	$c = 25{,}000 + 75f$
T	$c = 60{,}000 + 50f$
V	$c = 40{,}000 + 65f$

Based on these formulas, in which community would it cost the least to build a home with an area of 1,450 square feet?

A T

B S

C R

D V

Answer Key: page 236

Objective 2

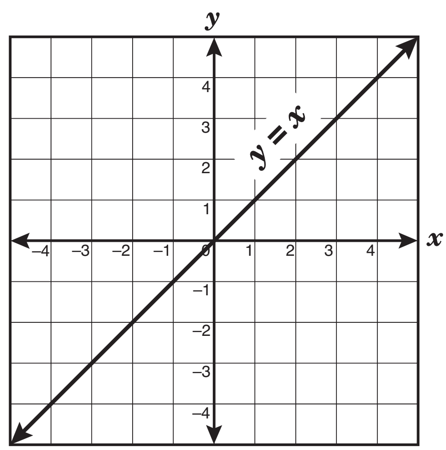
The student will demonstrate an understanding of the properties and attributes of functions.

For this objective you should be able to

- use the properties and attributes of functions;
- use algebra to express generalizations and use symbols to represent situations; and
- manipulate symbols to solve problems and use algebraic skills to simplify algebraic expressions and solve equations and inequalities in problem situations.

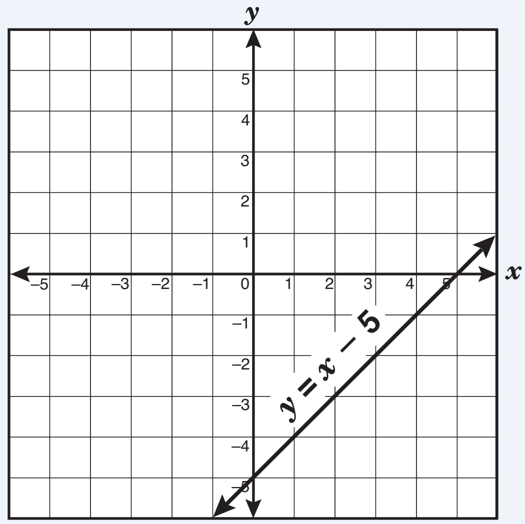
What Are Parent Functions?

The simplest linear function, $y = x$, is the **linear parent function**.

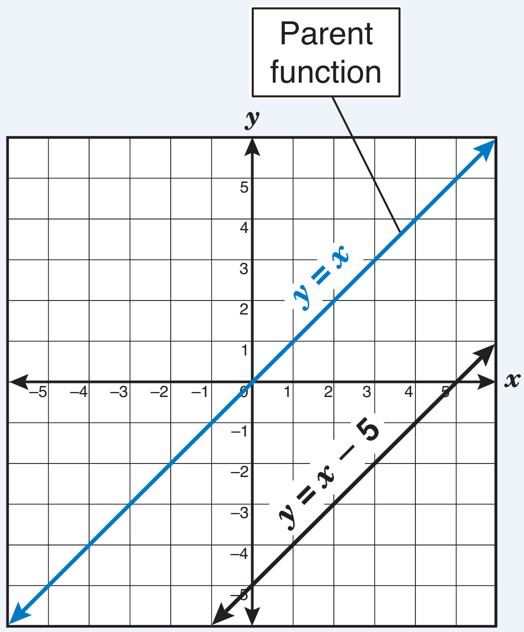


- If the graph of any function is a line, then its parent function is $y = x$.
- If a function can be written in the form $y = mx + b$, then it is linear.
- A linear function never has variables raised to a power other than 1.
- If a function is linear, then its parent function is $y = x$.
- An equation in the form $x = a$ is a linear equation, but it is not a function. Its graph is a vertical line.

What is the parent function of this graph?



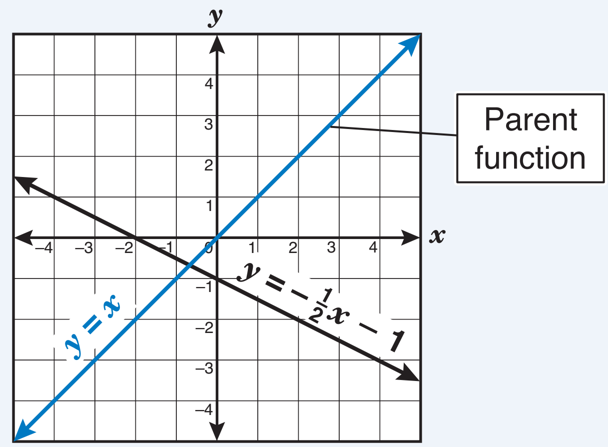
Since the graph of $y = x - 5$ is a line, its parent function is the linear parent function, $y = x$.



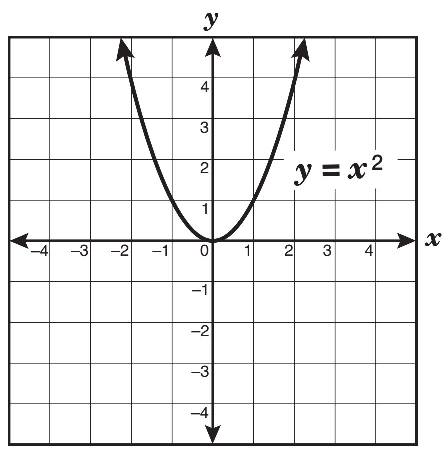
Objective 2

What is the parent function of the equation $y = -\frac{1}{2}x - 1$?

Since the equation $y = -\frac{1}{2}x - 1$ is a linear function, its graph is a line. Its parent function is the linear parent function, $y = x$.

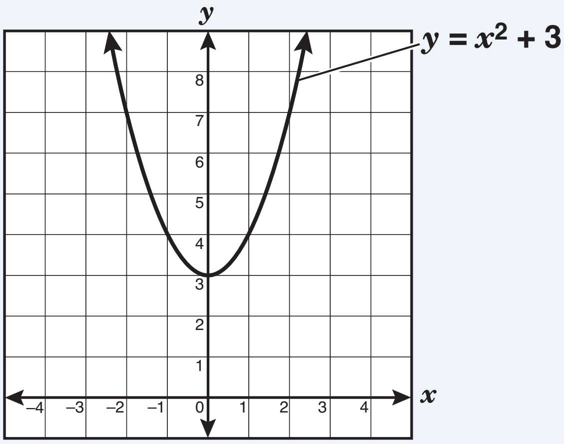


The simplest quadratic function, $y = x^2$, is the **quadratic parent function**.

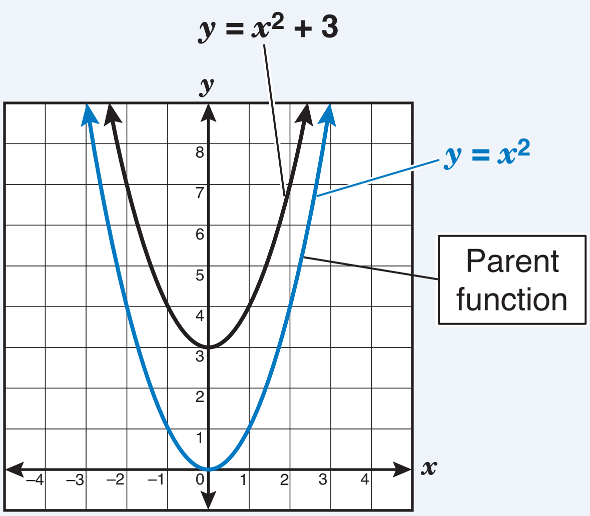


- If the graph of any function is a parabola, then its parent function is $y = x^2$.
- If an equation can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$, then it is quadratic.
- If an equation can be written in this form, then its parent function is $y = x^2$.

What is the parent function of this graph?

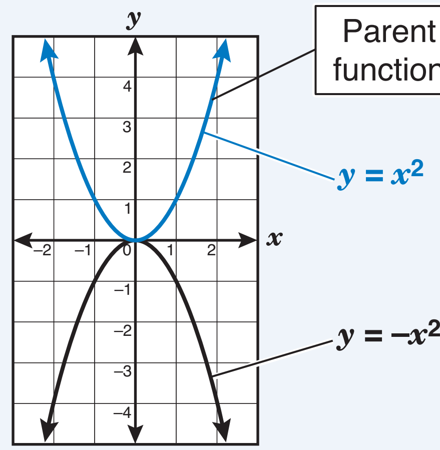


Since the graph of $y = x^2 + 3$ is a parabola, its parent function is the quadratic parent function, $y = x^2$.



Objective 2

What is the parent function of $y = -x^2$?



The equation $y = -x^2$ is a quadratic function; therefore, its parent function is the quadratic parent function, $y = x^2$.

What Are the Domain and Range of a Function?

A **function** is a set of ordered pairs of numbers (x, y) such that no x-values are repeated. The domain and range of a function are sets that describe those ordered pairs.

See Objective 1, page 10, for more information about functions.

	Definition	Example {(0,1), (2, 6), (3, 5)}
Domain	The set of all the x-coordinates in the function's ordered pairs	{0, 2, 3}
Range	The set of all the y-coordinates in the function's ordered pairs	{1, 5, 6}

- The **domain** is the set of all the values of the independent variable, the x-coordinate.
- The **range** is the set of all the values of the dependent variable, the y-coordinate.

Identify the domain and range of the function below.

{(3, 9), (5, 39), (9, 23), (6, 14)}

The domain is the set of x-coordinates in the ordered pairs: {($\underline{3}$, 9), ($\underline{5}$, 39), ($\underline{9}$, 23), ($\underline{6}$, 14)}. The domain is {3, 5, 6, 9}.

The range is the set of y-coordinates in the ordered pairs: {(3, $\underline{9}$), (5, $\underline{39}$), (9, $\underline{23}$), (6, $\underline{14}$)}. The range is {9, 14, 23, 39}.

Try It

What are the domain and range of the function below?

$$\{(4, 9), (-5, 16), (6, 25), (7, -36)\}$$

The domain of a function is the set of all _____-coordinates.

The domain of this function is {_____, _____, _____, _____}.

The range of a function is the set of all _____-coordinates.

The range of this function is {_____, _____, _____, _____}.

The domain of a function is the set of all x-coordinates. The domain of this function is $\{-5, 4, 6, 7\}$. The range of a function is the set of all y-coordinates. The range of this function is $\{-36, 9, 16, 25\}$.

The domain and range of algebraic functions are usually assumed to be the set of all real numbers. In some cases, however, the domain or range of a function may be a subset of the real numbers because certain numbers would not make sense in a real-life problem situation.

Consider the function $l = 4h$, in which l equals the number of legs on h horses. Are there any values that would not be reasonable to include in the domain or range of this function?

- The domain of this function is the set of values you may choose for h, the independent variable. Would it be reasonable to let $h = 1.2$? No. The variable h represents a number of horses; it must be a nonnegative integer. The domain is the set of nonnegative integers, $\{0, 1, 2, 3, \ldots\}$.

It would not be reasonable to include any other numbers in the domain.

- The range of this function is the set of values you will obtain for the dependent variable, l, the number of legs for a group of h horses. Is it possible to get 6 as a value for l? Could a group of horses normally have 6 legs? No, 6 is not a reasonable value for the range of this function. Since 1 horse has 4 legs, 2 horses have 8 legs, and so on, the range of this function is the set of multiples of 4, or $\{0, 4, 8, 12, \ldots\}$.

It would not be reasonable to include any other numbers in the range.

Objective 2

The total volume in sales generated by a television commercial is a function of the number of people who watch the commercial. Wilson Electronics executives estimate that s, the dollars they will generate in sales for the number of people, n, who watch their commercial, is described by the function $s = 0.0035n$.

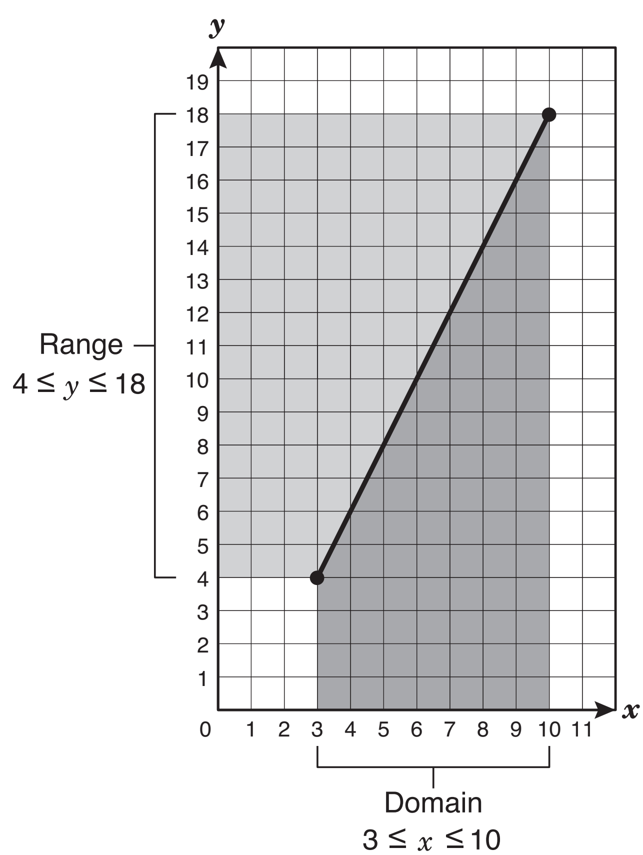
What set of numbers best represents the domain for this function: the integers, the real numbers, the rational numbers, or the whole numbers?

The domain of this function is the number of people who watch the commercial. The number of people cannot be a fraction or a negative number. Of the choices given, the only set of numbers that does not contain any fractions or negative numbers is the whole numbers.

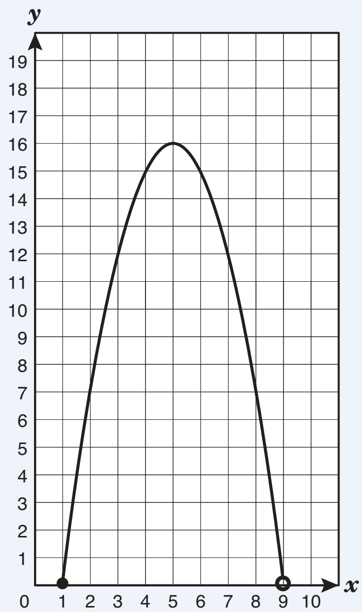
The set of whole numbers would be an appropriate domain for this function.

The graph of a function also can tell you about its domain and range.

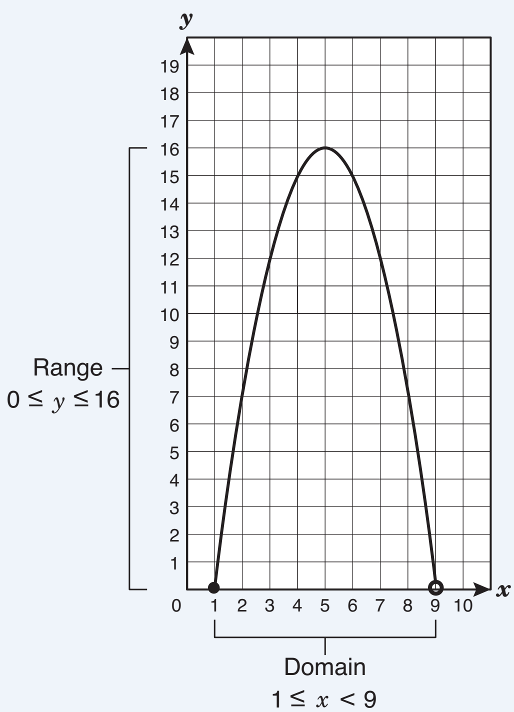
- The domain of a function is the set of all the x-coordinates in the function's graph.
- The range of a function is the set of all the y-coordinates in the function's graph.

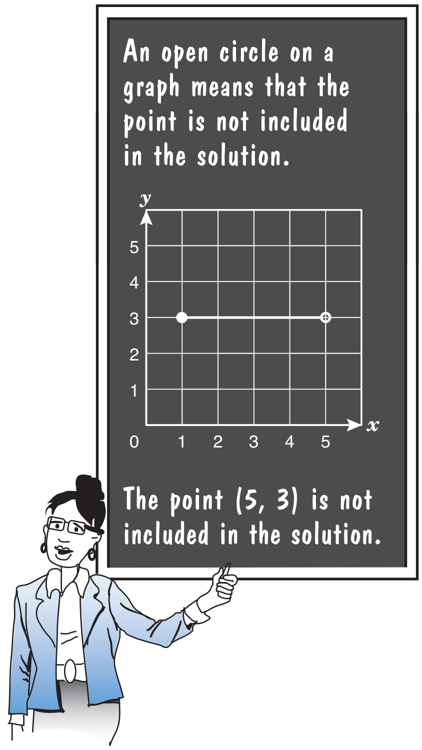


What inequalities best describe the domain and range of the function represented in this graph?

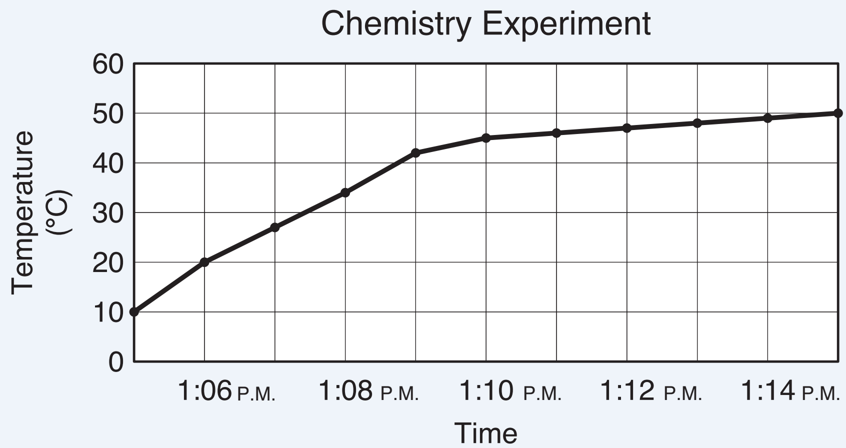


- The domain of this function is the set of *x*-values in the graph.
- The range of this function is the set of *y*-values in the graph.





In a chemistry experiment two chemicals are poured into a beaker to react. The temperature of the solution is taken every minute for 10 minutes beginning at 1:05 P.M. The graph below shows the temperature of the solution from this experiment.



What is a reasonable range for this function?

The range of the function is the set of all the y-coordinates in the function. The y-coordinates represent the temperature of the mixed chemicals. The temperature during the 10-minute interval begins at 10°C, and it ends at 50°C.

A reasonable range for this function is $10 \leq y \leq 50$.

Try It

The number of pounds of potato salad, p, that will be needed for a company picnic is given by the function $p = 0.25n + 4$, in which n equals the number of people who will attend the picnic. Each employee in the company can attend the picnic, and each can bring 3 guests. A total of 12 employees and guests have already signed up to attend the picnic. If the company employs a total of 40 people, what is a reasonable range for this function?

The range of the function is the set of all the possible values for the __________________ variable in the function, the amount of potato salad to be purchased.

To determine the range of the function, first determine the minimum and __________________ number of people who will attend the picnic.

The minimum number of people is ________, since that many people have already signed up.

The company has _________ employees. If every employee attends and brings 3 guests, then the maximum number of people is _________ because 40 • _________ = _________.

Use the function p = _____________________________________ to find the number of pounds of potato salad that will be needed.

If 12 people attend, then p = _________ • _________ + 4; _________ pounds of potato salad will be needed.

If 160 people attend, then p = _________ • _________ + 4; _________ pounds of potato salad will be needed.

The number of pounds of potato salad that will be needed is between _________ pounds and _________ pounds.

A reasonable range for this function is _________ $\leq p \leq$ _________.

The range of the function is the set of all the possible values for the dependent variable in the function, the amount of potato salad to be purchased. To determine the range of the function, first determine the minimum and maximum number of people who will attend the picnic. The minimum number of people is 12, since that many people have already signed up. The company has 40 employees. If every employee attends and brings 3 guests, then the maximum number of people is 160 because 40 • 4 = 160. Use the function $p = 0.25n + 4$ to find the number of pounds of potato salad that will be needed. If 12 people attend, then $p = 0.25 \cdot 12 + 4$; 7 pounds of potato salad will be needed. If 160 people attend, then $p = 0.25 \cdot 160 + 4$; 44 pounds of potato salad will be needed. The number of pounds of potato salad that will be needed is between 7 pounds and 44 pounds. A reasonable range for this function is $7 \leq p \leq 44$.

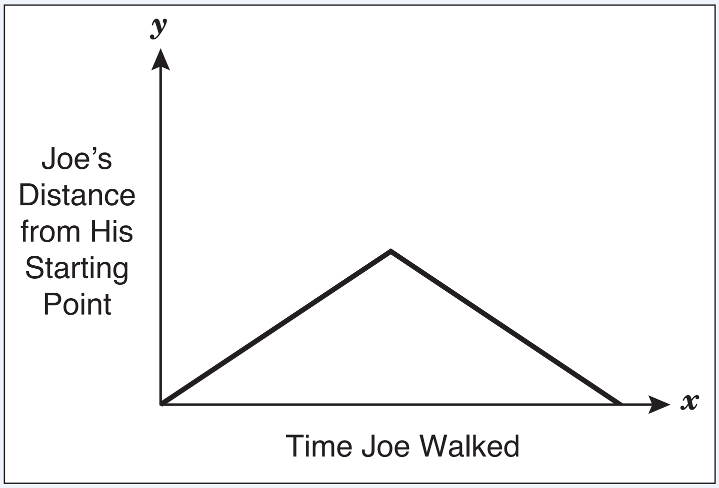
Objective 2

How Can You Interpret a Problem Situation from a Graph?

To interpret a problem situation described in terms of a graph, follow these guidelines.

- Identify the quantities that are being compared.
- Understand what relationship the graph is describing.
- Look at the scales used on the axes of the graph.
- Identify the domain or range of the function graphed.
- Look for patterns in the data—increases, decreases, or data that remain constant.

Joe walked at a constant speed.



Give a reasonable description of the route Joe walked.

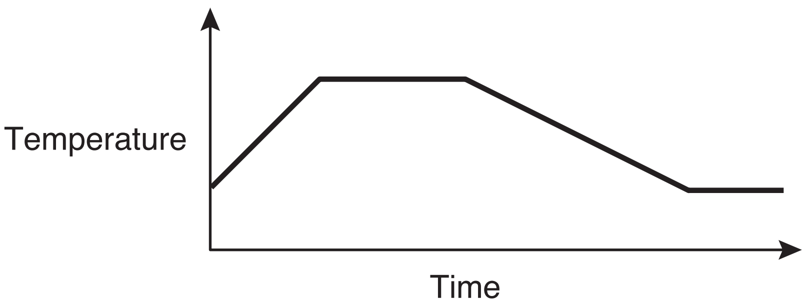
The y-axis represents Joe's distance from his starting point. The x-axis represents his walking time. Did Joe get farther and farther away from his starting point as time went by? No, not for the entire time graphed.

At first Joe's distance from his starting point was 0. As time went by, this distance increased, but then it returned to 0. He ended his walk where he began it.

A reasonable description of the route Joe walked is that he started at a certain point, walked for a while in one direction, and then turned around and returned to his starting point. The graph represents his distance from his starting point in terms of the time he walked.

Try It

Terri placed a pot of water on the stove to boil. The temperature of the water in terms of the time it was on the stove is represented by the graph.



What is a reasonable interpretation of the graph?

At first the water temperature ________________.

Then the water temperature remained ________________ for a while.

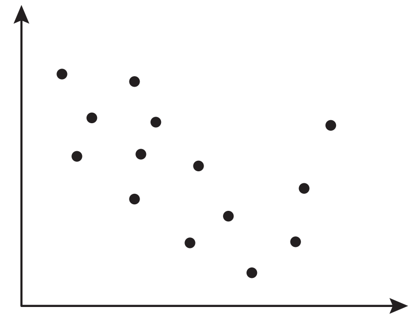
Finally the water temperature ________________ slowly.

A reasonable interpretation of the graph is that the water temperature ________________ until the water boiled. Then it remained at a ________________ temperature until Terri turned the stove off. Finally it ________________ slowly to room temperature, where it remained constant.

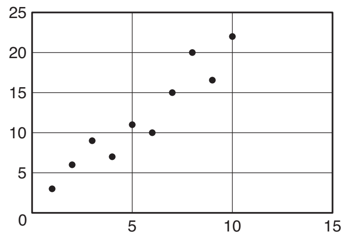
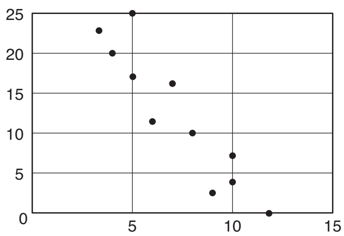
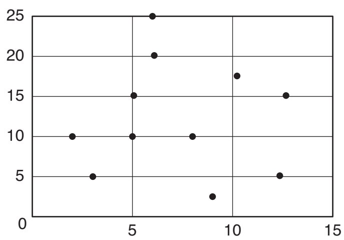
At first the water temperature increased. Then the water temperature remained constant for a while. Finally the water temperature decreased slowly. A reasonable interpretation of the graph is that the water temperature increased until the water boiled. Then it remained at a constant temperature until Terri turned the stove off. Finally it cooled slowly to room temperature, where it remained constant.

What Is a Correlation in a Scatterplot?

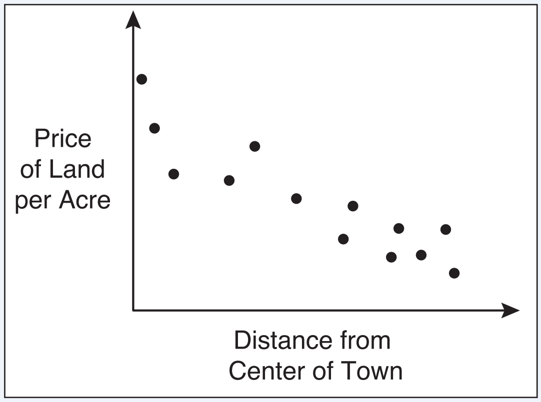
One way to represent a set of related data is to graph the data using a scatterplot. In a **scatterplot** each pair of corresponding values in the data set is represented by a point on a graph.



To make predictions using a scatterplot, look for a **correlation**, or pattern, in the data. The pattern may not be true for every point, but look for the overall pattern the data seem to best fit.

As you move from left to right on the graph, if the data points …	as shown in this scatterplot …	they show this type of correlation:
move up		positive correlation.
move down		negative correlation.
show no pattern		no correlation.

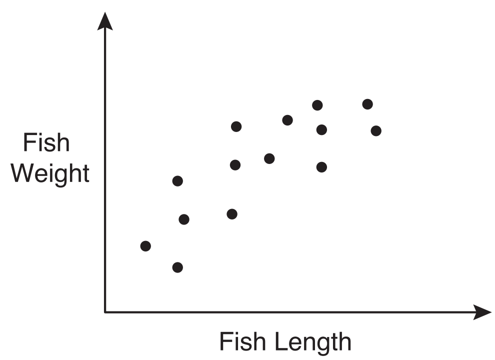
In a survey of property values, p, the price of an acre of land, was compared to d, the distance of the land from the center of town. The data were graphed in a scatterplot. Describe the correlation between the cost of an acre of land and the land's distance from the center of town.



Look for a pattern in the graph. The general tendency is for the price of an acre of land to decrease as the land's distance from the center of town increases. The price of an acre of land has a negative correlation to the land's distance from the center of town.

Try It

Raúl is a sport fisherman. He weighs each fish he catches, and he measures its length. He graphed his data in a scatterplot.



Describe the correlation between the lengths and weights of the fish Raúl caught.

As the lengths of the fish ____________________, their weights generally ____________________.

This is a ____________________ correlation.

As the lengths of the fish increase, their weights generally increase. This is a positive correlation.

How Do You Use Symbols to Represent Unknown Quantities?

Represent unknown quantities with **variables**, or letters such as x or y. Use variables in expressions, equations, or inequalities.

Jacob, Barbara, and Felix all have part-time jobs after school. Jacob earns \$5 more per week than Felix, and Barbara earns twice as much per week as Jacob. The combined earnings of these three students are \$115 per week. Write an equation that can be used to find how much each student earns per week.

- Represent the number of dollars each student earns per week.

x = Felix's earnings

$x + 5$ = Jacob's earnings (\$5 more than Felix's earnings)

$2(x + 5)$ = Barbara's earnings (twice Jacob's earnings)

- Represent the sum of their earnings.

Felix + Jacob + Barbara

$$x + (x + 5) + 2(x + 5)$$

- Write an equation setting this sum equal to \$115.

$$x + (x + 5) + 2(x + 5) = 115$$

Quattro plans to paint his living room walls. The rectangular room is 3 feet longer than it is wide. The walls are 8 feet high. If a can of paint covers approximately 200 ft^2, what expression can be used to represent the minimum number of cans of paint Quattro will need?

- Represent the area of each wall.

Smaller Walls	Larger Walls
x = width of the room	$x + 3$ = length of the room
8 = height of the room	8 = height of the room
$8x$ = area	$8(x + 3)$ = area

- Represent the sum of the areas of the four walls.

2(smaller wall) + 2(larger wall)

$$2(8x) + 2[8(x + 3)]$$
$$16x + 16(x + 3)$$
$$16x + 16x + 48$$
$$32x + 48$$

The total area of the four walls is represented by the expression $32x + 48$. Since the paint in each can covers 200 ft^2, divide the area by 200 to represent the number of cans of paint needed: $\frac{32x + 48}{200}$.

How Do You Represent Patterns in Data Algebraically?

To represent patterns in data algebraically, follow these guidelines.

- Identify what quantities the data represent.
- Identify the relationships between those quantities.
- Look for patterns in the data.
- Use symbols to translate the patterns into an algebraic expression or equation.

Ronald and Jaime make weekly deposits into their savings accounts. The table below shows the opening balances and the balances for each account after the first four weekly deposits.

Account Balances

Name	Opening Balance	Week 1	Week 2	Week 3	Week 4
Ronald	$100	$124	$148	$172	$196
Jaime	$180	$198	$216	$234	$252

If the pattern continues, what expression can be used to represent the balance in Jaime's account after *n* weeks?

Jaime's opening balance is $180. His balance increases by $18 each week. His balance in *n* weeks can be represented by the expression $18n + 180$. See whether this expression works for all the known values.

Week	$18n + 180$	Balance (dollars)	Yes/No
1	$18 \cdot 1 + 180 = 198$	198	Yes
2	$18 \cdot 2 + 180 = 216$	216	Yes
3	$18 \cdot 3 + 180 = 234$	234	Yes
4	$18 \cdot 4 + 180 = 252$	252	Yes

If the pattern continues, the balance in Jaime's account after *n* weeks can be represented by the expression $18n + 180$.

The table below represents a functional relationship between x, the lengths of the bases of a series of triangles, and y, their heights. If the pattern continues, what expression can be used to express their heights in terms of their bases?

x	y
1	2
2	5
3	10
4	17

Change in x-values: +1, +1, +1

Change in y-values: +3, +5, +7

The differences between the y-values are not constant. For example, $5 - 2 = 3$, but $10 - 5 = 5$. This is not a linear relationship.

To determine whether the relationship is quadratic, compare the y-values to the square of the x-values.

x	x^2	y
1	1	2
2	4	5
3	9	10
4	16	17

The y-values appear to be 1 more than the squares of the x-values.

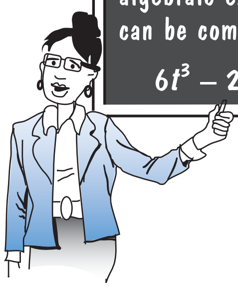
See whether all the coordinates satisfy the rule $y = x^2 + 1$. Substitute the values of x and y from the table into the equation.

x	$y = x^2 + 1$	y	Yes/No
1	$2 \stackrel{?}{=} (1)^2 + 1$ $2 \stackrel{?}{=} 1 + 1$ $2 = 2$	2	Yes
2	$5 \stackrel{?}{=} (2)^2 + 1$ $5 \stackrel{?}{=} 4 + 1$ $5 = 5$	5	Yes
3	$10 \stackrel{?}{=} (3)^2 + 1$ $10 \stackrel{?}{=} 9 + 1$ $10 = 10$	10	Yes
4	$17 \stackrel{?}{=} (4)^2 + 1$ $17 \stackrel{?}{=} 16 + 1$ $17 = 17$	17	Yes

All the ordered pairs satisfy the equation $y = x^2 + 1$.

Since every pair in the table satisfies the equation, then $y = x^2 + 1$ expresses the triangles' heights in terms of their bases.

Objective 2



Like terms are terms in an algebraic expression that use the same variable raised to the same power.

For example, in the expression $6t^3 - 2t^3$, $6t^3$ and $2t^3$ are like terms because they are both expressed in terms of the same variable, t, raised to the same power, 3.

Like terms in an algebraic expression can be combined.

$$6t^3 - 2t^3 = 4t^3$$

How Do You Simplify Algebraic Expressions?

You can use the commutative, associative, and distributive properties to simplify algebraic expressions. Use these properties to remove parentheses and combine like terms.

Name	Property (If a, b, and c are any three real numbers, then…)	Examples
Commutative Property	$a + b = b + a$ or $ab = ba$	$4 + 2x = 2x + 4$ or $5y^2x = 5xy^2$
Associative Property	$a + (b + c) = (a + b) + c$ or $a(bc) = (ab)c$	$(5 + y) + 2y = 5 + (y + 2y)$ $= 5 + 3y$ or $5(3m) = (5 \cdot 3)m$ $= 15m$
Distributive Property	$a(b + c) = ab + ac$	$7(2x + 3) = 7(2x) + 7(3)$ $= 14x + 21$

- Use the commutative property to change the order of the terms in addition and multiplication.
- Use the associative property to change the groupings in addition or multiplication.
- Use the distributive property to remove the parentheses by multiplying the term outside the parentheses by each term inside the parentheses.

Objective 2

Simplify the expression $5y^3 + 3 - 2y^3 + 1$.

$$\begin{aligned} 5y^3 + 3 - 2y^3 + 1 &= (5y^3 - 2y^3) + (3 + 1) \\ &= 3y^3 + 4 \end{aligned}$$

When simplified, the expression $5y^3 + 3 - 2y^3 + 1 = 3y^3 + 4$.

If the length of a rectangle is represented by l and its width is 3 units less than its length, does the expression $l^2 - 3l$ represent its area?

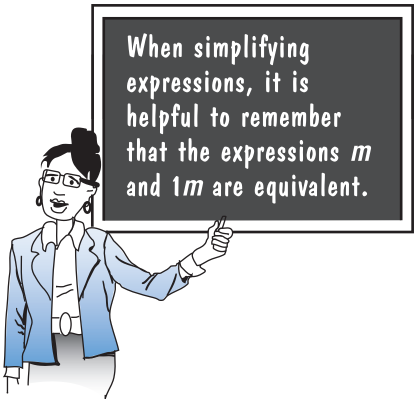
- Let l represent the length of the rectangle.
- Let $l - 3$ represent the width (3 units less than the length).
- Then $l(l - 3)$ represents the area of the rectangle, $A = lw$.

Simplify the expression $l(l - 3)$.

$$\begin{aligned} l(l - 3) &= l \cdot l - l \cdot 3 \\ &= l^2 - 3l \end{aligned}$$

Yes, the expression $l^2 - 3l$ represents the area of the rectangle.

Objective 2



The lengths of the three sides of a triangle are represented by the expressions $2m + 5$, $m - 1$, and $7m + 3$. Write an expression in terms of m that can be used to represent the perimeter of the triangle.

- The perimeter of a triangle is the sum of the lengths of its three sides. The perimeter of the triangle can be represented by the expression $2m + 5 + m - 1 + 7m + 3$.
- Simplify this expression.

$$\begin{aligned} 2m + 5 + m - 1 + 7m + 3 &= (2m + 1m + 7m) + (5 - 1 + 3) \\ &= 10m + 7 \end{aligned}$$

The perimeter of the triangle can be represented by the expression $10m + 7$.

Do you see that . . .

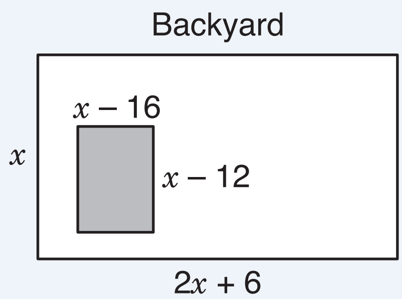
Simplify the expression $(4b^2 + 6) - (2b^2 + 3)$.

When parentheses are preceded by a negative sign, it means the quantity in parentheses is multiplied by -1.

$$\begin{aligned} (4b^2 + 6) - (2b^2 + 3) &= (4b^2 + 6) + -1(2b^2 + 3) \\ &= 4b^2 + 6 - 2b^2 - 3 \\ &= (4b^2 - 2b^2) + (6 - 3) \\ &= 2b^2 + 3 \end{aligned}$$

When simplified, the expression $(4b^2 + 6) - (2b^2 + 3) = 2b^2 + 3$.

Mr. and Mrs. Seymour have a dog pen in their backyard, as shown by the shaded figure in the diagram below.



Write an expression that can be used to represent the area of the yard that does not include the dog pen.

- The area of the entire yard, a rectangle, is equal to its width times its length.

  The area of the rectangle is $x(2x + 6)$. Simplify.

$$x(2x + 6) = 2x^2 + 6x$$

- The area of the dog pen is equal to its length times its width. The area of the smaller rectangle is $(x - 12)(x - 16)$. Simplify by using the FOIL method.

$$(x - 12)(x - 16) = (x \cdot x) + (x \cdot -16) + (-12 \cdot x) + (-12 \cdot -16)$$
$$= x^2 + (-16x - 12x) + 192$$
$$= x^2 - 28x + 192$$

- Subtract to find the area of the yard not including the dog pen.

$$\text{Area of yard} - \text{Area of pen}$$
$$(2x^2 + 6x) - (x^2 - 28x + 192)$$

- Simplify by multiplying the quantities in the second parentheses by -1.

$$(2x^2 + 6x) + -1(x^2 - 28x + 192) = 2x^2 + 6x - x^2 + 28x - 192$$
$$= (2x^2 - x^2) + (6x + 28x) - 192$$
$$= x^2 + 34x - 192$$

The expression $x^2 + 34x - 192$ represents the area of the yard that does not include the dog pen.

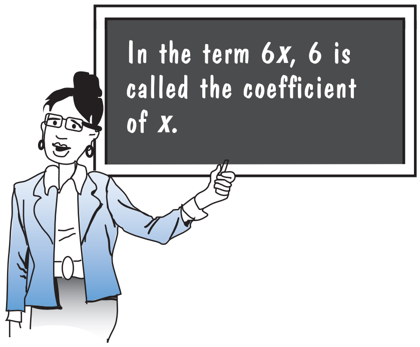
See Objective 5, page 125, for more information about the FOIL method.

Objective 2

How Do You Solve Algebraic Equations?

To solve algebraic equations, follow these guidelines.

- Simplify any expressions in the equation.
- Add or subtract on both sides of the equation to get variable terms on one side and constant terms on the other.
- Simplify again if necessary.
- Multiply or divide to obtain an equation that has the variable isolated with a coefficient of 1.



Solve the equation $3(y + 2) = -9$.

$$\begin{aligned} 3(y + 2) &= -9 \\ 3y + 3(2) &= -9 \\ 3y + 6 &= -9 \\ -6 &= -6 \\ \hline 3y &= -15 \\ \frac{3y}{3} &= \frac{-15}{3} \\ y &= -5 \end{aligned}$$

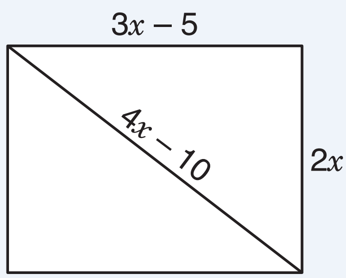
In this equation, $y = -5$.

Solve the equation $2x - 5 = x + 4$.

$$\begin{array}{rcl} 2x - 5 &=& x + 4 \\ -1x \quad\; &=& -1x \\ \hline x - 5 &=& 4 \\ +5 &=& +5 \\ \hline x &=& 9 \end{array}$$

In this equation, $x = 9$.

In the figure below, the perimeter of the rectangle is 20 units greater than the perimeter of each triangle. Find the length of the diagonal of the rectangle.



- Represent the perimeter of the rectangle.

$$\begin{aligned} P &= 2l + 2w \\ &= 2(3x - 5) + 2(2x) \end{aligned}$$

- Simplify this expression.

$$\begin{aligned} 2(3x - 5) + 2(2x) &= 2(3x) - 2(5) + (2 \cdot 2)x \\ &= 6x - 10 + 4x \\ &= (6x + 4x) - 10 \\ &= 10x - 10 \end{aligned}$$

- Represent the perimeter of a triangle.

$$\begin{aligned} P &= s_1 + s_2 + s_3 \\ &= (3x - 5) + (2x) + (4x - 10) \end{aligned}$$

- Simplify this expression.

$$\begin{aligned} (3x - 5) + (2x) + (4x - 10) &= 3x - 5 + 2x + 4x - 10 \\ &= (3x + 2x + 4x) + (-5 - 10) \\ &= 9x - 15 \end{aligned}$$

- Write an equation.

Perimeter of rectangle = Perimeter of triangle + 20

$$\begin{aligned} 10x - 10 &= 9x - 15 + 20 \\ 10x - 10 &= 9x + 5 \\ -9x \quad &= -9x \\ \hline x - 10 &= 5 \\ +10 &= +10 \\ \hline x &= 15 \end{aligned}$$

Objective 2

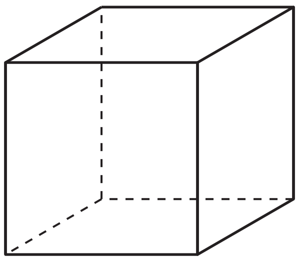
- Use this value for x to find the length of the diagonal of the rectangle. The diagonal is represented by the expression $4x - 10$. Replace x with 15.

$$\begin{aligned} 4x - 10 &= 4(15) - 10 \\ &= 60 - 10 \\ &= 50 \end{aligned}$$

The diagonal of the rectangle is 50 units long.

Try It

A manufacturer builds cubical containers. The equation that models c, the cost of building a container, is $c = 3.6s + 3.20$, in which s represents the length of each side of the cubical container in feet.



Find the length of each side of a cubical container that costs $35.60 to manufacture.

Substitute ________ for c in the equation that models the cost of building a container.

Solve the equation for ________, the length of a side.

$$35.60 = 3.6s + 3.20$$
$$-\square = \quad -\square$$
$$\underline{\qquad} = 3.6s$$
$$\frac{32.40}{\square} = \frac{3.6s}{\square}$$
$$\underline{\qquad} = s$$

Each side of the cubical container is ________ feet long.

Substitute 35.60 for c in the equation that models the cost of building a container. Solve the equation for s, the length of a side.

$$35.60 = 3.6s + 3.20$$
$$-3.20 = \quad -3.20$$
$$32.40 = 3.6s$$
$$\frac{32.40}{3.6} = \frac{3.6s}{3.6}$$
$$9 = s$$

Each side of the cubical container is 9 feet long.

Objective 2

As was mentioned earlier, function notation can be used to describe a function: $f(x) = 3x - 2$ represents the same relationship as $y = 3x - 2$ does. For any x, the function f creates ordered pairs $(x, 3x - 2)$. The y-value in these ordered pairs is $3x - 2$, thus the equation $y = 3x - 2$ also represents this function.

Look at this function:

$$y = \frac{1}{2}x^2 + 4$$

Can we rewrite this equation in function notation?

The equation tells us that for any ordered pair (x, y) belonging to this function, the y-coordinate can be represented by $\frac{1}{2}x^2 + 4$.

$$(x, y)$$

$$\downarrow$$

$$\left(x, \frac{1}{2}x^2 + 4\right)$$

$$\downarrow$$

$$f(x) = \frac{1}{2}x^2 + 4$$

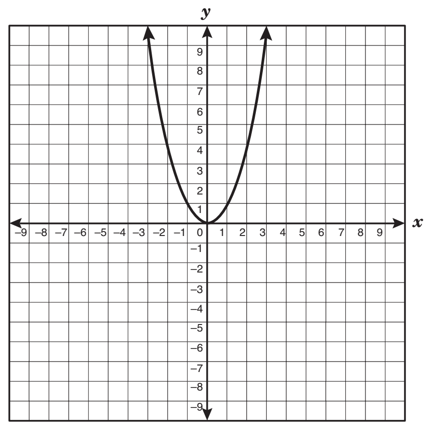
Thus, this function consists of ordered pairs $\left(x, \frac{1}{2}x^2 + 4\right)$. Therefore $f(x) = \frac{1}{2}x^2 + 4$ represents the same relationship that $y = \frac{1}{2}x^2 + 4$ does.

Now practice what you've learned.

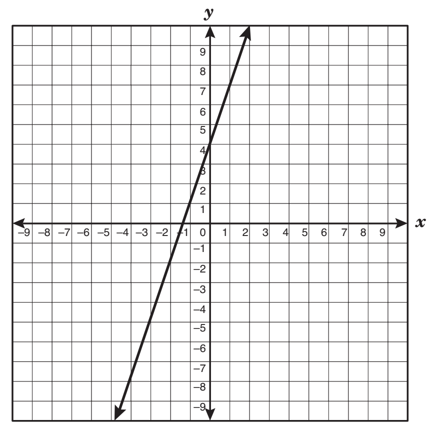
Question 14

Which graph below best represents the parent function of $y = 3x^2 + 4$?

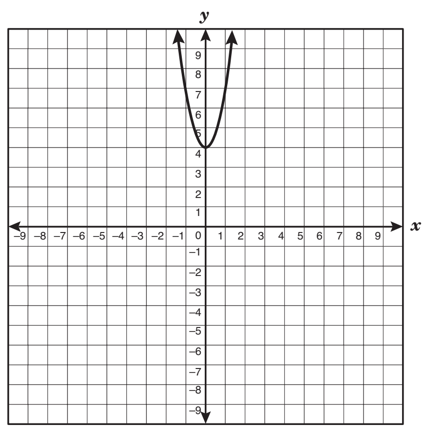
A



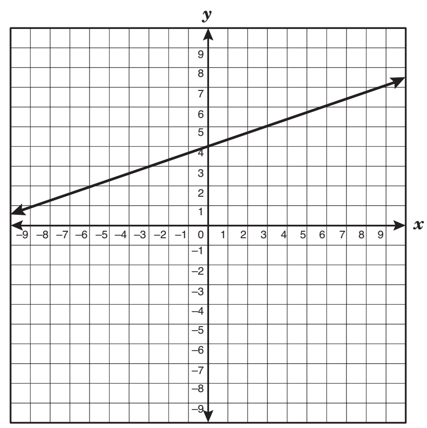
C



B



D



Answer Key: page 237

Question 15

Marcy deposits $550 in a savings account at 3% simple annual interest. The value of this account, v, is given by the function $v = 550 + 16.5t$, in which t is the number of years the money is in the bank. What is the range of this function?

A $0 \leq v \leq 550$

B $v \geq 550$

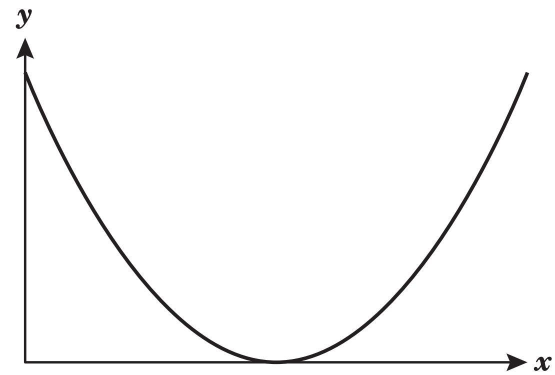
C $v \leq 550$

D $0 \leq v \leq 16.5$

Answer Key: page 237

Question 16

Which of the following situations is best represented by the graph below?



A The speed of a roller coaster as it goes from a high point on its track to a low point on the track and then back up to another high point

B The price of a stock that drops to half of its original value and then goes back to its original value

C The speed of a bullet that is fired straight up and then falls back to the ground

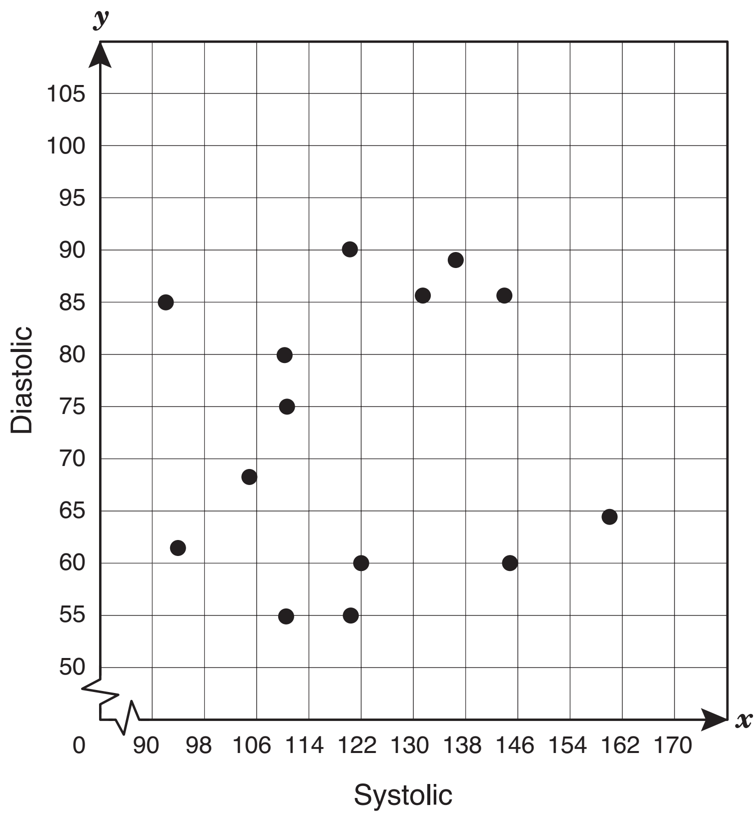
D The speed of a race car that starts from the starting line, races several laps, and then makes a refueling stop

Answer Key: page 237

Question 17

A nurse checked the blood pressure of 14 people at a health clinic. The table and graph below show the relationship between the systolic and diastolic blood-pressure readings.

Systolic	Diastolic
92	85
94	62
105	68
110	80
111	55
111	75
121	55
121	90
122	60
132	86
136	89
144	86
145	60
160	64



Which of the following is not a true statement about the systolic and diastolic blood-pressure readings for this set of data?

A Systolic pressure is always higher than diastolic pressure.

B There appears to be no trend between systolic and diastolic blood pressures.

C Systolic blood pressure ranges from 92 to 160.

D There appears to be a negative trend between systolic and diastolic pressures.

Answer Key: page 237

Question 18

Alex and Millie are selling kites from their stand on the beach. Each day more people stop to look at and buy the kites. Alex and Millie kept the following record comparing the number of customers that stopped to look at their kites and the money they collected in sales each day.

Kite Sales

Number of customers	12	18	24	30
Amount of sales (dollars)	180	210	240	270

If this trend continues, how many customers will need to stop and look at Alex and Millie's kites in order for their sales in one day to reach \$330?

A 42

B 60

C 66

D 48

Answer Key: page 237

Question 19

Frieda wants to buy a refrigerator that is 6 feet tall. The refrigerator's width is 1.75 times its depth. Which equation best describes V, the volume of the refrigerator in terms of its depth, x?

A $V = 6x + 10.5$

B $V = x^2 + 1.75x$

C $V = 1.75x^2 + 6x$

D $V = 10.5x^2$

Answer Key: page 237

Question 20

Which expression generates the set $\{0, 2, 8, 18, \ldots\}$ from the set $n = \{0, 1, 2, 3, \ldots\}$?

A n

B $2n$

C $2n^2$

D $n^2 + 2$

Answer Key: page 237

Question 21

Which algebraic expression best represents the relationship between the terms in the following sequence and n, their position in the sequence?

$$1, 3, 5, 7, 9, \ldots$$

A $n + 2$

B $2n + 1$

C $2n - 1$

D $n + 1$

Answer Key: page 237

Question 22

Mr. Jones runs a bakery. His monthly profit in dollars, $P(x)$, is given by the function $P(x) = \frac{1}{2}x - 2$, in which x represents the number of loaves of bread sold in a month. What is the minimum number of loaves he must sell next month if he is to have a profit of at least \$3000?

Record your answer and fill in the bubbles. Be sure to use the correct place value.

				.			
0	0	0	0		0	0	0
1	1	1	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Answer Key: page 238

Question 23

Which of these is equivalent to the expression below?

$$2x + 2(3x - 4) + 3(8x - 4)$$

A $\frac{8x - 5}{4}$

B $32x - 8$

C $32x - 20$

D $4(8x + 5)$

Answer Key: page 238

Question 24

Maria was given the equation $y = 2x - 1$ to graph. Which of the following would be an appropriate label for her graph?

A $f(x) = 2y - 1$

B $f(x) = 2x - 1$

C $y = 2x$

D $x = 2y - 1$

Answer Key: page 238

Objective 3

The student will demonstrate an understanding of linear functions.

For this objective you should be able to

- represent linear functions in different ways and translate among their various representations; and
- interpret the meaning of slope and intercepts of a linear function and describe the effects of changes in slope and *y*-intercept in real-world and mathematical situations.

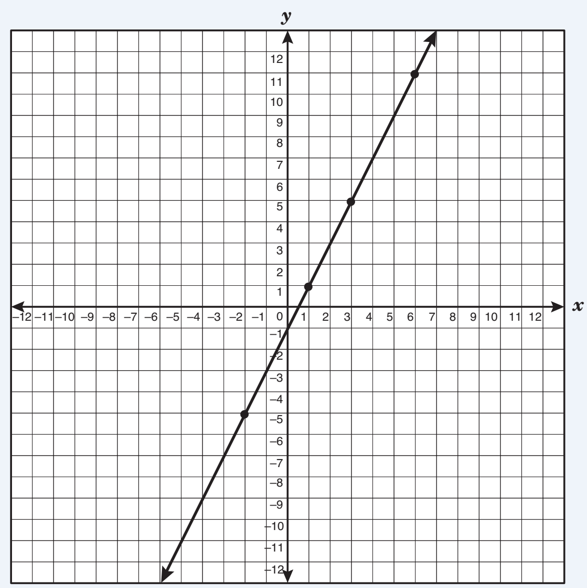
What Is a Linear Function?

A **linear function** is any function whose graph is a nonvertical line.

Is the function represented by the following table of values a linear function?

x	*y*
−2	−5
1	1
3	5
6	11

Graph the ordered pairs in the table on a coordinate grid.



The points lie on a line. Therefore, the function they represent is a linear function.

Try It

The table below describes a linear relationship.

x	y
0	1
1	$\frac{5}{3}$
3	3

Does the equation $3y = 2x + 3$ represent the same linear function?

To confirm that the equation $3y = 2x + 3$ represents the same linear function as the table, substitute the ordered pairs from the table into the equation.

x	y	Does $3y = 2x + 3$?	Yes/No
0	1	3(______) $\stackrel{?}{=}$ 2(______) + 3 ______ $\stackrel{?}{=}$ ______ + 3 ______ = ______	______
1	$\frac{5}{3}$	3(______) $\stackrel{?}{=}$ 2(______) + 3 ______ $\stackrel{?}{=}$ ______ + 3 ______ = ______	______
3	3	3(______) $\stackrel{?}{=}$ 2(______) + 3 ______ $\stackrel{?}{=}$ ______ + 3 ______ = ______	______

The table and the equation represent the same function.

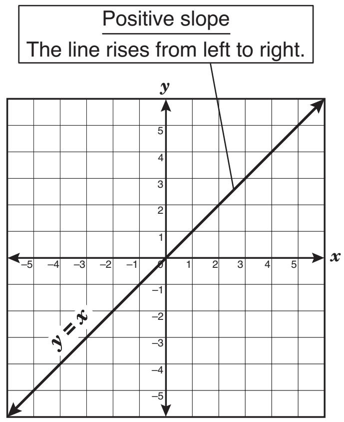
---

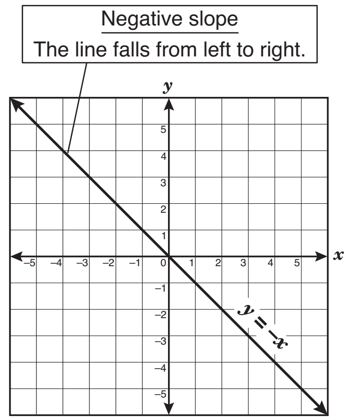
x	y	Does $3y = 2x + 3$?	Yes/No
0	1	$3(1) \stackrel{?}{=} 2(0) + 3$ $3 \stackrel{?}{=} 0 + 3$ $3 = 3$	Yes
1	$\frac{5}{3}$	$3\left(\frac{5}{3}\right) \stackrel{?}{=} 2(1) + 3$ $5 \stackrel{?}{=} 2 + 3$ $5 = 5$	Yes
3	3	$3(3) \stackrel{?}{=} 2(3) + 3$ $9 \stackrel{?}{=} 6 + 3$ $9 = 9$	Yes

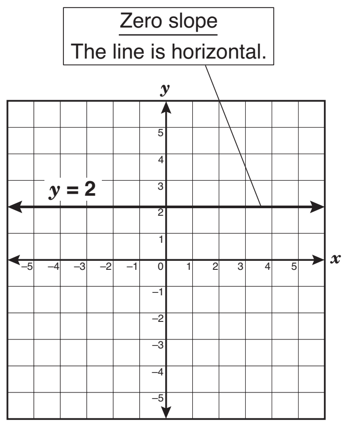
Objective 3

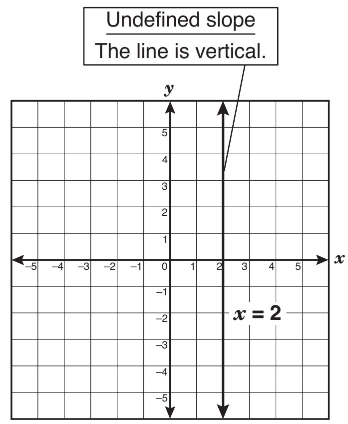
What Is Slope?

The **slope** of a linear graph is its rate of change. The slope shows how fast the graph increases or decreases. The slope of a line can also be described as its steepness, how fast the line rises or falls.









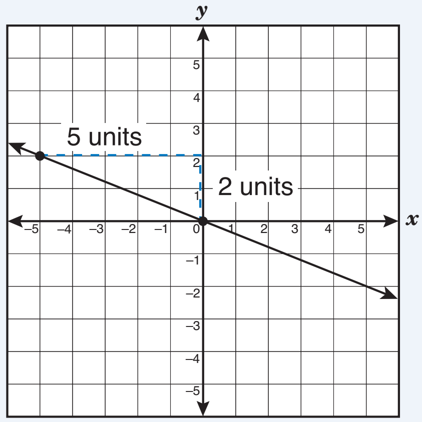
The rate of change of a line is the ratio that compares the change in y-values to the corresponding change in x-values for any two points on the graph.

A graph's slope is often described as its **rise** (change in y) over **run** (change in x). To find the slope of a line from its graph:

- Pick any two points on the graph.
- Find the change in y-values, $y_2 - y_1$, or the rise.
  Count the number of units up or down between the two points.
- Find the change in x-values, $x_2 - x_1$, or the run.
  Count the number of units left to right between them.
- Determine whether the slope is positive or negative.
  As you go from left to right:
  if the line points up, the slope is positive.
  if the line points down, the slope is negative.
- Write the slope as a ratio: $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$.

Do you see that . . .

What is the slope of the line in the graph below?



Find the slope by counting the change in y-values and the corresponding change in x-values between any two points.

The y-value changes 2 units for every 5 units the x-value changes. As you go from left to right, the line points down, so the slope is negative.

The slope of the graph is $\frac{\text{rise}}{\text{run}} = -\frac{2}{5}$.

Another way to find the slope is to use the slope formula.

Slope Formula

For any two points (x_1, y_1) and (x_2, y_2) on a graph, the slope, m, of the line that passes through them is:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

← change in y-values
← change in x-values

What is the slope of the line passing through the points (1, 1) and (3, 5)?

Let (x_1, y_1) be (1, 1) and (x_2, y_2) be (3, 5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{3 - 1} = \frac{4}{2} = 2$$

The slope of the line is 2.

Objective 3

The set of ordered pairs below represents a linear function. What is the rate of change of the function?

$$\{(0, 1), (1, 3), (2, 5), (3, 7), (4, 9)\}$$

Since this is a linear function, you can choose any two ordered pairs belonging to the function to find its rate of change.

- Use the ordered pairs (0, 1) and (2, 5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 1}{2 - 0} = \frac{4}{2} = 2$$

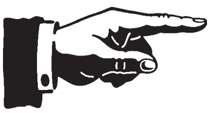
The rate of change of the function is 2.

- Use the ordered pairs (1, 3) and (4, 9).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{4 - 1} = \frac{6}{3} = 2$$

The rate of change of the function is still 2.

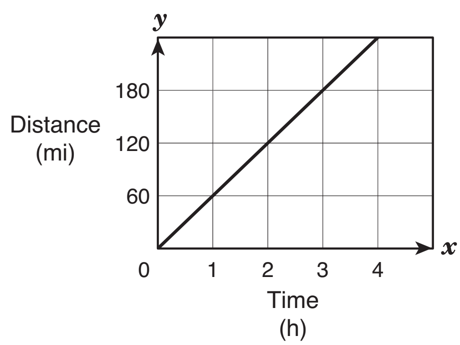
Do you see that . . .



The rate of change, or slope, of a linear function does not depend on the points you pick to calculate the slope.

Try It

Suppose you graphed the number of miles you drove on a trip.



Based on this graph, at what speed did you drive?

The slope of the graph represents your _______________, the number of miles per hour you drove.

To find the slope, compare the change in _______________ and _______________ between any two points on the graph.

You could use the following two points: (1, _______) and (_______, 240).

$$m = \frac{240 - \square}{\square - 1}$$

$$= \frac{\square}{\square}$$

$$= \_\_\_\_\_\_$$

Your speed was ________ miles per hour.

The slope of the graph represents your speed, the number of miles per hour you drove. To find the slope, compare the change in y-values and x-values between any two points on the graph. You could use the following two points: (1, 60) and (4, 240).

$$m = \frac{240 - 60}{4 - 1}$$

$$= \frac{180}{3}$$

$$= 60$$

Your speed was 60 miles per hour.

What Is the Slope-Intercept Form of a Linear Function?

One form of the equation of a linear function is $y = mx + b$. This form is called the **slope-intercept form** of a linear function.

When the equation of a linear function is written in the form $y = mx + b$:

- ***m*** is the **slope** of the graph of the function; m is the value by which the x-term is being multiplied.
- ***b*** is the y-coordinate of the ***y*-intercept** of the graph of the function; b is the value that is being added to the x-term.

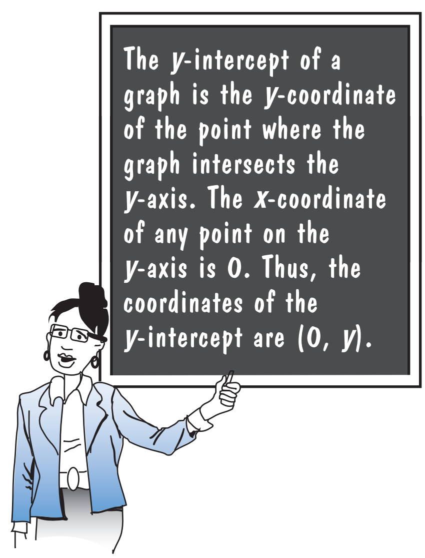
What are the slope and the y-intercept of the graph of $y = 4x + 1$?

The equation is in slope-intercept form, $y = mx + b$. Read the values of m and b from the equation.

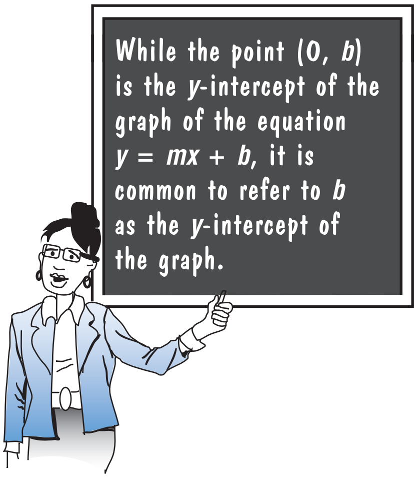
- The value by which the x-term is being multiplied is 4, so $m = 4$.
- The value that is being added to the x-term is 1, so $b = 1$.

The slope, m, of the graph of the function is 4.

The y-coordinate of the y-intercept, b, of the graph of the function is 1. Therefore, the coordinates of the y-intercept are (0, 1).



Objective 3



What are the slope and y-intercept of the graph of $6x + y = 10$?

- To determine the slope and y-intercept, transform the equation $6x + y = 10$ into slope-intercept form, $y = mx + b$.

$$\begin{aligned} 6x + y &= 10 \\ -6x \quad &= -6x \\ \hline y &= -6x + 10 \end{aligned}$$

  The equation is now in the form $y = mx + b$.

- Read the values of m and b from the revised equation. For this function, $m = -6$, and $b = 10$.

The slope, m, of the graph of the function is -6.

The y-intercept, b, of the graph of the function is 10.

Try It

Find the slope and y-intercept of the graph of $3y = 6x - 9$.

To determine the slope and y-intercept of the graph, transform the equation $3y = 6x - 9$ into slope-intercept form, $y = mx + b$.

$$3y = 6x - 9$$

$$\frac{3y}{\square} = \frac{6x}{\square} - \frac{9}{\square}$$

$$y = \_\_\_\_\_\_ x - \_\_\_\_\_\_$$

Read the values of ______ and ______ from the revised equation.

For this function, $m =$ ______ and $b =$ ______.

The slope, m, of the graph of the function is ______.

The y-coordinate of the y-intercept, b, of the graph of the function is ______.

$$3y = 6x - 9$$

$$\frac{3y}{3} = \frac{6x}{3} - \frac{9}{3}$$

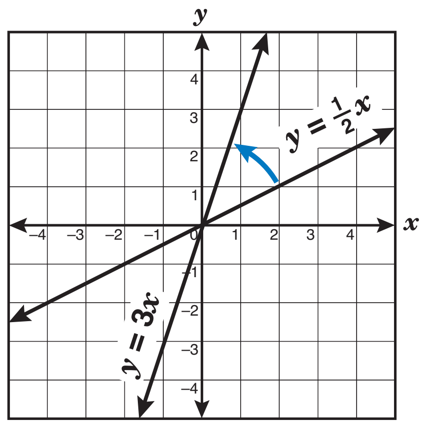
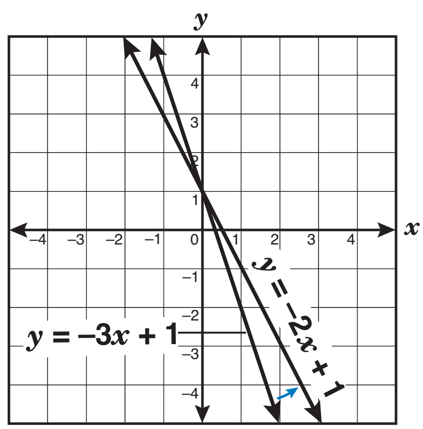
$$y = 2x - 3$$

Read the values of m and b from the revised equation. For this function, $m = 2$ and $b = -3$. The slope, m, of the graph of the function is 2. The y-coordinate of the y-intercept, b, of the graph of the function is -3.

What Are the Effects on the Graph of a Linear Function If the Values of *m* and *b* Are Changed in the Equation $y = mx + b$?

In the equation $y = mx + b$, m represents the slope of the graph, and b represents the y-intercept of the graph. Changing either of these two constants, m or b, will produce a new graph. The new graph is related to the original graph in predictable ways.

Change in Slope, *m*

Function 1	Function 2	Effect
$y = \frac{1}{2}x$	$y = 3x$	 Since $\lvert 3\rvert > \lvert\frac{1}{2}\rvert$, the graph of function 2 is steeper than the graph of function 1.
$y = -3x + 1$	$y = -2x + 1$	 Since $\lvert -2\rvert < \lvert -3\rvert$, the graph of function 2 is less steep than the graph of function 1.

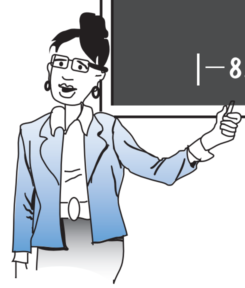
The absolute value of a number indicates its distance from 0 on a number line. The symbol for the absolute value of x is $|x|$.

For example,

$$|-3| = 3$$

$$|5| = 5$$

$$|-8.2| = 8.2$$



Objective 3

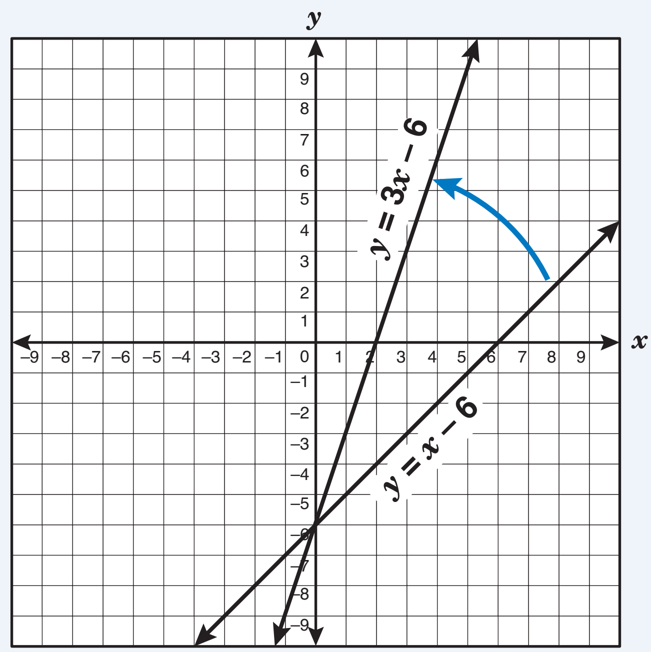
Do you see that . . .

If the function $y = x - 6$ were changed to $y = 3x - 6$, how would the graph of the new function compare to the original graph?

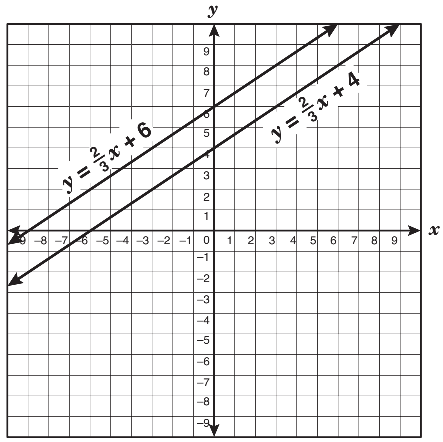
The equations are both in slope-intercept form, $y = mx + b$. In this form, m represents the graph's slope.

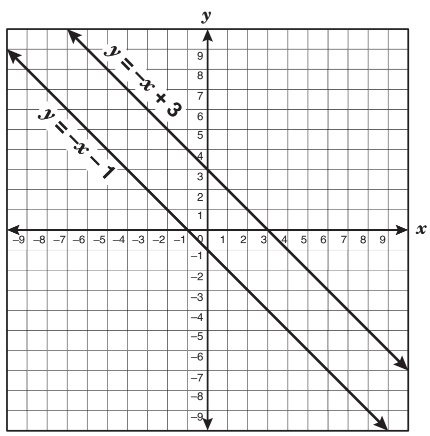
- The slope of the original function is 1.
- If the equation were changed to $y = 3x - 6$, the new value for m would be 3.

The graph of the new function would be steeper because $|3| > |1|$.



Change in *y*-intercept, *b*

<table>
<tr><th>Function 1</th><th>Function 2</th><th>Effect</th></tr>
<tr><td>$y = \frac{2}{3}x + 4$</td><td>$y = \frac{2}{3}x + 6$</td><td>

Function 2 intersects the *y*-axis at a higher point.</td></tr>
<tr><td>$y = -x + 3$</td><td>$y = -x - 1$</td><td>

Function 2 intersects the *y*-axis at a lower point.</td></tr>
</table>

If the *y*-intercept of the function $y = \frac{1}{2}x - 4$ were decreased by 3, what would be the equation of the new function?

The equation is in slope-intercept form, $y = mx + b$. In this form, b represents the graph's *y*-intercept.

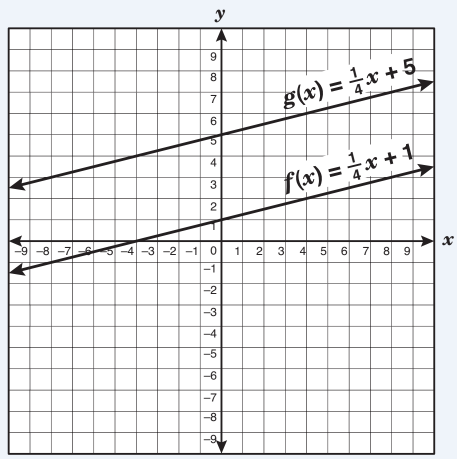
- The *y*-intercept of the given function is -4.
- If the *y*-intercept of the function were decreased by 3, it would be $-4 - 3 = -4 + (-3) = -7$. The new value for b would be -7.

The equation of the new function would be $y = \frac{1}{2}x - 7$.

The functions $f(x) = \frac{1}{4}x + 1$ and $g(x) = \frac{1}{4}x + 5$ are graphed below. How does the graph of $g(x)$ compare to the graph of $f(x)$?

- The value for b in $f(x)$ is 1.
  Its graph intersects the y-axis at (0, 1).
- The value for b in $g(x)$ is 5.
  Its graph intersects the y-axis at (0, 5).

Since $5 - 1 = 4$, the graph of $g(x)$ is 4 units above the graph of $f(x)$.



Try It

If the y-intercept of the function $y = 3.2x - 1.8$ is increased by 4.4 to create a new function, what are the characteristics of the new graph?

The equation of the given function is in slope-intercept form.

In this form, ______ represents the slope of the line, and the y-coordinate of the y-intercept is ______.

If the y-intercept were increased by 4.4, the new y-intercept would be equal to ______ + ______ = ______.

The new line would cross the y-axis at a ____________ point than the original line.

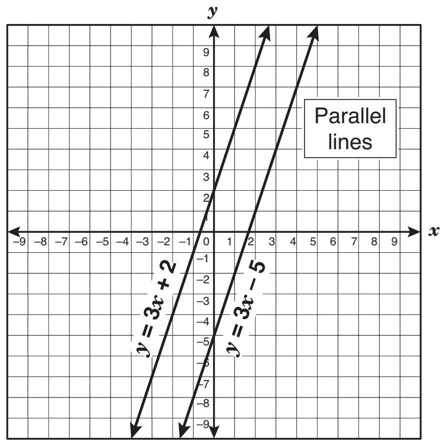
The new line would have the same slope, ______, as the original line.

The equation of the given function is in slope-intercept form. In this form, 3.2 represents the slope of the line, and the y-coordinate of the y-intercept is -1.8. If the y-intercept were increased by 4.4, the new y-intercept would be equal to $-1.8 + 4.4 = 2.6$. The new line would cross the y-axis at a higher point than the original line. The new line would have the same slope, 3.2, as the original line.

Slopes of lines can tell you whether the lines are parallel or perpendicular.

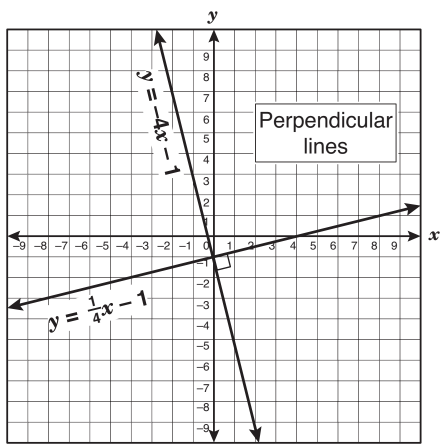
- If two lines are parallel, then they have the same slope, and their equations have the same value for m.

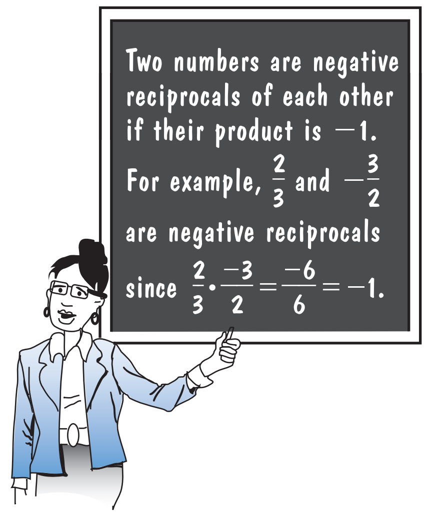
  Look at the graphs of these two equations: $y = 3x - 5$ and $y = 3x + 2$.



- If two lines are perpendicular, then they have negative reciprocal slopes, and their equations have negative reciprocal values for m.

  Look at the graphs of these two equations: $y = -4x - 1$ and $y = \frac{1}{4}x - 1$.





Objective 3

How Do You Write Linear Equations?

You can write linear equations in slope-intercept form, $y = mx + b$, or in **standard form**, $Ax + By = C$. In standard form, A, B, and C are integers, and A is usually greater than zero.

You can find the equation of a line given any of the following information:

- the slope and the y-intercept of the graph
- the slope and a point on the graph
- two points on the graph

Given the slope and the y-intercept

Identify the values for both m, the slope, and b, the y-intercept. Write the equation in slope-intercept form, $y = mx + b$, using these values.

What is the equation of the line with a slope of $\frac{1}{3}$ and a y-intercept of -4?

Find the values you should substitute into the equation $y = mx + b$.

- If the slope is $\frac{1}{3}$, then $m = \frac{1}{3}$.
- If the y-intercept is -4, then $b = -4$.

The equation of the line is:

$$y = mx + b$$
$$y = \frac{1}{3}x - 4$$

Try It

Write the equation of the line with a slope of -2 and a y-coordinate of the y-intercept of $\frac{3}{5}$.

If the slope is -2, then $m =$ ______.

If the y-coordinate of the y-intercept is $\frac{3}{5}$, then $b = \frac{\square}{\square}$.

The equation of the function is:

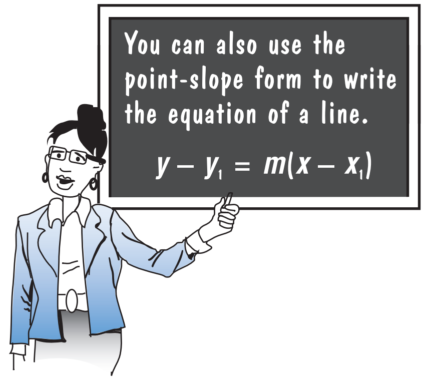
$$y = mx + b$$
$$y = \_\_\_\_ x + \frac{\square}{\square}$$

If the slope is -2, then $m = -2$. If the y-coordinate of the y-intercept is $\frac{3}{5}$, then $b = \frac{3}{5}$. The equation of the function is:

$$y = -2x + \frac{3}{5}$$

Given the slope and a point on the graph

- Substitute the given values (the *x*- and *y*-coordinates of the given point and *m*, the slope) into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for *b*.
- Substitute the values for *m* and *b* into the slope-intercept form, $y = mx + b$.



What is the equation of a line with a slope of 9, passing through the point (3, 4)?

- Substitute $x = 3$, $y = 4$, and $m = 9$ into the slope-intercept form of the equation.

$$y = mx + b$$
$$4 = 9(3) + b$$

- Solve for *b*.

$$\begin{aligned} 4 &= 9(3) + b \\ 4 &= 27 + b \\ -27 &= -27 \\ \hline -23 &= b \end{aligned}$$

- Substitute the given value for *m*, 9, and the value you found for *b*, −23, into the slope-intercept form of the equation.

$$y = mx + b$$
$$y = 9x - 23$$

The equation of the line is $y = 9x - 23$.

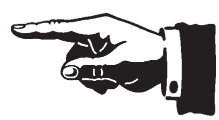
This equation could also be written in standard form, $Ax + By = C$, in which *A* is usually positive.

$$\begin{aligned} y &= 9x - 23 \\ -9x &= -9x \\ \hline -9x + y &= -23 \end{aligned}$$

To change −9 to a positive value, multiply both sides of the equation by −1.

$$9x - y = 23$$

In standard form, the equation of this line is $9x - y = 23$.



Do you see that . . .

Objective 3

Try It

Write the equation of the linear graph that has a slope of 3 and contains the point (6, 5).

Begin by substituting the given values into the slope-intercept form.

The line passes through the point (______, ______). Therefore,

$x =$ ______ and $y =$ ______.

Since the slope equals 3, $m =$ ______.

$$y = mx + b$$

$$\_\_\_\_\_\_ = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_ + b$$

Find the value for ______ that makes the equation true.

$$\_\_\_\_\_\_ = \_\_\_\_\_\_ + b$$

$$b = \_\_\_\_\_\_$$

Finally, substitute the given value for m, ______, and the value you found for b, ______, into the slope-intercept form of the equation.

$$y = mx + b$$

$$y = \_\_\_\_\_ x + \_\_\_\_\_\_$$

or

$$y = \_\_\_\_\_ x - \_\_\_\_\_\_$$

Begin by substituting the given values into the slope-intercept form. The line passes through the point (6, 5). Therefore, $x = 6$ and $y = 5$. Since the slope equals 3, $m = 3$.

$$5 = 3 \cdot 6 + b$$

Find the value for b that makes the equation true.

$$5 = 18 + b$$

$$b = -13$$

Finally, substitute the given value for m, 3, and the value you found for b, -13, into the slope-intercept form of the equation.

$$y = 3x + -13$$

or

$$y = 3x - 13$$

Given two points on the graph

- Use the x-coordinates and y-coordinates of the two given points to find the slope, m, of the line.
- Substitute the coordinates of one of the known points and the slope you just found into the slope-intercept form of the equation, $y = mx + b$.
- Solve the equation for b.
- Substitute m and b into the slope-intercept form, $y = mx + b$.

Find the equation of the line passing through the points (1, 2) and (5, 3).

- Find the slope of the graph using the slope formula. The line passes through the points (1, 2) and (5, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{5 - 1} = \frac{1}{4}$$

If m is $\frac{1}{4}$, the slope of the line is $\frac{1}{4}$.

- Substitute the coordinates of either of the given points and the value you found for m, $\frac{1}{4}$, into the slope-intercept form of the equation. Find the value of b.

Use (1, 2) or (5, 3).

$x = 1, y = 2$	$x = 5, y = 3$
$y = mx + b$	$y = mx + b$
$2 = \frac{1}{4}(1) + b$	$3 = \frac{1}{4}(5) + b$
$2 = \frac{1}{4} + b$	$3 = \frac{5}{4} + b$
$-\frac{1}{4} = -\frac{1}{4}$	$-\frac{5}{4} = -\frac{5}{4}$
$1\frac{3}{4} = b$	$1\frac{3}{4} = b$

- Substitute the values of m and b into the slope-intercept form of the equation.

$$y = mx + b$$
$$y = \frac{1}{4}x + 1\frac{3}{4}$$

The equation of the line passing through points (1, 2) and (5, 3) is $y = \frac{1}{4}x + 1\frac{3}{4}$.

Objective 3

Try It

Write the equation of the linear function that contains the points (4, 1) and (2, 3).

Find the __________ of the graph.

The line passes through the points (_____, _____) and (_____, _____).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square - \square}{\square - \square} = \frac{\square}{\square} = \_\_\_\_\_$$

The slope of the line is _____.

Substitute the coordinates of one of the given points, (4, _____), and the slope you found, _____, into the slope-intercept form of the equation of a line to find the value of b.

$$y = mx + b$$
$$\_\_\_\_\_ = \_\_\_\_\_ \cdot \_\_\_\_\_ + b$$
$$\_\_\_\_\_ = \_\_\_\_\_ + b$$
$$b = \_\_\_\_\_$$

Substitute the values of m and b into the slope-intercept form.

$$y = \_\_\_\_\_ + \_\_\_\_\_$$

Find the slope of the graph. The line passes through the points (4, 1) and (2, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{2 - 4} = \frac{2}{-2} = -1$$

The slope of the line is -1.

Substitute the coordinates of one of the given points, (4, 1), and the slope you found, -1, into the slope-intercept form of the equation of a line to find the value of b.

$$1 = -1 \cdot 4 + b$$
$$1 = -4 + b$$
$$b = 5$$

Substitute the values of m and b into the slope-intercept form.

$$y = -x + 5$$

How Do You Find the *x*-Intercept and *y*-Intercept of the Graph of an Equation?

Finding the x-intercept

The x-intercept is the point where the graph of a line intersects the x-axis. The x-intercept has the coordinates $(x, 0)$.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x. The value of x is the x-intercept. The graph of the line intersects the x-axis at the point $(x, 0)$.

What is the x-intercept of the graph of $5x - y = 10$?

Substitute $y = 0$ into the equation and solve for x.

$$\begin{aligned} 5x - y &= 10 \\ 5x - 0 &= 10 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

The x-coordinate of the x-intercept of the graph is 2. The graph of the line intersects the x-axis at $(2, 0)$.

Finding the y-intercept

The y-intercept is the point where the graph of a line intersects the y-axis. The y-intercept has the coordinates $(0, y)$.

One way to find the y-intercept is to write the equation in slope-intercept form, $y = mx + b$.

The value of b is the y-coordinate of the y-intercept. The graph of the line intersects the y-axis at the point $(0, b)$.

Another way to find the y-intercept is to substitute $x = 0$ into the equation and solve for y.

Here are two ways to find the y-intercept of the graph of $y + 4 = 2x$.

Write the equation in slope-intercept form, $y = mx + b$.

$$\begin{aligned} y + 4 &= 2x \\ -4 &= - 4 \\ \hline y &= 2x - 4 \end{aligned}$$

Therefore, $b = -4$.

Substitute $x = 0$ and solve for y.

$$\begin{aligned} y + 4 &= 2x \\ y + 4 &= 2(0) \\ y + 4 &= 0 \\ y &= -4 \end{aligned}$$

The y-intercept of the graph is -4. The graph intersects the y-axis at $(0, -4)$.

Objective 3

What are the x- and y-intercepts of the line of $4x + 3y = 24$?

To find the y-intercept, substitute $x = 0$ into the equation and solve for y.

$$4x + 3y = 24$$
$$4(0) + 3y = 24$$
$$0 + 3y = 24$$
$$3y = 24$$
$$y = 8$$

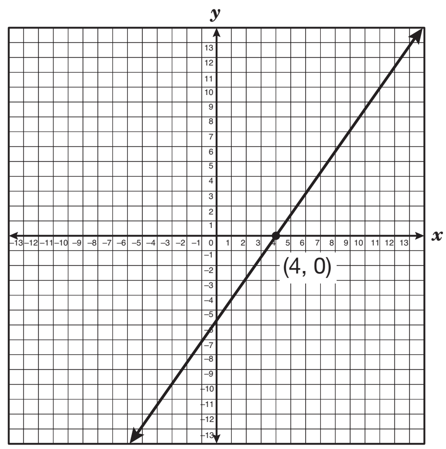
The y-intercept is 8.

To find the x-intercept, substitute $y = 0$ into the equation and solve for x.

$$4x + 3y = 24$$
$$4x + 3(0) = 24$$
$$4x + 0 = 24$$
$$4x = 24$$
$$x = 6$$

The x-intercept is 6.

For any equation, such as $y = \frac{3}{2}x - 6$, a **root** is a value of x that makes $y = 0$. In a function, a value of x that makes $f(x)$ equal to zero is called a **zero of the function**. Both the root of an equation and the zero of a function can be found by locating the x-coordinate of the x-intercept of the graph of a function, the point at which the graph intercepts the x-axis.

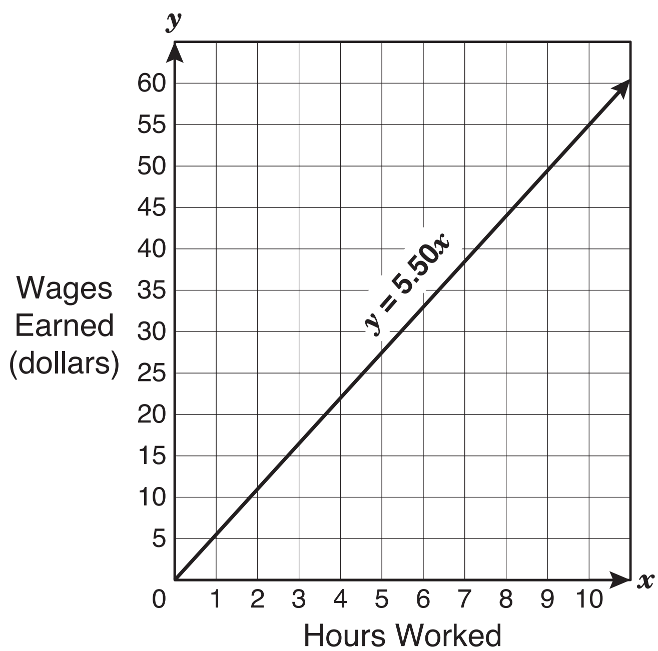
Roots of Equations	Zeros of Functions	x-Intercepts of Graphs
$y = \frac{3}{2}x - 6$	$f(x) = \frac{3}{2}x - 6$	$y = \frac{3}{2}x - 6$
$0 = \frac{3}{2}x - 6$ $+6 = \quad +6$ $6 = \frac{3}{2}x$ $\frac{2}{3} \cdot 6 = \frac{2}{3} \cdot \frac{3}{2}x$ $4 = x$ $x = 4$	$0 = \frac{3}{2}x - 6$ $+6 = \quad +6$ $6 = \frac{3}{2}x$ $\frac{2}{3} \cdot 6 = \frac{2}{3} \cdot \frac{3}{2}x$ $4 = x$ $x = 4$	
The root of $y = \frac{3}{2}x - 6$ is 4.	The zero of $f(x) = \frac{3}{2}x - 6$ is 4.	The coordinate of the x-intercept of $y = \frac{3}{2}x - 6$ is (4, 0).

How Do You Interpret the Meaning of Slopes and Intercepts?

To interpret the meaning of the slope or of the x- or y-intercept of a function in a real-life problem, follow these guidelines:

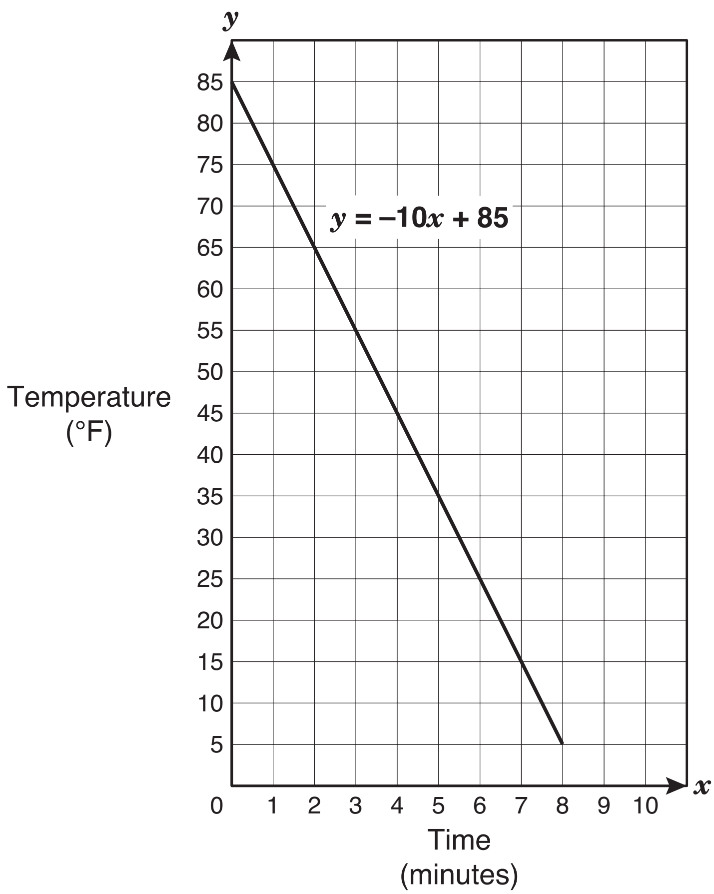
- The slope of the function is the function's rate of change. A graph's slope tells you how fast the function's dependent variable is changing for every unit change in the independent variable.

  For example, if a graph compares wages earned in dollars to hours worked, the slope tells the rate at which you are paid, or how much you make per hour.



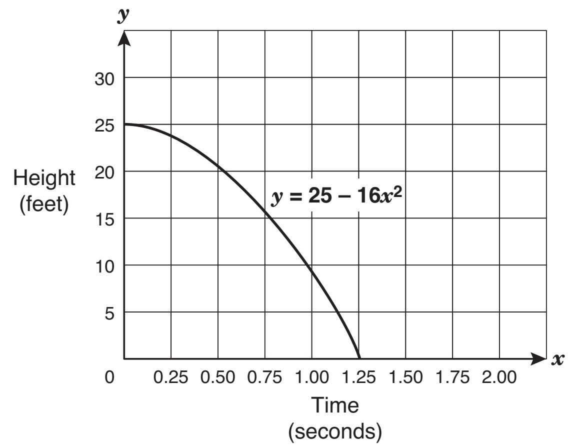
- The y-intercept is the point where the graph of a function intersects the y-axis. The y-intercept has the coordinates $(0, y)$. It is the point in the function where the independent quantity, x, has a value of 0. The y-intercept is often the starting point in a problem situation.

  For example, if a graph describes a constant temperature drop of 10°F per minute from $t = 0$ to $t = 8$ minutes, the y-intercept tells you what the temperature was at $t = 0$. The y-intercept of the graph is (0, 85), so the initial temperature was 85°F.

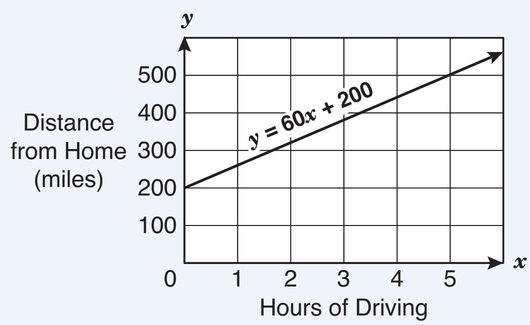


- The x-intercept is the point where the graph of the function intersects the x-axis. The x-intercept has the coordinates $(x, 0)$. It is the point in the problem where the dependent quantity, y, has a value of 0.

  For example, if a graph compares the height of a falling object to the number of seconds it has fallen, the x-intercept (when the object's height above the ground is 0) tells you how many seconds it will take for the object to hit the ground.



Gracie took a long driving trip. She started her journey at home and drove due west for two days. The graph below represents the number of miles Gracie was from her home on the second day of her trip. The slope of the graph is 60, and the y-intercept is (0, 200). Interpret the meaning of the slope and the y-intercept.

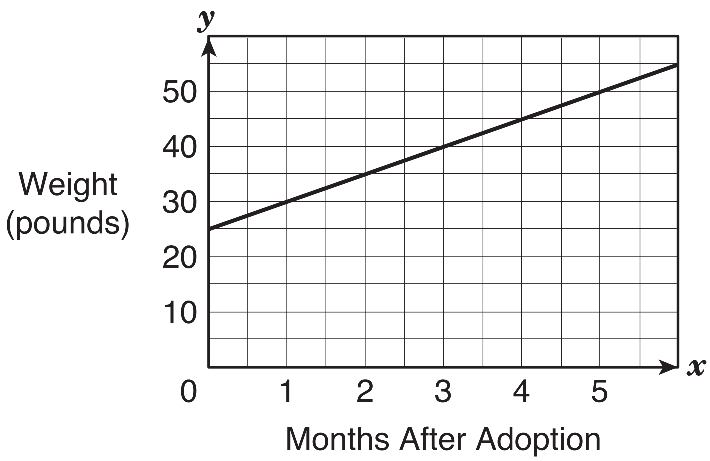


The slope of the graph is the function's rate of change. It compares the number of miles Gracie was from home to the number of hours she was driving, or miles per hour. A slope of 60 means she was driving at the rate of 60 miles per hour, or that her speed was 60 mph.

The y-intercept is the point where the graph intersects the y-axis, when hours traveled equals 0. This point is the time when Gracie started driving on the second day. A y-intercept of 200 means she was already 200 miles from home when she began driving on the second day of her trip.

Try It

The graph below shows the weight of Denise's dog Elmo over the 6-month period after she adopted him. What was the dog's weight when she adopted him? How many pounds did he gain each month during that 6-month period?



The y-intercept of the graph is at the point (0, ______).

This means Elmo weighed ______ pounds when Denise adopted him.

The number of pounds Elmo gained each month is the graph's rate of ____________, or its ____________.

To find the slope of the graph, identify the coordinates of ______ points on the graph.

For example, (2, ______) and (______, 45) are points on the graph.

The slope of the graph is the change in the ______-coordinates between any two points compared to the corresponding change in their ______-coordinates.

The slope of the graph is $\frac{45 - \square}{4 - \square} = \frac{\square}{\square} =$ ______.

Elmo gained ______ pounds per month during the 6-month period.

The y-intercept of the graph is at the point (0, 25). This means Elmo weighed 25 pounds when Denise adopted him. The number of pounds Elmo gained each month is the graph's rate of change, or its slope. To find the slope of the graph, identify the coordinates of two points on the graph. For example, (2, 35) and (4, 45) are points on the graph. The slope of the graph is the change in the y-coordinates between any two points compared to the corresponding change in their x-coordinates. The slope of the graph is $\frac{45 - 35}{4 - 2} = \frac{10}{2} = 5$. Elmo gained 5 pounds per month during the 6-month period.

Objective 3

How Do You Predict the Effects of Changing Slopes and *y*-Intercepts in Applied Situations?

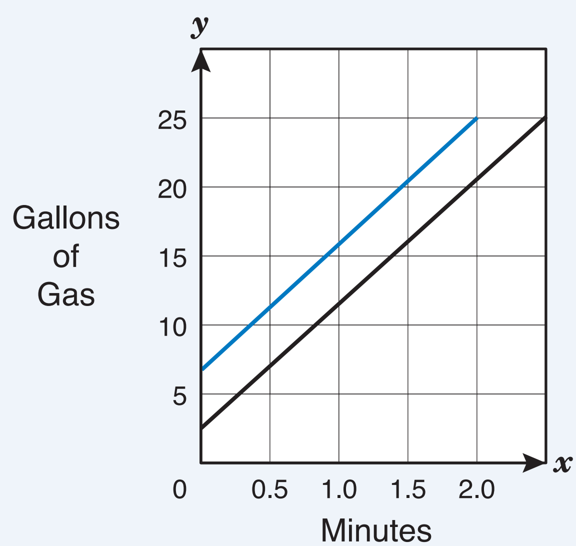
Many real-life problems can be modeled with linear functions. To analyze such problems, it is often helpful to identify the slope and the *y*-intercept of the linear function. Interpreting the meaning of these values will help you predict the effect that changing them will have on the quantities in the problem.

- If the slope is changed, a rate of change in the problem will increase or decrease.
- If the *y*-intercept is changed, an initial condition will change.

Tess is filling the gas tank of her car. The graph represents the gallons of gas in her tank in terms of the number of minutes she pumps gas.

The graph intersects the *y*-axis at the point (0, 2.5). The graph suggests that Tess had 2.5 gallons of gasoline in her car when she started to pump gas.

If her car had had 7 gallons of gas in it when she started pumping gas, how would the graph be affected?



If her gas tank had had 7 gallons instead of 2.5 gallons when she started pumping gas, the graph would pass through the *y*-axis at the point (0, 7) instead of (0, 2.5).

The graphs would be parallel lines.

Try It

Zippy's, a package-delivery service, charges $12 plus $0.08 per mile to deliver a package within the city. How would the graph of the cost of delivering a package change if Zippy's increased its mileage charge to $0.09 per mile?

Write an equation that represents the cost, c, of delivering a package n miles.

$$c = \_\_\_\_\_\_ n + \_\_\_\_\_\_$$

In this equation 0.08 represents the ____________ of the graph of the equation.

If the mileage charge were changed from 0.08 to 0.09, the slope of the graph would ____________.

The new line will be ____________.

Write an equation that represents the cost, c, of delivering a package n miles.

$$c = 0.08n + 12$$

In this equation 0.08 represents the slope of the graph of the equation. If the mileage charge were changed from 0.08 to 0.09, the slope of the graph would increase. The new line will be steeper.

Objective 3

How Do You Solve Problems Involving Direct Variation or Proportional Change?

If a quantity y varies directly with a quantity x, then the linear equation representing the relationship between the two quantities is $y = kx$. In this equation, k is called the **proportionality constant**.

To say "y varies directly with x" is to say "y is directly proportional to x."

If the equation $y = kx$ were graphed, k would be the slope of the graph.

The price of milk varies directly with the number of quarts of milk purchased. If 5 quarts of milk cost $6.25, what would 3 quarts of milk cost?

- Write an equation that compares the number of quarts purchased to the cost.

  Let q = the number of quarts purchased.

  Let c = the cost.

  The direct variation equation is $c = kq$.

- Substitute the known values for c and q to find the proportionality constant, k. Five quarts of milk ($q = 5$) cost $6.25 ($c = 6.25$).

$$c = kq$$
$$6.25 = k(5)$$
$$1.25 = k$$

  If $k = 1.25$, then the proportionality constant is $1.25.
  If $k = 1.25$, then the slope of the graph is 1.25.

- Find the cost of 3 quarts of milk by substituting $k = 1.25$ and $q = 3$ into the equation $c = kq$.

$$c = kq$$
$$c = 1.25(3)$$
$$c = 3.75$$

Three quarts of milk cost $3.75.

Now practice what you've learned.

Question 25

Which of the following can best be described by a linear function?

A The area of a circle with radius r

B The perimeter of an equilateral triangle with side length s

C The surface area of a cube with side length s

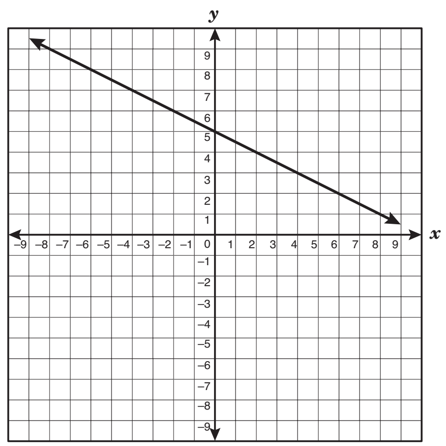
D The volume of a cylinder with radius r and height h

Answer Key: page 238

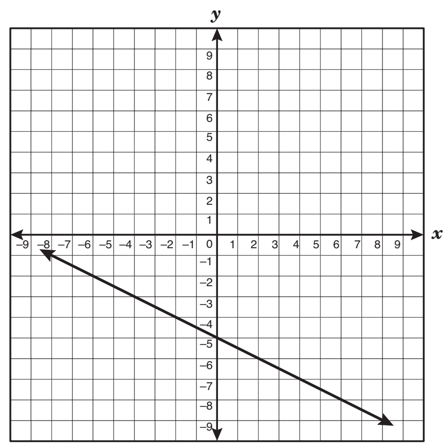
Question 26

Which graph best represents the equation $2y - x = 10$?

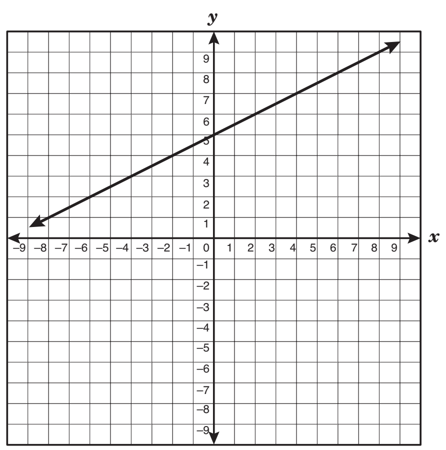
A



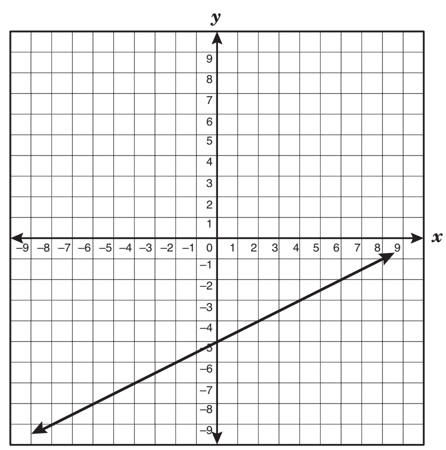
C



B



D

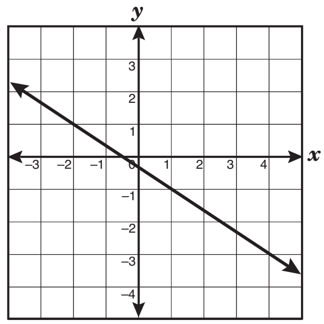


Answer Key: page 238

Question 27

Which of the following linear functions does not represent a rate of change of $-\frac{2}{3}$?

A



C

x	−3	1	5	9
y	10	4	−2	−8

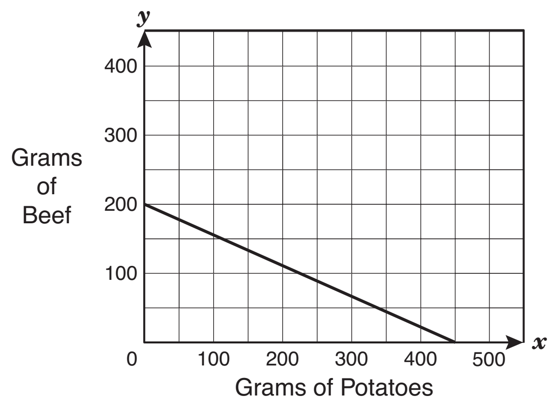
B $2x + 3y = 12$

D $y = -\frac{2}{3}x + 9$

Answer Key: page 238

Question 28

The graph below shows the number of grams of beef and the number of grams of potatoes you could eat to obtain approximately 500 calories of energy.



Which of the following numbers represents the maximum number of grams of potatoes you could eat to obtain approximately 500 calories?

A 200 g

B 450 g

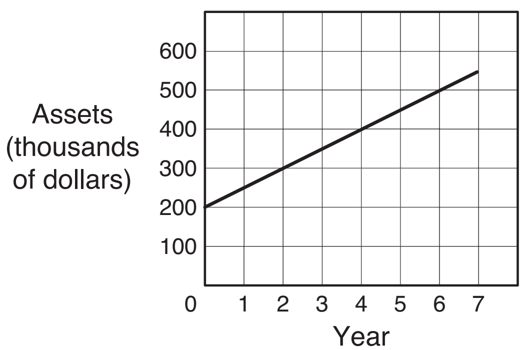
C 100 g

D 150 g

Answer Key: page 239

Question 29

The graph projects a business's growth in financial assets over a seven-year period.



Which of the following interpretations of the graph is true?

A The company's initial assets are \$200,000. The expected growth rate is \$50 per year.

B The company's initial assets are \$200. The expected growth rate is \$50,000 per year.

C The company's initial assets are \$200,000. The expected growth rate is \$50,000 per year.

D The company's initial assets are \$200. The expected growth rate is \$50 per year.

Answer Key: page 239

Question 30

Which equation describes the line that passes through the point (–3, –8) and is parallel to the line represented by the equation $4x - 5y = -2$?

A $y = \frac{4}{5}x - \frac{47}{5}$

B $y = \frac{4}{5}x - \frac{28}{5}$

C $y = \frac{4}{5}x + \frac{17}{5}$

D $y = \frac{4}{5}x - \frac{68}{5}$

Answer Key: page 239

Question 31

The line $y = \frac{3}{4}x - 4$ is drawn on a coordinate grid. A second line is drawn with a slope of 1.

Which statement best describes the relationship between these two graphs?

A The second line is steeper than the first line.

B The graphs are perpendicular lines.

C The second line is less steep than the first line.

D The graphs are parallel lines.

Answer Key: page 239

Question 32

Which equation is best represented by a line containing the points (2, −5) and (4, 3)?

A $x + 4y = 13$

B $y = 4x + 13$

C $y = -4x + 19$

D $-4x + y = -13$

Answer Key: page 239

Question 33

Find the coordinates of the x-intercept and the y-intercept of the line $2x = 9 - 3y$.

A x-intercept (3, 0); y-intercept $(0, \frac{9}{2})$

B x-intercept (0, 3); y-intercept $(\frac{9}{2}, 0)$

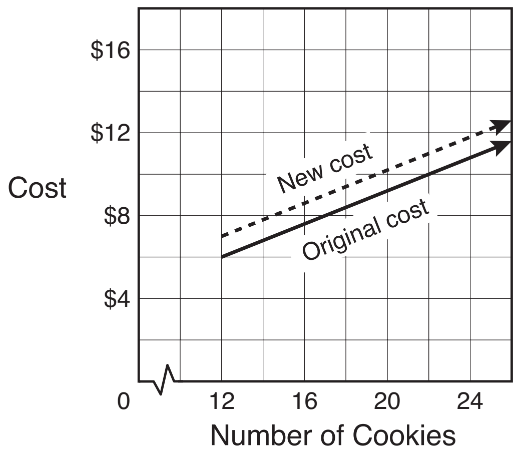
C x-intercept $(0, \frac{9}{2})$; y-intercept (3, 0)

D x-intercept $(\frac{9}{2}, 0)$; y-intercept (0, 3)

Answer Key: page 240

Question 34

A local bakery sells cookies by the dozen. If you want more than one dozen cookies, the bakery charges by the cookie for the additional cookies. The first graph below shows the original cost of buying a dozen or more cookies from the bakery. The second graph shows the cost of buying a dozen or more cookies after the bakery changed its prices.



Which statement describes how the bakery changed its price for a dozen or more cookies?

A The bakery increased the cost of the first dozen cookies.

B The bakery increased the cost per cookie after the first dozen.

C The bakery decreased the cost per cookie after the first dozen.

D The bakery decreased the cost of the first dozen cookies.

Answer Key: page 240

Question 35

The number of miles Sammie walks is directly proportional to the number of minutes she walks. If Sammie walks 3 miles in 45 minutes, what is the constant of proportionality, and how far would she walk in 2.5 hours?

A $\frac{1}{15}$; 10 miles

B $\frac{1}{15}$; 16.6 miles

C 15; 37.5 miles

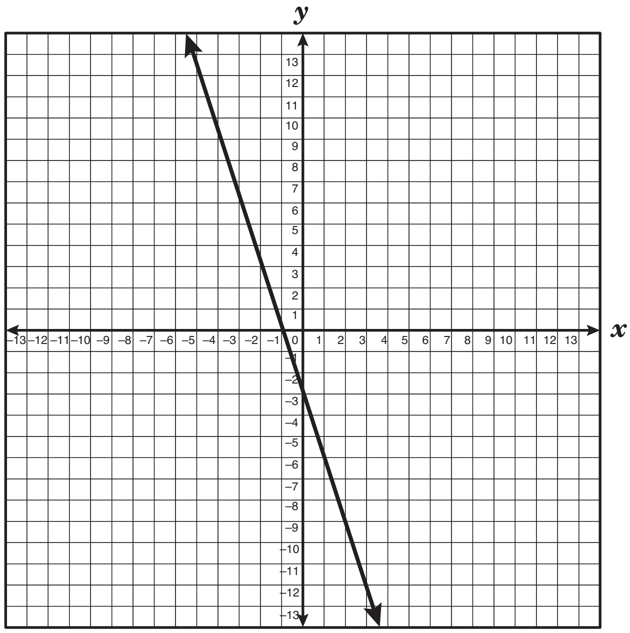
D 15; 225 miles

Answer Key: page 240

Question 36

Which of the following represents a function that has –3 as a zero?

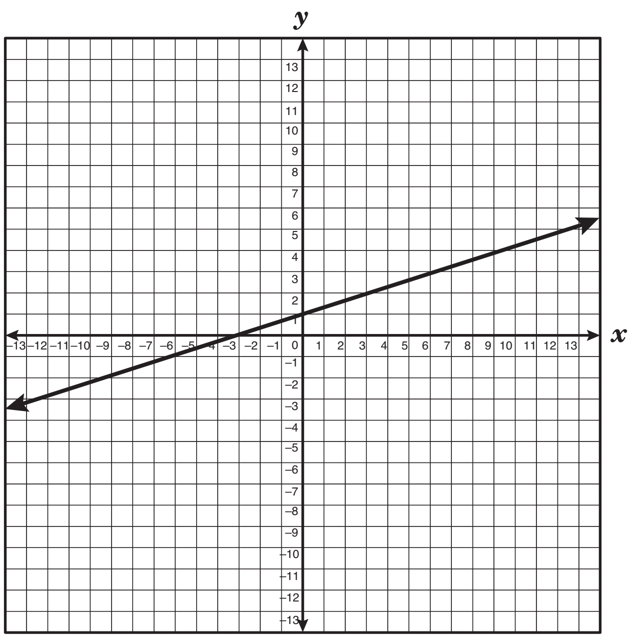
A



C

x	y
–2	6
–1	3
0	0
1	–3
2	–6

B



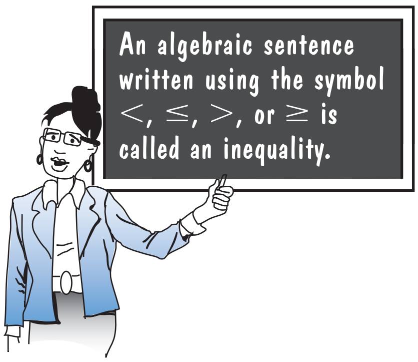
D

x	y
–3	–3
–2	0
–1	3
0	6
1	9

Answer Key: page 240

Objective 4

The student will formulate and use linear equations and inequalities.



For this objective you should be able to

- formulate linear equations and inequalities from problem situations, use a variety of methods to solve them, and analyze the solutions; and
- formulate systems of linear equations from problem situations, use a variety of methods to solve them, and analyze the solutions.

How Do You Solve Problems Using Linear Equations or Inequalities?

Many real-life problems can be solved using either a linear equation or an inequality. To solve the equation or inequality, follow these steps:

- Simplify the expressions in the equation or inequality by removing parentheses and combining like terms.
- Isolate the variable as a single term on one side of the equation by adding or subtracting expressions on both sides of the equation or inequality.
- Use multiplication or division to produce a coefficient of 1 for the variable term.

Do you see that . . .

- When solving an inequality, you must reverse the inequality symbol if you multiply or divide both sides by a negative number.

$$-2x > 10$$

$$\frac{-2x}{-2} < \frac{10}{-2} \quad \text{Divide both sides by a negative number.}$$

$$x < -5$$

  The inequality symbol reversed; it went from $>$ to $<$.

- Use the solution of the equation or inequality to find the answer to the question asked.
- See whether your answer is reasonable.

The combined weight of 3 people in a small plane cannot safely exceed 620 pounds. The pilot weighs 185 pounds, and Ramon weighs $1\frac{1}{2}$ times as much as his wife Grace. Altogether they do not exceed the weight limit. Write an inequality that could be used to find Grace's maximum possible weight.

- Represent the quantities involved with variables or expressions.
  You know that Ramon's weight is 1.5 times Grace's weight.
  Represent Grace's weight with g.
  Ramon weighs 1.5 times his wife's weight, or 1.5g.

- Write the inequality.
  Grace's weight + Ramon's weight + the pilot's weight must be less than or equal to 620 pounds.

$$g + 1.5g + 185 \leq 620$$
$$1g + 1.5g + 185 \leq 620$$
$$2.5g + 185 \leq 620$$

The inequality $2.5g + 185 \leq 620$ could be used to find Grace's maximum possible weight.

In the school Adopt-a-Highway program, Trent picked up twice as many empty soda cans as Susan did but only one-third as many as Ginger collected. Together, the team picked up 135 cans. How many cans did Trent pick up?

- Represent the unknown quantities with variables or expressions.
  The number of cans each team member picked up is described in terms of the number of cans Trent picked up. Represent the number of cans Trent picked up using the variable t.

  Susan picked up $\frac{1}{2}$ as many cans as Trent.

  Ginger picked up 3 times as many cans as Trent.

Trent	Susan	Ginger
t	$0.5t$	$3t$

- Write an equation that can be used to solve the problem.
  Together the team picked up 135 cans.

$$t + 0.5t + 3t = 135$$

- Simplify the expression by combining like terms. Then solve the equation for t.

$$1t + 0.5t + 3t = 135$$
$$(1 + 0.5 + 3)t = 135$$
$$4.5t = 135$$
$$t = 30$$

In this problem, the solution to the equation, $t = 30$, is also the answer to the problem. Trent picked up 30 empty soda cans.

Objective 4

The width of a rectangle is between 5 and 10 inches. If its length is 1 inch more than twice its width, what is a reasonable range for the rectangle's perimeter?

Its width can be represented by w.
Its length can be represented by $2w + 1$.
Its perimeter can be represented by P.

$$P = 2l + 2w$$

$$P = 2(2w + 1) + 2w$$

$$P = 4w + 2 + 2w$$

$$P = 6w + 2$$

Find a reasonable range for P.

Substitute $w = 5$		Substitute $w = 10$

$$(6w + 2) < P < (6w + 2)$$

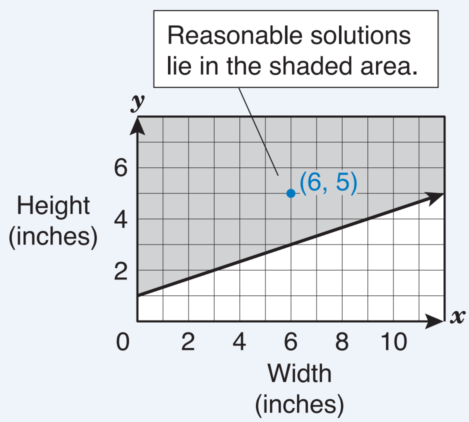
$$(6 \cdot 5 + 2) < P < (6 \cdot 10 + 2)$$

$$(30 + 2) < P < (60 + 2)$$

$$32 < P < 62$$

Any perimeter between 32 inches and 62 inches would be reasonable.

An illustrator wants to produce a series of pen-and-ink drawings. The height of each drawing must be at least 1 inch more than $\frac{1}{3}$ its width. This relationship is represented in the graph below.



If a drawing 6 inches wide is 5 inches tall, does it meet the requirements?

Find the 6 on the x-axis and then go up to the shaded region in the graph. Is the point (6, 5) in the shaded region? Yes.

Then a 6-inch-by-5-inch drawing meets the requirements.

Try It

The length of a rectangle is 5 inches greater than its width. If the width of the rectangle is represented by w and its perimeter is represented by P, write an equation in terms of P and w that could be used to find the dimensions of the rectangle.

Represent the width of the rectangle by w.

The length of the rectangle is 5 inches ________________ its width.

Represent the length by w + ________.
Write an equation and simplify it.

$$P = 2l + 2w$$

$$P = 2(w + \_\_\_\_\_\_) + 2w$$

$$P = 2w + \_\_\_\_\_\_ + 2w$$

$$P = \_\_\_\_\_\_ w + \_\_\_\_\_\_$$

The equation P = ________ w + ________ could be used to find the dimensions of the rectangle.

The length of the rectangle is 5 inches greater than its width. Represent the length by $w + 5$.

$$P = 2l + 2w$$

$$P = 2(w + 5) + 2w$$

$$P = 2w + 10 + 2w$$

$$P = 4w + 10$$

The equation $P = 4w + 10$ could be used to find the dimensions of the rectangle.

Try It

Saul used two different types of trim molding on a woodworking project. He used 6 feet more of the molding that cost \$2.50 per foot than he used of the molding that cost \$1.50 per foot. If the combined cost of the molding used to complete his project was \$55, how many feet of each type of molding did Saul use?

Represent the number of feet of \$1.50-per-foot molding with m.

Represent the number of feet of \$2.50-per-foot molding with

$m +$ ________.

Represent the cost of the \$1.50 molding used.

____

Represent the cost of the \$2.50 molding used.

2.50(________ + ________)

Write an equation that shows the total cost as \$55 and solve for m.

1.50m + 2.50(________ + ________) = 55

1.50m + 2.50m + ________ = 55

________m + 15 = 55

________m = ________

m = ________

Saul used ________ feet of molding that cost \$1.50 per foot.

He used $m + 6 =$ ________ + 6 = ________ feet of molding that cost \$2.50 per foot.

Represent the number of feet of \$2.50-per-foot molding with $m + 6$. Represent the cost of the \$1.50 molding used: $1.50m$. Represent the cost of the \$2.50 molding used: $2.50(m + 6)$.

$$1.50m + 2.50(m + 6) = 55$$

$$1.50m + 2.50m + 15 = 55$$

$$4m + 15 = 55$$

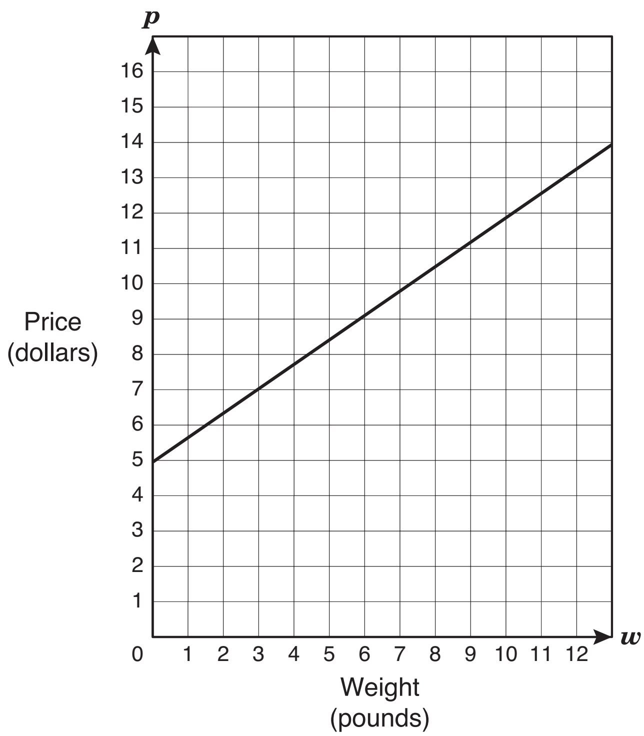
$$4m = 40$$

$$m = 10$$

Saul used 10 feet of molding that cost \$1.50 per foot. He used $m + 6 = 10 + 6 = 16$ feet of molding that cost \$2.50 per foot.

Try It

A farmer sells peaches and decorative baskets at a roadside stand. He sells the baskets for $4.95 each and the peaches for $0.69 per pound. The equation $p = 0.69w + 4.95$ represents the price, p, of a basket containing w pounds of peaches. The graph of this relationship is shown below.



Use the graph to find a reasonable value for the number of pounds of peaches in a decorative basket that would sell for $10.00 altogether.

Go to the point on the ________________ axis where the cost is approximately $10.00.

Go ________________ until you reach the graph.

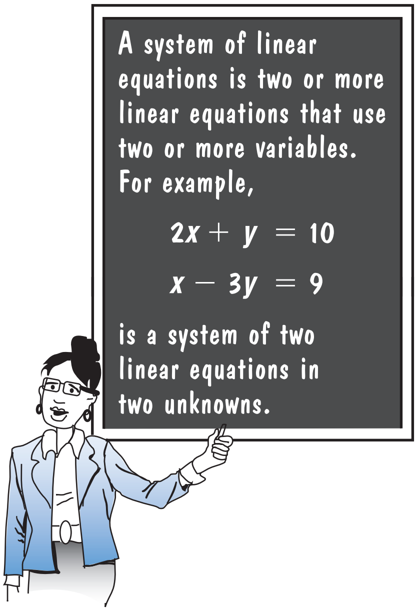
Go ________________ to the ________________ axis and read the value there.

The corresponding value is greater than ________.

A reasonable value for the number of pounds of peaches in a basket costing $10.00 is about ________ pounds.

Go to the point on the vertical axis where the cost is approximately $10.00. Go across until you reach the graph. Go down to the horizontal axis and read the value there. The corresponding value is greater than 7. A reasonable value for the number of pounds of peaches in a basket costing $10.00 is about $7\frac{1}{3}$ pounds.

Objective 4



How Do You Represent Problems Using a System of Linear Equations?

Many real-life problems can be solved using a system of two or more linear equations.

To represent a problem using a system of linear equations, follow these guidelines.

- Identify the quantities involved and the relationships between them.
- Represent the quantities involved with two different variables or with expressions involving two variables.
- Write two independent equations that can be used to solve the problem.

The sum of two numbers is 100. Twice the first number plus three times the second number is 275. Write a system of two equations in two unknowns that could be used to find the two numbers.

The problem involves two numbers. One number is not described in terms of the other number, so it makes sense to represent them using two different variables.

- Represent the first number with the variable x.
- Represent the second number with the variable y.

You know two different relationships between the numbers.

- Their sum is 100.

$$x + y = 100$$

- Twice the first number plus three times the second number is 275.

$$\text{twice the first} + \text{three times the second} = 275$$

$$2x \quad + \quad 3y \quad = 275$$

The following system of linear equations could be used to find the two numbers.

$$x + y = 100$$

$$2x + 3y = 275$$

Try It

Daniel put all the pennies and nickels from his pocket change into a jar. At the end of the month, there were 210 coins in his jar, worth $3.30 in all. Write a system of two equations that can be used to find p, the number of pennies, and n, the number of nickels, in the jar at the end of the month.

The value of a group of mixed coins depends on the value of each type of coin.

The number of any one type of coin times its ________________ gives the total value of all the coins of that type.

Represent the number of coins of each type Daniel had in his jar.
Represent the number of pennies with the variable p.
Represent the number of nickels with the variable n.

Represent the total value in cents of each type of coin in his jar.

Represent the value of the pennies with the expression _________.

Represent the value of the nickels with the expression _________.

Write two equations that describe the relationships between these quantities:

The number of pennies + the number of nickels = _________ coins in the jar:

_________ + _________ = _________

The value of the pennies + the value of the nickels = _________ cents in the jar:

_________ + _________ = _________

The following system of equations could be used to find the number of each type of coin Daniel has in his jar:

$p + n =$ _________

_______ $p +$ _______ $n =$ _________

The number of any one type of coin times its value gives the total value of all the coins of that type. Represent the value of the pennies with the expression $1p$. Represent the value of the nickels with the expression $5n$. The number of pennies + the number of nickels = 210 coins in the jar: $p + n = 210$. The value of the pennies + the value of the nickels = 330 cents in the jar: $1p + 5n = 330$. The following system of equations could be used to find the number of each type of coin Daniel has in his jar:

$$p + n = 210$$
$$1p + 5n = 330$$

How Do You Solve a System of Linear Equations?

You can solve a system of linear equations algebraically and graphically. Two algebraic methods are substitution and elimination.

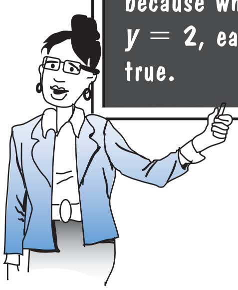
A graph can show you how many solutions a system of equations has.

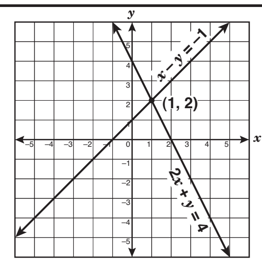
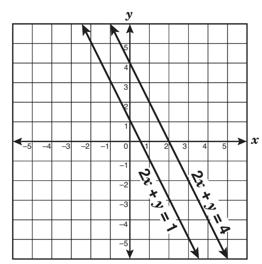
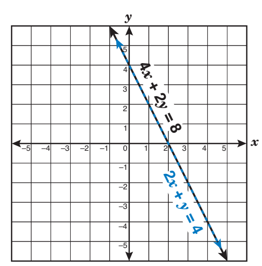
The solution to a system of linear equations is a pair of numbers that makes both equations true.

For example, the ordered pair (4, 2) is a solution to the following system of equations:

$$2x + y = 10$$
$$x - y = 2$$

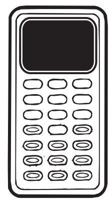
because when $x = 4$ and $y = 2$, each equation is true.



If the two lines …	… as shown by this graph …	… then the system of equations …
intersect at a single point (intersecting lines)		has one solution: $x = 1$ and $y = 2$ or (1, 2).
do not intersect (parallel lines)		has no solution.
intersect at every point (coincident lines)		has infinitely many solutions.

You can also determine the number of solutions a system of equations has by writing the equations in slope-intercept form, $y = mx + b$.

If the two equations …	… as shown by this system …	… then the system of equations …
have different values for m	$2x + y = 4 \rightarrow y = -2x + 4$ $x - y = -1 \rightarrow y = x + 1$	has one solution: $x = 1$ and $y = 2$.
have different values for b but the same value for m	$2x + y = 4 \rightarrow y = -2x + 4\ (b = 4)$ $2x + y = 1 \rightarrow y = -2x + 1\ (b = 1)$	has no solution.
have the same values for m and b	$2x + y = 4 \rightarrow y = -2x + 4$ $4x + 2y = 8 \rightarrow y = -2x + 4$	has infinitely many solutions.

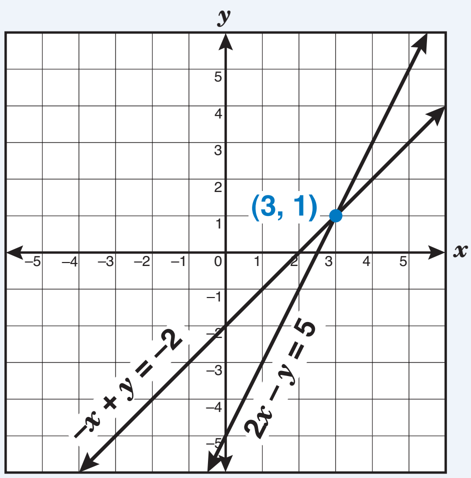


Solve this system of equations using the graphical method.

$$-x + y = -2$$

$$2x - y = 5$$

The two equations are graphed below.



The coordinates of the point where the two lines intersect, (3, 1), is the solution to the system of equations. The coordinates satisfy both equations.

	$-x + y = -2$	$2x - y = 5$
$x = 3$ and $y = 1$	$-x + y = -2$ $-(3) + (1) = -2$ $-2 = -2$	$2x - y = 5$ $2(3) - (1) = 5$ $5 = 5$

The solution to the system of equations is (3, 1).

Objective 4

When you use the substitution method to solve a system of equations, solve one equation for one of the two variables. Then substitute into the second equation.

Solve the following system of equations using the substitution method.

$$a + b = 30$$
$$2a - 3b = 10$$

This system of equations lends itself to being solved using substitution because the first equation can be solved easily for a in terms of b.

To solve the first equation for a, subtract b from both sides.

$$\begin{array}{r} a + b = 30 \\ -\,b = \quad -\,b \\ \hline a = 30 - b \end{array}$$

Now substitute $(30 - b)$ for a in the second equation.

The result will be a linear equation with just one variable.

Solve that equation.

$2a - 3b = 10$	
$2(30 - b) - 3b = 10$	Substitute $(30 - b)$ for a.
$60 - 2b - 3b = 10$	Remove parentheses; multiply by 2.
$60 - 5b = 10$	Combine like terms: $-2b - 3b = -5b$.
$60 = 10 + 5b$	Add $5b$ to both sides.
$50 = 5b$	Subtract 10 from both sides.
$10 = b$	Divide both sides by 5.

Now that you know the value of b, you can use this value to find the value of a by substituting the value of b into either of the two original equations.

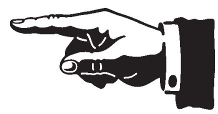
In this system, the first equation is the easier equation to use.

$a + b = 30$	
$a + 10 = 30$	Substitute $b = 10$.
$a = 20$	Subtract 10 from both sides.

The solution to the system of equations is $a = 20$ and $b = 10$, or $(20, 10)$.

Objective 4

The elimination method is also called the addition method because you eliminate one of the variables by adding. Before you add, you may need to multiply one or both equations so that one of the variables has opposite coefficients.



Do you see that . . .

Solve the same system of equations using the elimination method.

$$\begin{aligned} a + b &= 30 \\ 2a - 3b &= 10 \end{aligned}$$

In the given system of equations, if you multiply both sides of the first equation by 3, you get the following equations:

$$\begin{aligned} 3(a + b) = 3(30) &\longrightarrow 3a + 3b = 90 \\ 2a - 3b = 10 &\longrightarrow 2a - 3b = 10 \end{aligned}$$

The variable b has a coefficient of 3 in the new equation and an opposite coefficient of -3 in the original second equation. When you add those two equations, the term containing b will disappear because it will have a 0 coefficient.

$$\begin{aligned} 3a + 3b &= 90 \\ +\ 2a - 3b &= 10 \\ \hline 5a \qquad &= 100 \\ a \qquad &= 20 \end{aligned}$$

To find b, substitute 20 for a into the first equation.

$$\begin{aligned} a + b &= 30 \\ 20 + b &= 30 \\ b &= 10 \end{aligned}$$

The solution to the system of equations is $a = 20$ and $b = 10$, or (20, 10). This is the same solution obtained by using the substitution method.

Try It

At the concession stand a can of soda costs $0.25 more than a bottle of water. If John bought 3 bottles of water and 2 cans of soda for $8, how much did each type of drink cost?

Let s represent the cost of a can of soda.
Let w represent the cost of a bottle of water.

If a can of soda costs $0.25 more than a bottle of water, then an equation that can be used to represent this is

$$s = w + \_\_\_\_\_\_\_\_.$$

If 3 bottles of water and 2 cans of soda cost $8, then an equation that can be used to represent this is

$$3\_\_\_\_\_\_\_\_ + 2\_\_\_\_\_\_\_\_ = 8.$$

Solve the system of equations using the substitution method. Substitute $w + 0.25$ for s in the second equation and solve.

$$\begin{aligned} 3w + 2s &= 8.00 \\ 3w + 2(\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_) &= 8.00 \\ 3w + \_\_\_\_\_\_\_\_ w + \_\_\_\_\_\_\_\_ &= 8.00 \\ \_\_\_\_\_\_\_\_ w + 0.50 &= 8.00 \\ \_\_\_\_\_\_\_\_ w &= 7.50 \\ w &= \_\_\_\_\_\_\_\_ \end{aligned}$$

A bottle of water costs $________.

A can of soda costs $s = w + 0.25 =$ ________ $+ 0.25 =$ $________.

If a can of soda costs $0.25 more than a bottle of water, then an equation that can be used to represent this is $s = w + 0.25$. If 3 bottles of water and 2 cans of soda cost $8, then an equation that can be used to represent this is $3w + 2s = 8$.

$$\begin{aligned} 3w + 2s &= 8.00 \\ 3w + 2(w + 0.25) &= 8.00 \\ 3w + 2w + 0.50 &= 8.00 \\ 5w + 0.50 &= 8.00 \\ 5w &= 7.50 \\ w &= 1.50 \end{aligned}$$

A bottle of water costs $1.50.
A can of soda costs $s = w + 0.25 = 1.50 + 0.25 =$ $1.75.

Now practice what you've learned.

Question 37

Jeanne wants to build a fence to enclose an area for a new rose garden. She can afford 150 feet of fencing. The length of the rectangular garden will be 5 feet more than the width, w. Which inequality best describes the possible width of her garden?

A $w + (w + 5) \leq 150$

B $2w + 2(w + 5) \geq 150$

C $2w + 2(w + 5) \leq 150$

D $w + (w + 5) \geq 150$

Answer Key: page 240

Question 38

Sharon kept track of her expenses last week. She spent $4 more on movie rentals than she did on lunches. She spent five times as much fixing her car as she did on movie rentals. If Sharon spent a total of $80 last week on these three expenses, how much did her car repairs cost?

A $8

B $12

C $60

D $14

Answer Key: page 241

Question 39

The sum of two numbers is 59. The difference between 2 times the first number and 6 times the second is -34. Find the two numbers.

A 40 and 19

B 38 and 97

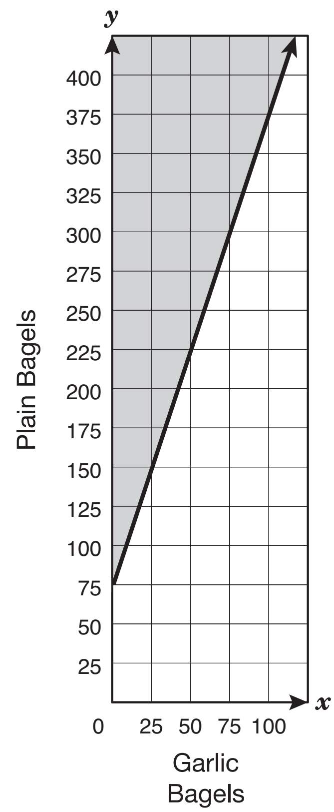
C -38 and 97

D -40 and -19

Answer Key: page 241

Question 40

Each morning at his bagel shop, Sid makes at least three times as many plain bagels as he does onion bagels and 25 more onion bagels than garlic bagels. The graph below represents the relationship between the number of plain bagels and the number of garlic bagels Sid prepares each day.



Which statement below does not satisfy this inequality relationship?

A Sid made 100 garlic bagels and 390 plain bagels.

B Sid made 50 garlic bagels and 225 plain bagels.

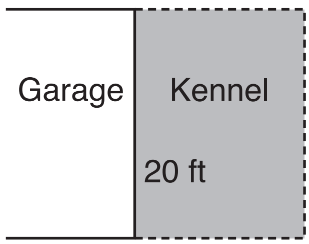
C Sid made 25 garlic bagels and 125 plain bagels.

D Sid made 75 garlic bagels and 300 plain bagels.

Answer Key: page 241

Question 41

Ira wants to build a rectangular dog kennel adjacent to the back wall of his garage.



Using the garage wall as the fourth side of the kennel allows him to fence only three sides of the kennel. The fencing material he is using costs \$4 per foot. Ira has \$120 to spend on the project. If the back wall of the garage is 20 feet long, what is the maximum width Ira can make the kennel?

A 10 ft

B 60 ft

C 100 ft

D 5 ft

Answer Key: page 241

Question 42

Which describes the solution to this system of linear equations?

$$6x - 2y = 7$$
$$-9x + 3y = 5$$

A This system of linear equations has only two solutions.

B This system of linear equations has no solution.

C This system of linear equations has only one solution.

D This system of linear equations has an infinite number of solutions.

Answer Key: page 242

Question 43

Sandra spent \$48.40 on tickets for a movie sneak preview. She bought 3 adult tickets and 5 child tickets. If the cost of an adult ticket, a, is twice as much as the cost of a child ticket, c, what is the cost of each kind of ticket?

A a = \$8.80
c = \$4.40

B a = \$13.82
c = \$6.91

C a = \$4.40
c = \$2.20

D a = \$6.91
c = \$3.46

Answer Key: page 242

Question 44

An ice-cream store projects that the profit, p, it earns on a total sales volume of s dollars is given by the formula $p = 0.25(s - 3000)$. If sales for the next month are projected to be between \$5000 and \$7000, what range best represents the total profit the store can expect for that month?

A $500 \le p \le 1000$

B $5000 \le p \le 7000$

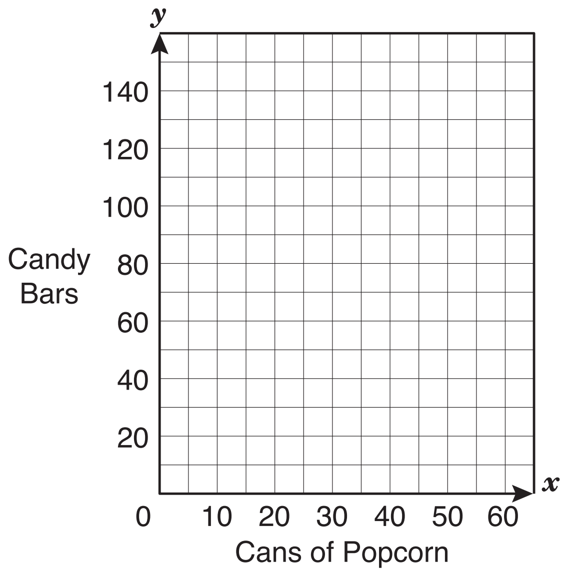
C $0 \le p \le 2000$

D $2000 \le p \le 4000$

Answer Key: page 242

Question 45

The members of a school choir had a fund-raising drive last month. They sold candy bars for $2 each and cans of popcorn for $5 each. Brent sold more than $300 worth of candy and popcorn altogether. Brent's sales can be represented by the inequality $5x + 2y > 300$.



Which of the following points could not reasonably represent the number of candy bars and cans of popcorn sold by Brent last month?

A (30, 90)

B (40, 80)

C (20, 50)

D (50, 40)

Answer Key: page 242

Question 46

The Beachfront Resort charges its guests according to the number of nights they stay at the resort and the number of meals they eat there. A guest can stay 2 nights and have 5 meals for $395, or a guest can stay 5 nights and have 11 meals for $959. Which system of equations can be used to find n, the cost of a night's stay, and m, the cost of a meal?

A $5n + 2m = 395$
$11n + 5m = 959$

B $5n + 2m = 959$
$11n + 5m = 395$

C $2n + 5m = 959$
$5n + 11m = 395$

D $2n + 5m = 395$
$5n + 11m = 959$

Answer Key: page 242

Question 47

Rachel is selling watermelons for $2 each and cantaloupes for $1 each. A customer bought a total of 13 watermelons and cantaloupes for $20. Which system of equations best describes the number of watermelons, w, and the number of cantaloupes, c, the customer bought?

A $w + c = 20$
$w + 2c = 13$

B $w + c = 13$
$w + 2c = 20$

C $w + c = 20$
$2w + c = 13$

D $w + c = 13$
$2w + c = 20$

Answer Key: page 243

Objective 5

The student will demonstrate an understanding of quadratic and other nonlinear functions.

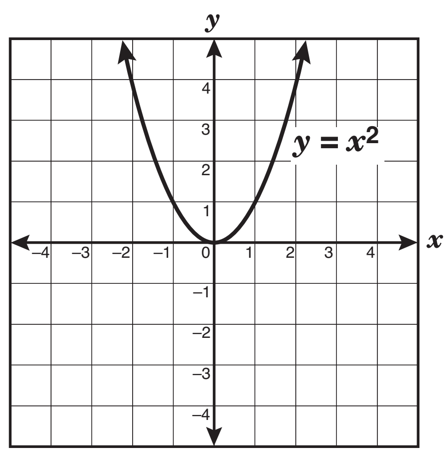
For this objective you should be able to

- interpret and describe the effects of changes in the parameters of quadratic functions;
- solve quadratic equations using appropriate methods; and
- apply the laws of exponents in problem-solving situations.

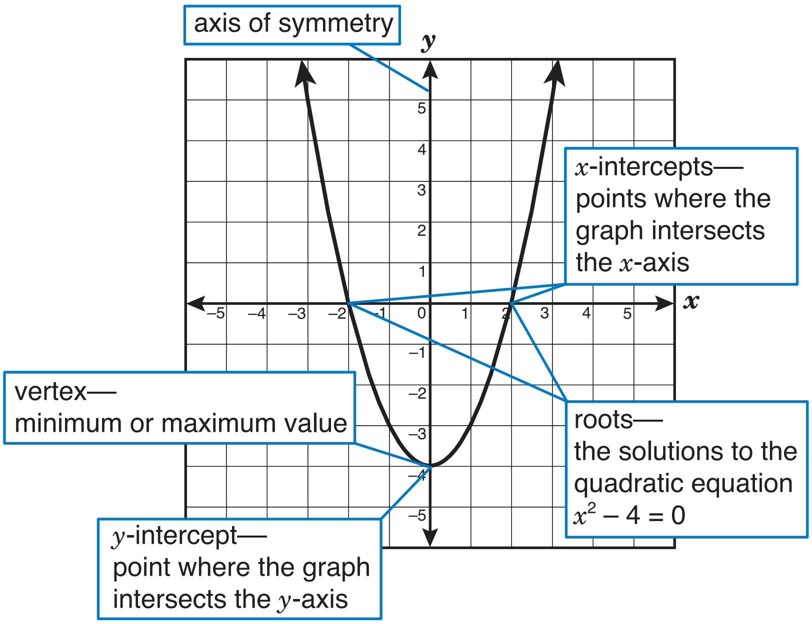
What Is a Quadratic Function?

- A **quadratic function** is any function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. Its graph is a parabola. When you know the values of a, b, and c, they help you describe the shape and location of the parabola.
- A **quadratic equation** is any equation that can be written in the form $ax^2 + bx + c = 0$. The constants a, b, and c are called the **parameters** of the equation.

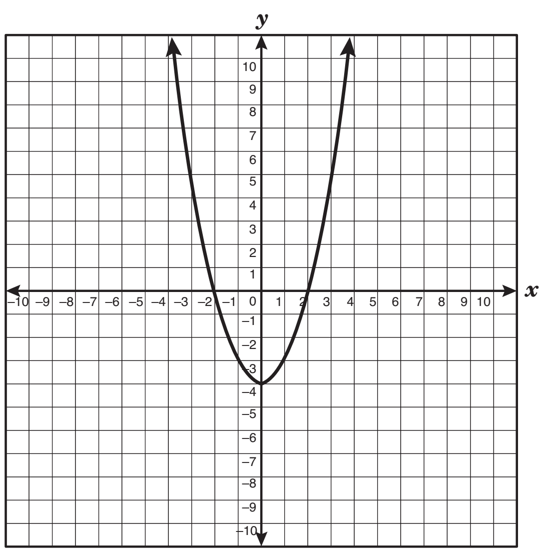
The simplest quadratic function is $y = x^2$. It is the quadratic parent function.



The graph of the quadratic function $y = x^2 - 4$ is shown below.



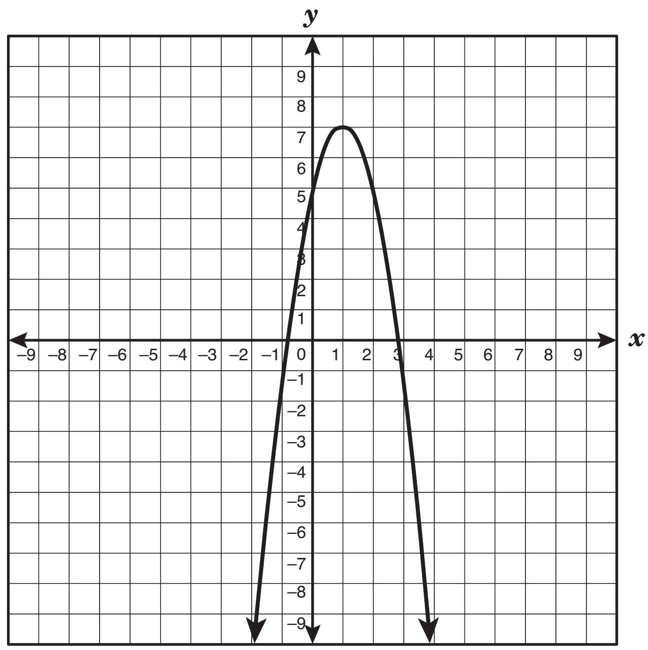
The characteristics of the graph of $y = x^2 - 4$ are shown below.



Vertex	(0, −4)
Roots	$x = -2$ and $x = 2$
Zeros	$x = -2$ and $x = 2$
x-intercepts	(−2, 0) and (2, 0)
y-intercept	(0, −4)
Axis of Symmetry	$x = 0$

Try It

The graph of $f(x) = -2x^2 + 4x + 5$ is shown below.



What are the characteristics of this graph?

The __________ of the function are between –1 and 0 and between 2 and 3.

The vertex is at __________.

The y-intercept of the graph is at __________.

The axis of symmetry is __________.

The zeros of the function are between –1 and 0 and between 2 and 3. The vertex is at (1, 7). The y-intercept of the graph is at (0, 5). The axis of symmetry is $x = 1$.

What Happens to the Graph of $y = ax^2$ When a Is Changed?

If two quadratic functions of the form $y = ax^2$ differ only in the sign of the coefficient of x^2, then one graph will be a reflection of the other graph across the x-axis.

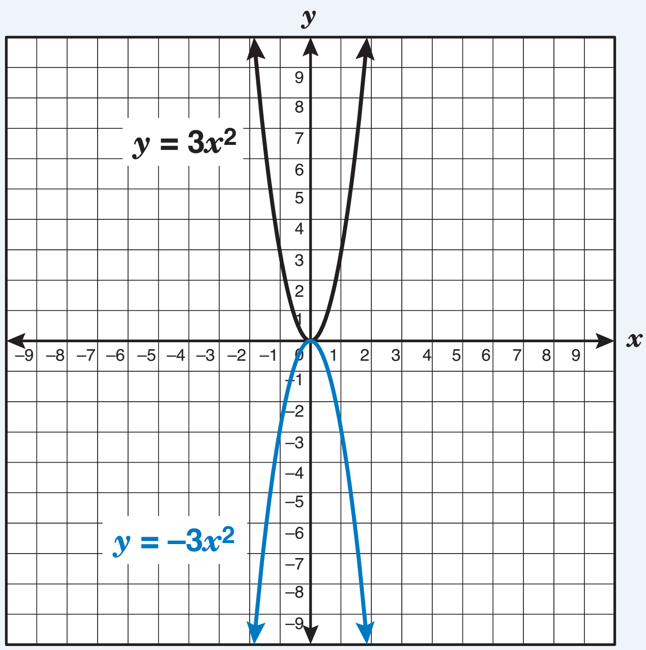
- If $a > 0$, then the parabola opens upward.
- If $a < 0$, then the parabola opens downward.

How do the graphs of $y = 3x^2$ and $y = -3x^2$ compare?

In one function, $a = 3$. In the other function, $a = -3$. Each graph is a reflection of the other across the x-axis.

The graph of $y = 3x^2$ opens upward because $3 > 0$.

The graph of $y = -3x^2$ opens downward because $-3 < 0$.



Objective 5

See Objective 3, page 69, for more information about absolute value.

If two quadratic functions of the form $y = ax^2$ have different coefficients of x^2, then one graph will be wider than the other. The smaller the absolute value of a, the coefficient of x^2, the wider the graph.

Which of these three functions produces the narrowest graph?

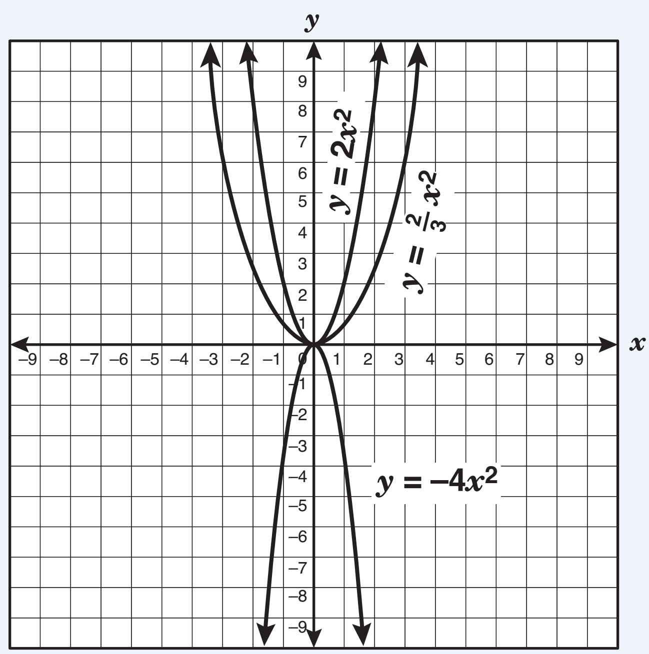
$$y = \frac{2}{3}x^2 \qquad y = 2x^2 \qquad y = -4x^2$$

Compare the absolute value of the three coefficients of x^2.

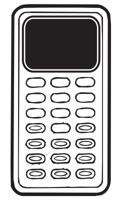
- In the first function the coefficient of x^2 is $\frac{2}{3}$.

  In the second function the coefficient of x^2 is 2.

  In the last function it is -4.
- Since $\left|\frac{2}{3}\right| = \frac{2}{3}$, then $\frac{2}{3}$ has the least absolute value. The graph of $y = \frac{2}{3}x^2$ produces the widest parabola.
- Since $|-4| = 4$, then -4 has the greatest absolute value. The graph of $y = -4x^2$ produces the narrowest parabola.

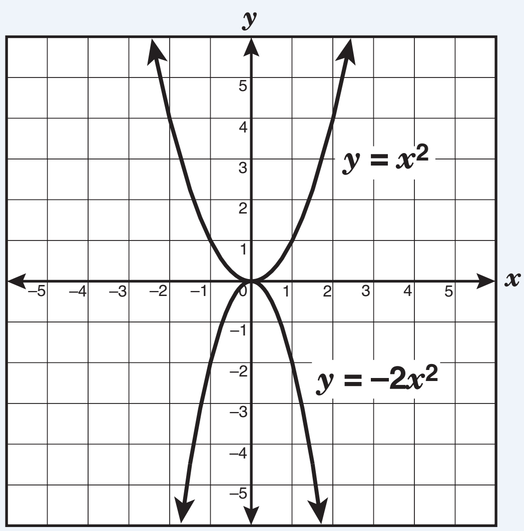


If the coefficient of x^2 in the function $y = x^2$ is changed to -2, how does the new graph compare to the original graph?



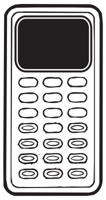
First find the effect of changing the sign of a. Then find the effect of changing its value.

- If the coefficient of x^2 changes from positive 1 to negative 1, the new graph will be a reflection of the original graph. The graph of $y = -x^2$ will be a reflection of the graph of $y = x^2$ across the x-axis.
- If the coefficient of x^2 changes from -1 to -2, the graph becomes narrower because $|-2| > |-1|$. The graph of $y = -2x^2$ will be narrower than the graph of $y = -1x^2$.



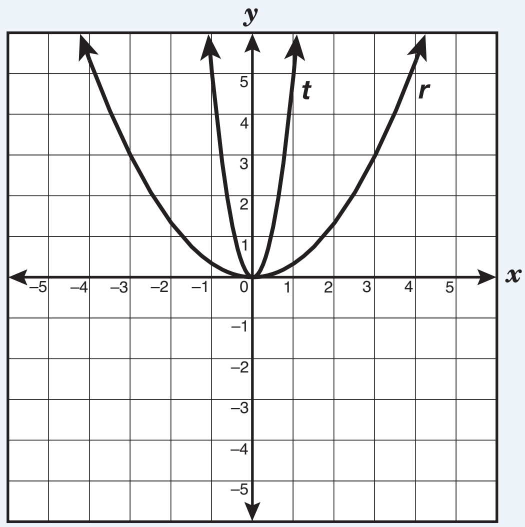
The graph of $y = -2x^2$ is narrower than the graph of $y = x^2$, and it opens down, not up.

Objective 5



Two quadratic functions are graphed below.

$y = \frac{1}{3}x^2$ $\qquad$ $y = 5x^2$



Which parabola is the graph of $y = \frac{1}{3}x^2$?

Which parabola is the graph of $y = 5x^2$?

Compare the absolute values of the coefficients of x^2.

- Since $\frac{1}{3}$ is the smaller value, $y = \frac{1}{3}x^2$ produces the wider graph.
- Since 5 is the greater value, $y = 5x^2$ produces the narrower graph.

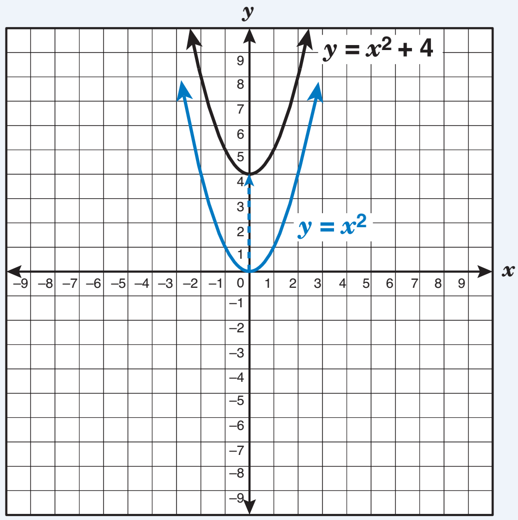
Parabola r, the wider graph, is the graph of $y = \frac{1}{3}x^2$, and parabola t, the narrower graph, is the graph of $y = 5x^2$.

What Happens to the Graph of $y = x^2 + c$ When c Is Changed?

If two quadratic functions of the form $y = x^2 + c$ have different constants, c, then one graph will be a translation up or down of the other graph.

How does the graph of $y = x^2 + 4$ compare to the graph of $y = x^2$?

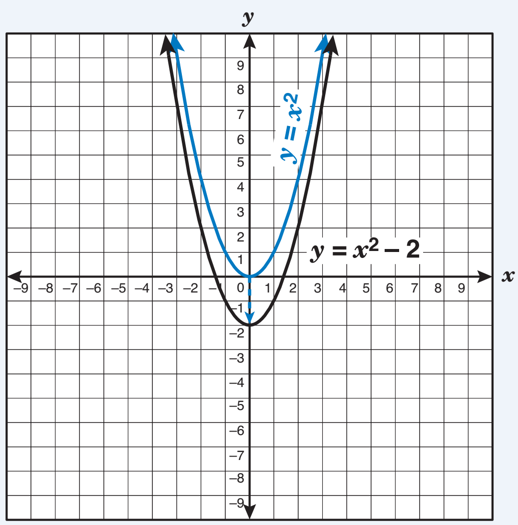
In the function $y = x^2 + 4$, the constant 4 has been added to the parent function $y = x^2$.



The graph of $y = x^2 + 4$ is 4 units above the graph of $y = x^2$.

How does the graph of $y = x^2 - 2$ compare to the graph of $y = x^2$?

In the function $y = x^2 - 2$, the constant -2 has been added to the parent function $y = x^2$.

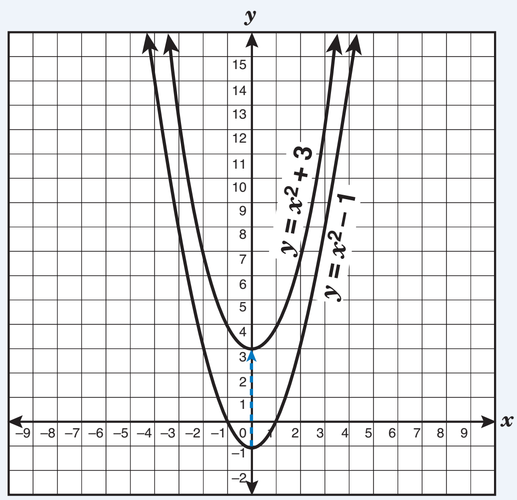


The graph of $y = x^2 - 2$ is 2 units below the graph of $y = x^2$.

Objective 5

How many units apart are the vertices of the graphs of $y = x^2 + 3$ and $y = x^2 - 1$?

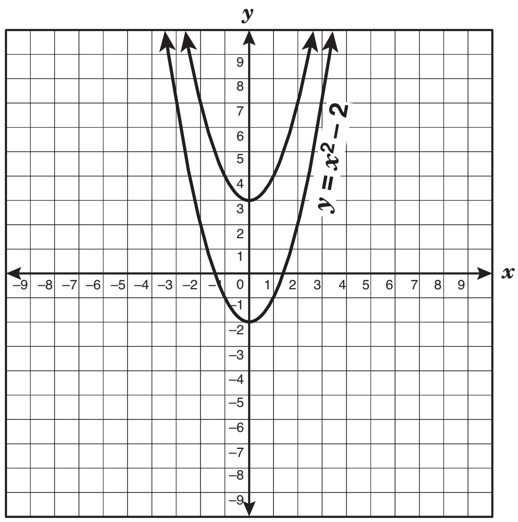
- Adding the constant 3 to the parent function $y = x^2$ causes the parent function to be translated 3 units up.
- Adding the constant -1 to the parent function $y = x^2$ causes the parent function to be translated 1 unit down.
- Look at the graphs of the two functions.



The vertex of the graph of $y = x^2 + 3$ is 4 units higher than the vertex of the graph of $y = x^2 - 1$.

Try It

The graph of the function $y = x^2 - 2$ is translated 5 units up. The equation $y = x^2 - 2$ and the translated function are graphed below.



What is the equation of the translated graph?

In the equation $y = x^2 - 2$, $c =$ __________.

If the graph is translated 5 units up, then the value of c increases __________ units.

The value of c goes from __________ to __________.

The equation of the translated function is ________________________.

In the equation $y = x^2 - 2$, $c = -2$. If the graph is translated 5 units up, then the value of c increases 5 units. The value of c goes from -2 to $+3$. The equation of the translated function is $y = x^2 + 3$.

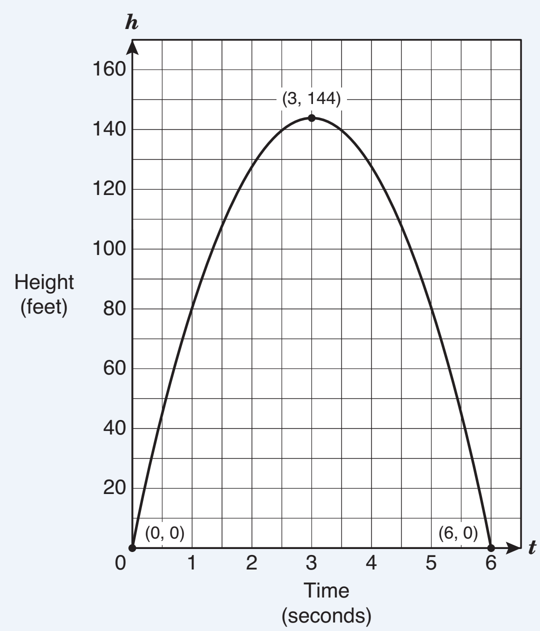
Objective 5

How Do You Draw Conclusions from the Graphs of Quadratic Functions?

To analyze graphs of quadratic functions and draw conclusions from them, consider the following.

- Understand the problem. Identify the quantities involved and the relationship between them.
- Identify the quantities represented on the graph by using the horizontal and vertical axes and looking at the scales used.
- Find the x-intercepts and y-intercept of the graph and determine what these values represent in the problem.
- Decide whether the graph has a minimum or maximum point and determine what this value represents in the problem.

A golf ball was hit into the air. The graph below shows the height of the ball t seconds after it was hit.



What conclusions about the ball's path can you draw from the graph?

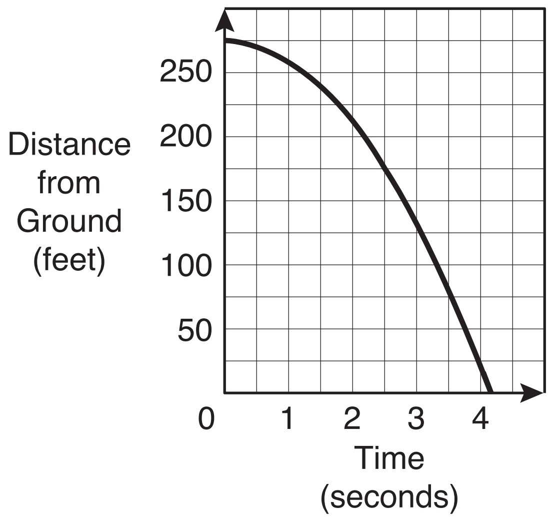
The horizontal axis represents time in seconds. The vertical axis represents the height of the ball in feet.

- The point (0, 0) on the graph tells you that at 0 seconds the height of the ball was 0 feet.
- The greatest value of the function is at the vertex, (3, 144). This means that the maximum height of the ball was 144 feet and that it took 3 seconds for the ball to reach this height.
- The point (6, 0) on the graph tells you that at 6 seconds the ball was back on the ground, at 0 feet. The ball was in the air a total of 6 seconds.

Try It

If a stone is dropped from a height, its distance, d, from the ground is modeled by the quadratic equation $d = -16t^2 + h$, where t is the number of seconds it falls and h is the height in feet from which it was dropped.

The graph below models the distance from the ground of a stone dropped from the top of a tall building.



From what height was the stone dropped?

The stone was dropped when $t =$ __________.

On this graph, $t = 0$ at the point (0, __________).

The stone was dropped from a height of __________ feet.

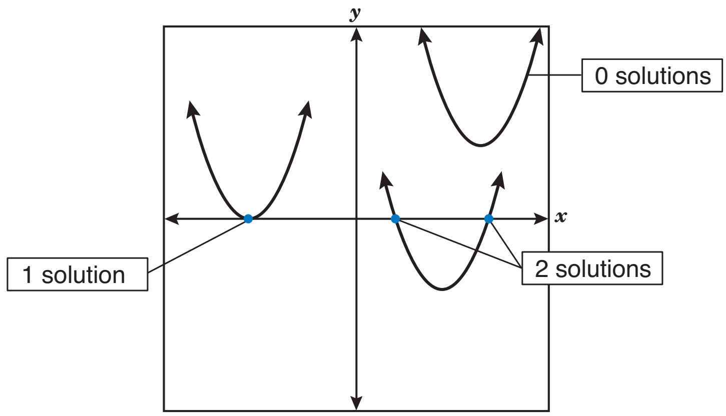
The stone was dropped when $t = 0$. On this graph, $t = 0$ at the point (0, 275). The stone was dropped from a height of 275 feet.

Objective 5

How Can You Solve a Quadratic Equation Graphically?

To find solutions to the quadratic equation $ax^2 + bx + c = 0$, you can look at the graph of the related quadratic function, $y = ax^2 + bx + c$.

A quadratic equation can have 0, 1, or 2 unique solutions. The number of solutions is shown by the graph of the related quadratic function.

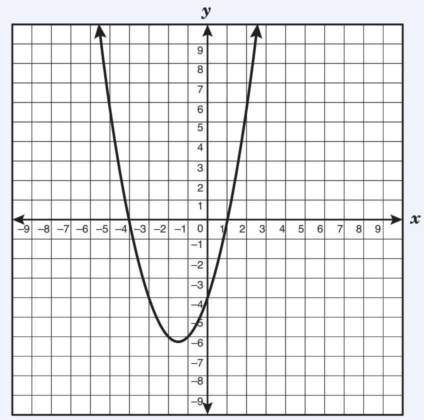


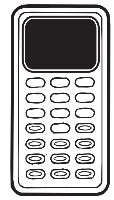
The solutions are called:

- the **roots** of the quadratic equation
- the **zeros** of the quadratic function
- the ***x*-intercepts** of the graph of the function

What are the solutions to the equation $x^2 + 3x - 4 = 0$?

The graph of $y = x^2 + 3x - 4$ is shown below.





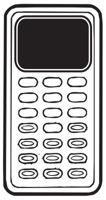
- The points where the graph intersects the x-axis are the points (1, 0) and (−4, 0). The x-coordinates of these points are 1 and −4.
- The zeros of the function $y = x^2 + 3x - 4$ are 1 and −4.
- Therefore, the roots of the equation $x^2 + 3x - 4 = 0$ are −4 and 1.

You can verify that these numbers are solutions by replacing x with their value in the quadratic equation.

Substitute $x = 1$	Substitute $x = -4$
$x^2 + 3x - 4 = 0$	$x^2 + 3x - 4 = 0$
$(1)^2 + 3(1) - 4 \stackrel{?}{=} 0$	$(-4)^2 + 3(-4) - 4 \stackrel{?}{=} 0$
$1 + 3 - 4 \stackrel{?}{=} 0$	$16 - 12 - 4 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

Both numbers make the equation true. Both 1 and −4 are solutions.

Objective 5



How Can You Solve a Quadratic Equation by Using a Table?

You can use the values representing a quadratic function in a table to find solutions to a quadratic equation.

- Identify the points in the table that have y-values of 0.
- The x-values of those points are the solutions to the equation.

The table below models the function $f(x) = x^2 + 6x + 5$. Find solutions to the equation $x^2 + 6x + 5 = 0$.

x	y
0	5
–1	0
–2	–3
–3	–4
–4	–3
–5	0
–6	5

The zeros of the function are the x-coordinates of the points where the y-coordinate is 0.

Look for any points in the table where the y-coordinate is 0.

The points $(-1, 0)$ and $(-5, 0)$ are points where the y-coordinate is 0. The x-values of these points are -1 and -5.

The solution set of the quadratic equation $x^2 + 6x + 5 = 0$ is $\{-5, -1\}$. The roots of the equation are -5 and -1.

You can confirm that these are the solutions by replacing x with these values in the equation $x^2 + 6x + 5 = 0$.

Substitute $x = -1$

$$x^2 + 6x + 5 = 0$$
$$(-1)^2 + 6(-1) + 5 \stackrel{?}{=} 0$$
$$1 - 6 + 5 \stackrel{?}{=} 0$$
$$0 = 0$$

Substitute $x = -5$

$$x^2 + 6x + 5 = 0$$
$$(-5)^2 + 6(-5) + 5 \stackrel{?}{=} 0$$
$$25 - 30 + 5 \stackrel{?}{=} 0$$
$$0 = 0$$

Both numbers make the equation true. Both -1 and -5 are solutions.

Objective 5

How Can You Solve a Quadratic Equation by Factoring?

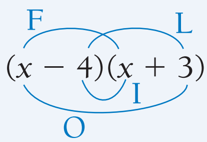
A quadratic equation can be solved by factoring the quadratic expression and then setting its factors equal to zero.

Find the solution to the quadratic equation $x^2 - x - 12 = 0$.

This quadratic equation can be solved by factoring because the quadratic expression $x^2 - x - 12$ can be written as the product of two factors, $x - 4$ and $x + 3$.

$$x^2 - x - 12 = (x - 4)(x + 3)$$

To verify that this equation is true, use the FOIL method to multiply the two binomials.



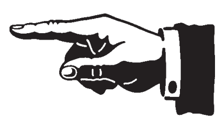
First	$x \cdot x = x^2$
Outer	$x \cdot 3 = 3x$
Inner	$-4 \cdot x = -4x$
Last	$-4 \cdot 3 = -12$
FOIL	$x^2 + 3x - 4x - 12$
	$x^2 - x - 12$

Write the left side of the equation as a product of these two factors.

$$x^2 - x - 12 = 0$$

$$(x - 4)(x + 3) = 0$$

The product of two factors is 0 only if either of the factors is 0. Set each factor in the equation equal to 0 and solve for x.



Do you see that . . .

$$x - 4 = 0 \qquad x + 3 = 0$$

$$x = 4 \qquad x = -3$$

The solutions of the quadratic equation are 4 and −3.

Check both values of x to verify that the equation is true.

Substitute $x = 4$	Substitute $x = -3$
$x^2 - x - 12 = 0$	$x^2 - x - 12 = 0$
$(4)^2 - 4 - 12 \stackrel{?}{=} 0$	$(-3)^2 - (-3) - 12 \stackrel{?}{=} 0$
$16 - 4 - 12 \stackrel{?}{=} 0$	$9 + 3 - 12 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

Objective 5

In real-life problems modeled by quadratic equations, not all the solutions of the equation may make sense in the problem.

A rectangular garden is 1 foot longer than it is wide. Find the length and width of the garden if its area is 42 square feet.

- If the width is represented by w, then the length can be represented by $w + 1$.
- Substitute these expressions into the formula for the area of a rectangle to model the problem situation with an equation.

$$A = lw$$

$$42 = (w + 1)w$$

- To find the width of the garden, solve this equation for w. First write the equation in standard quadratic form, $ax^2 + bx + c = 0$.

$$w(w + 1) = 42$$

$$w^2 + w = 42$$

$$w^2 + w - 42 = 0$$

- The quadratic expression $w^2 + w - 42$ can be factored.

$$w^2 + w - 42 = (w + 7)(w - 6)$$

  Rewrite the equation with the quadratic expression factored.

$$(w + 7)(w - 6) = 0$$

- Set each factor equal to 0 and solve for w.

$$w + 7 = 0 \qquad\qquad w - 6 = 0$$

$$w = -7 \qquad\qquad w = 6$$

Do you see that . . .

- Width cannot be a negative value, so the solution $w = -7$ is not used. Use $w = 6$ as the solution to the equation.

The width of the garden is 6 feet. The length is $w + 1$, which is equal to $6 + 1$, or 7 feet.

How Can You Solve a Quadratic Equation by Using the Quadratic Formula?

Another method used to solve quadratic equations is the quadratic formula. This method can be used to solve all quadratic equations.

> **The Quadratic Formula**
>
> The solutions to a quadratic equation in the standard form $ax^2 + bx + c = 0$ are given by the formula
>
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
>
> where a, b, and c are the parameters of the quadratic equation.

When you find the square root of a number, remember to add the ± symbol in front of the square root symbol, $\sqrt{}$.

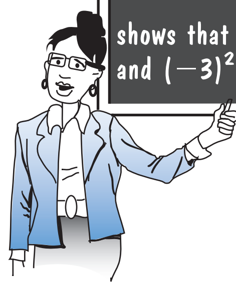
For example,

$$x^2 = 9$$

$$\sqrt{x^2} = \pm\sqrt{9}$$

$$x = \pm 3$$

shows that $(+3)^2 = 9$ and $(-3)^2 = 9$.



Find the solutions to the equation $x^2 + x - 2 = 0$.

- The quadratic equation $x^2 + x - 2 = 0$ is written in standard form. Identify the values of the constants a, b, and c.

$$a = 1$$

$$b = 1$$

$$c = -2$$

- Substitute these values for a, b, and c in the quadratic formula, simplify the expression, and represent the two solutions separately.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$$

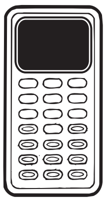
$$= \frac{-1 \pm \sqrt{9}}{2}$$

$$= \frac{-1 \pm 3}{2}$$

$$x = \frac{-1 + 3}{2} = \frac{2}{2} = 1 \text{ and } x = \frac{-1 - 3}{2} = \frac{-4}{2} = -2$$

The solutions to the equation are 1 and −2.

Objective 5



Find the solutions to the equation $2x^2 + 4x - 3 = 0$.

- The quadratic equation $2x^2 + 4x - 3 = 0$ is written in standard form. The values of the constants a, b, and c are its parameters.

$$a = 2$$
$$b = 4$$
$$c = -3$$

- Substitute these values for a, b, and c in the quadratic formula, simplify the expression, and represent the two roots separately.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{40}}{4}$$

$$x = \frac{-4 + \sqrt{40}}{4} \text{ and } x = \frac{-4 - \sqrt{40}}{4}$$

- To approximate the roots of the equation, evaluate the final expressions using $\sqrt{40} \approx 6.32$.

$$x \approx \frac{-4 + 6.32}{4} \approx 0.58$$

$$x \approx \frac{-4 - 6.32}{4} \approx -2.58$$

The roots of the equation are $x \approx 0.58$ and $x \approx -2.58$.

Try It

Estimate the roots of the equation $4x^2 + 1 = 8x$.

Write the quadratic equation in standard form.

$$\_\_\_\_\_\_ x^2 - \_\_\_\_\_\_ x + \_\_\_\_\_\_ = 0$$

In the equation above,

$a =$ ________, $b =$ ________, and $c =$ ________.

Substitute the values of a, b, and c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

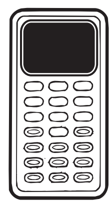
$$x = \frac{-\square \pm \sqrt{\square^2 - 4 \cdot \square \cdot \square}}{2 \cdot \square}$$

$$x = \frac{\square \pm \sqrt{\square - \square}}{8}$$

$$x = \frac{\square \pm \sqrt{\square}}{8}$$

$x = \frac{8 + \square}{8} \approx$ ______ and $x = \frac{8 - \square}{8} \approx$ ______

The approximate roots of the equation are ______ and ______.



$$4x^2 - 8x + 1 = 0$$

In the equation above, $a = 4$, $b = -8$, and $c = 1$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot 1}}{2 \cdot 4}$$

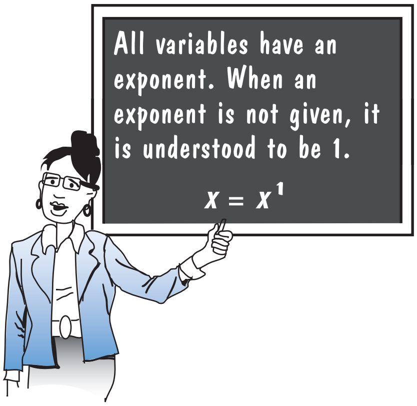
$$x = \frac{8 \pm \sqrt{64 - 16}}{8}$$

$$x = \frac{8 \pm \sqrt{48}}{8}$$

$x = \frac{8 + \sqrt{48}}{8} \approx 1.87$ and $x = \frac{8 - \sqrt{48}}{8} \approx 0.13$

The approximate roots of the equation are 1.87 and 0.13.

Objective 5



How Do You Apply the Laws of Exponents in Problem-Solving Situations?

When simplifying an expression with exponents, there are several rules, known as the **laws of exponents**, which must be followed.

- When multiplying terms with like bases, add the exponents.

$$x^a \cdot x^b = x^{(a+b)}$$

Example: $x^4 \cdot x^2 = x^{(4+2)} = x^6$

$$(x \cdot x \cdot x \cdot x) \cdot (x \cdot x) = xxxxxx = x^6$$

- When dividing terms with like bases, subtract the exponents.

$$\frac{x^a}{x^b} = x^{(a-b)}$$

Example: $\frac{x^8}{x^3} = x^{(8-3)} = x^5$

$$\frac{\cancel{x}\cancel{x}\cancel{x}xxxxx}{\cancel{x}\cancel{x}\cancel{x}} = x^5$$

Sometimes dividing variables with exponents produces negative exponents.

Example: $\frac{x^3}{x^5} = x^{(3-5)} = x^{-2}$

$$\frac{\cancel{x}\cancel{x}\cancel{x}}{\cancel{x}\cancel{x}\cancel{x}xx} = \frac{1}{x^2} = x^{-2}$$

- A term with a negative exponent is equal to the reciprocal of that term with a positive exponent.

$$x^{-a} = \frac{1}{x^a}$$

Example: $x^{-5} = \frac{1}{x^5}$

- When raising a term with an exponent to a power, multiply the exponents.

$$(x^a)^b = x^{ab}$$

Example: $(x^2)^7 = x^{2 \cdot 7} = x^{14}$

$$(x^2)^7 = (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx) \cdot (xx)$$

$$(x^2)^7 = (xxxxxxxxxxxxxx)$$

$$(x^2)^7 = x^{14}$$

- Any base other than zero raised to the zero power equals one.

$$x^0 = 1$$

Example: $8^0 = 1$

Try It

Simplify the expression $(4x^3)(-2x^4)$.

$$\begin{aligned}(4x^3)(-2x^4) &= (4 \cdot x^3)(-2 \cdot x^4)\\ &= (4 \cdot -2)(x^3 \cdot x^4)\\ &= -8 \cdot x^{(\square + \square)}\\ &= -8 \cdot x^{\square}\\ &= \underline{\qquad\qquad}\end{aligned}$$

$$\begin{aligned}(4x^3)(-2x^4) &= (4 \cdot x^3)(-2 \cdot x^4)\\ &= (4 \cdot -2)(x^3 \cdot x^4)\\ &= -8 \cdot x^{(3+4)}\\ &= -8 \cdot x^7\\ &= -8x^7\end{aligned}$$

Try It

Simplify the expression $(5a^3b^2)^2$.

$$\begin{aligned}(5a^3b^2)^2 &= 5^2 \cdot (a^3)^{\square} \cdot (b^2)^{\square}\\ &= 25 \cdot a^{(\square \cdot \square)} \cdot b^{(\square \cdot \square)}\\ &= 25 \cdot a^{\square} \cdot b^{\square}\\ &= 25a^{\square}b^{\square}\end{aligned}$$

$$\begin{aligned}(5a^3b^2)^2 &= 5^2 \cdot (a^3)^2 \cdot (b^2)^2\\ &= 25 \cdot a^{(3 \cdot 2)} \cdot b^{(2 \cdot 2)}\\ &= 25 \cdot a^6 \cdot b^4\\ &= 25a^6b^4\end{aligned}$$

Objective 5

Do you see that . . .

Simplify the following expression: $\left(\frac{x^7yz^3}{xy^5z}\right)^3$.

Simplify the exponents in an expression raised to a power by multiplying the exponents.

Each term in the parentheses is raised to the power of three, so the rule must be applied to each of the variables.

$$\left(\frac{x^7yz^3}{xy^5z}\right)^3 = \frac{(x^7)^3\,(y)^3\,(z^3)^3}{(x)^3\,(y^5)^3\,(z)^3} = \frac{x^{21}\,y^3\,z^9}{x^3\,y^{15}\,z^3}$$

Next divide the like variables with exponents by subtracting the exponents. This rule can be used only if the bases are the same.

$$\frac{x^{21}\,y^3\,z^9}{x^3\,y^{15}\,z^3} = \frac{x^{21}}{x^3} \cdot \frac{y^3}{y^{15}} \cdot \frac{z^9}{z^3}$$

$$= x^{(21-3)} \cdot y^{(3-15)} \cdot z^{(9-3)}$$

$$= x^{18}\,y^{-12}\,z^6$$

Write the expression using only positive exponents.

$$x^{18}\,y^{-12}\,z^6 = \frac{x^{18}\,z^6}{y^{12}}$$

Try It

Find the area of a triangle with base $3x^2y^2$ and height $2x^4y^3$.

Substitute the given expressions for base and height into the formula for the area of a triangle, $A = \frac{1}{2}\,bh$.

$$A = \frac{1}{2} \cdot \underline{\qquad\qquad} \cdot \underline{\qquad\qquad}$$

Simplify the expression.

$$A = (\frac{1}{2} \cdot \underline{\qquad\qquad} \cdot \underline{\qquad\qquad}) \cdot (x^{\square} \cdot x^{\square}) \cdot (y^{\square} \cdot y^{\square})$$

$$A = \frac{1}{2} \cdot \underline{\qquad\qquad} \cdot (x^{\square + \square}) \cdot (y^{\square + \square})$$

$$A = 3x^{\square}y^{\square}$$

$$A = \frac{1}{2} \cdot 3x^2y^2 \cdot 2x^4y^3$$

$$A = (\frac{1}{2} \cdot 3 \cdot 2) \cdot (x^2 \cdot x^4) \cdot (y^2 \cdot y^3)$$

$$A = \frac{1}{2} \cdot 6 \cdot (x^{2+4}) \cdot (y^{2+3})$$

$$A = 3x^6y^5$$

Now practice what you've learned.

Question 48

How does the graph of $y = 2x^2$ compare to the graph of $y = \frac{1}{2}x^2$?

A The graph of $y = 2x^2$ is narrower than the graph of $y = \frac{1}{2}x^2$.

B The graph of $y = 2x^2$ is wider than the graph of $y = \frac{1}{2}x^2$.

C The vertex of the graph of $y = 2x^2$ is above the vertex of the graph of $y = \frac{1}{2}x^2$.

D The vertex of the graph of $y = 2x^2$ is to the right of the vertex of the graph of $y = \frac{1}{2}x^2$.

Answer Key: page 243

Question 49

How does the graph of the quadratic function $y = -\frac{1}{2}x^2$ compare to the graph of the quadratic function $y = \frac{1}{2}x^2$?

A The graph of $y = -\frac{1}{2}x^2$ is the graph of $y = \frac{1}{2}x^2$ reflected across the y-axis.

B The graph of $y = -\frac{1}{2}x^2$ is the graph of $y = \frac{1}{2}x^2$ reflected across the x-axis.

C The graph of $y = -\frac{1}{2}x^2$ is wider than the graph of $y = \frac{1}{2}x^2$.

D The graph of $y = -\frac{1}{2}x^2$ is narrower than the graph of $y = \frac{1}{2}x^2$.

Answer Key: page 243

Question 50

How does the graph of $y = x^2 - 1$ differ from the graph of $y = x^2 + 6$?

A The vertex of $y = x^2 - 1$ is 1 unit below the vertex of $y = x^2 + 6$.

B The vertex of $y = x^2 + 6$ is 6 units above the vertex of $y = x^2 - 1$.

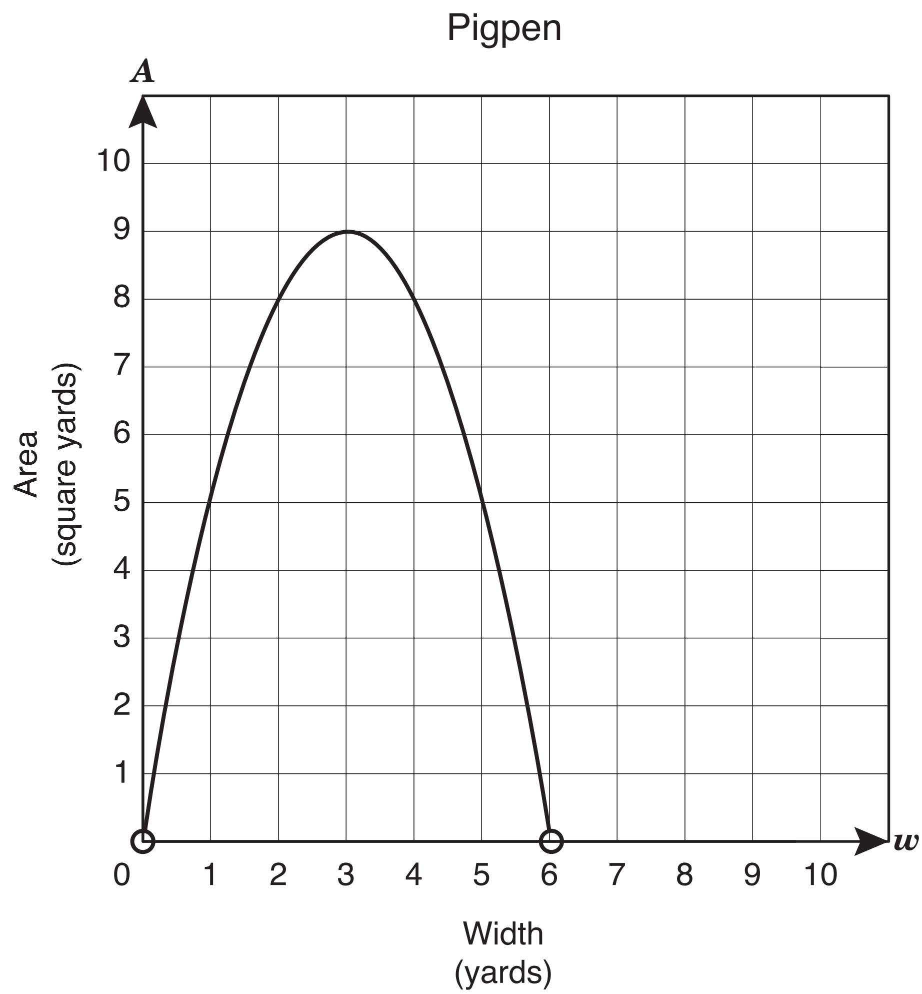
C The vertex of $y = x^2 - 1$ is 5 units below the vertex of $y = x^2 + 6$.

D The vertex of $y = x^2 + 6$ is 7 units above the vertex of $y = x^2 - 1$.

Answer Key: page 243

Question 51

Hank wants to build a rectangular pigpen using 12 linear yards of fencing. The possible area in square yards, A, for this pigpen is described by the function $A = w(6 - w)$, where w represents the width in yards of the rectangular pen. The graph of this function is shown below.



Which statement best represents the information in this graph?

A The pigpen with the maximum area has a width of 6 yards.

B The pigpen with the maximum area has a width of 3 yards.

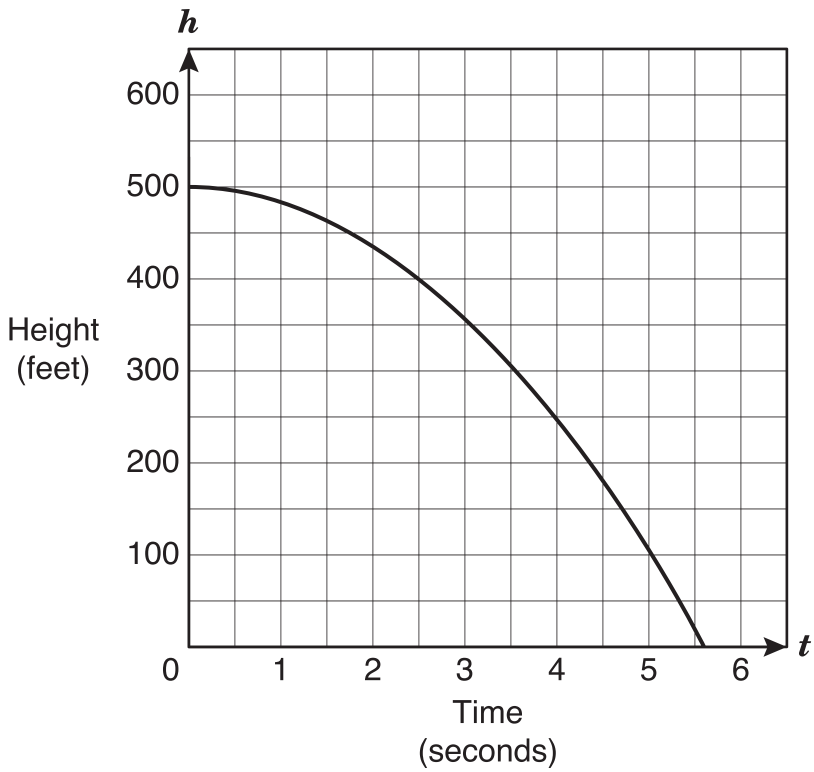
C The pigpen with the maximum area has a width of 9 yards.

D The pigpen with the maximum area has a width of 12 yards.

Answer Key: page 243

Question 52

A stone is dropped from a height of 500 feet above the ground. The graph shows the stone's distance from the ground at different times.



Which of the following is a correct interpretation of the graph?

A The stone lands about 5 feet from where it was dropped.

B The graph is a picture of the path of the stone as it falls.

C The stone hits the ground between 5 seconds and 6 seconds after it is dropped.

D The stone's distance from the ground decreases at a constant rate until the stone hits the ground.

Answer Key: page 243

Question 53

The table below shows selected values of a quadratic function.

x	y
−3	21
−2	7
−1	−1
0	−3
1	1
2	11

Based on the information in the table, between which two integers can one of the zeros of this function be found?

A −3 and −2

B −2 and −1

C −1 and 0

D 1 and 2

Answer Key: page 244

Question 54

What are the solutions to the following equation?

$$2x^2 - 15x - 16 = -11 - 24x$$

A $x = -5$ and $x = \frac{1}{2}$

B $x = -\frac{1}{2}$ and $x = 5$

C $x = -1$ and $x = -\frac{5}{2}$

D $x = \frac{5}{2}$ and $x = -1$

Answer Key: page 244

Question 55

Which of the following quadratic equations has the solutions −1 and 5?

A $x^2 - 4x + 5 = 0$

B $x^2 - 4x - 1 = 0$

C $x^2 + 4x - 1 = 0$

D $x^2 - 4x - 5 = 0$

Answer Key: page 244

Question 56

For the quadratic equation $x^2 - 4x + 2 = 0$, the smaller root is between which 2 integers?

A 0 and 1

B −1 and 0

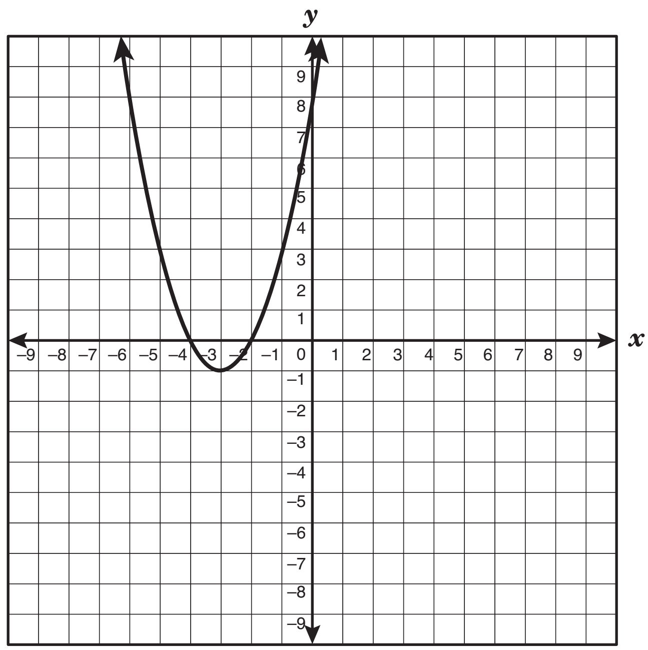
C 1 and 2

D −2 and −1

Answer Key: page 244

Question 57

The graph of $f(x) = x^2 + 6x + 8$ is shown below.



Which of the following statements does not appear to be true?

A The vertex is $(-3, 1)$.

B The axis of symmetry is $x = -3$.

C The zeros are $\{-4, -2\}$.

D The y-intercept is $(0, 8)$.

Answer Key: page 244

Question 58

The volume, V, of a sphere can be found using the following formula:

$$V = \frac{4}{3}\pi r^3$$

Which expression describes the volume of a sphere with radius $3x^2y$?

A $12x^5y^3\pi$

B $36x^5y^3\pi$

C $12x^6y^3\pi$

D $36x^6y^3\pi$

Answer Key: page 244

Objective 6

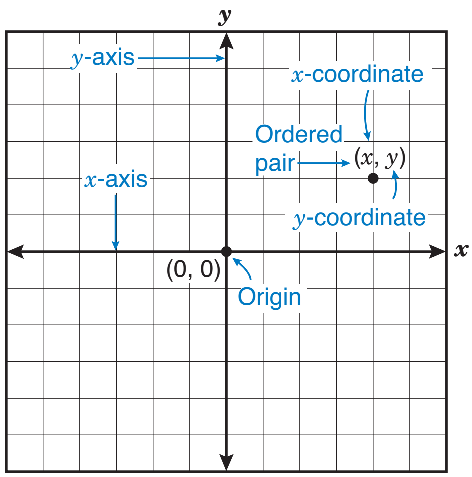
The student will demonstrate an understanding of geometric relationships and spatial reasoning.

For this objective you should be able to

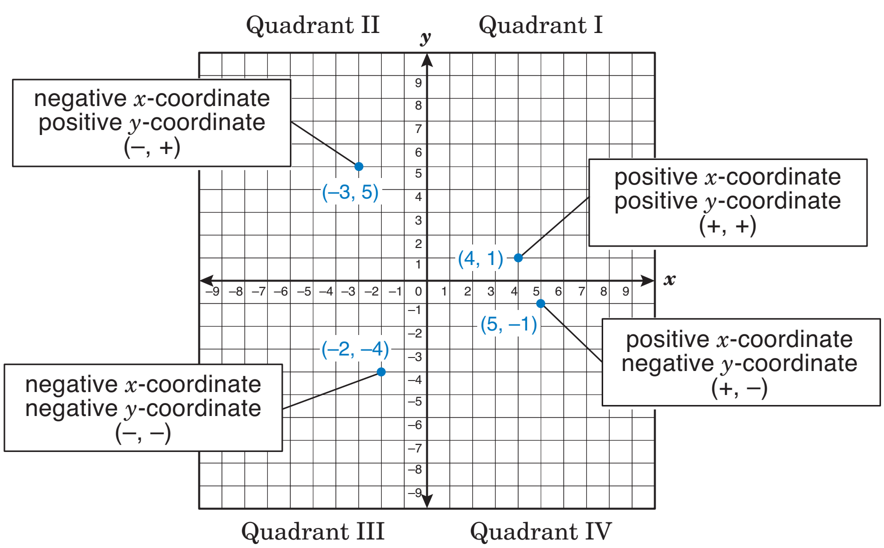
- use transformational geometry to develop spatial sense; and
- use geometry to model and describe the physical world.

How Do You Locate and Name Points on a Coordinate Plane?

A **coordinate grid** is used to locate and name points on a plane. A coordinate grid is formed by two perpendicular number lines.

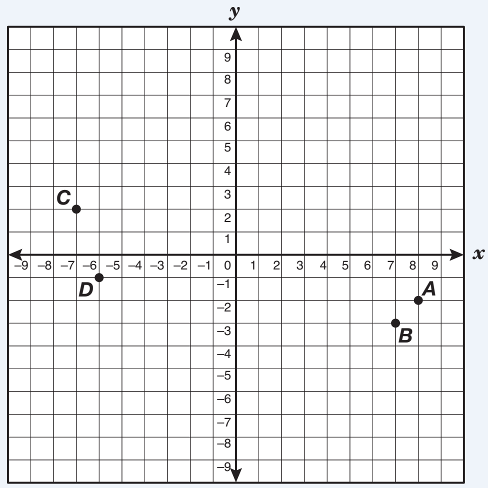


The *x*-axis and *y*-axis divide the coordinate plane into four regions, called **quadrants**. The quadrants are usually referred to by the Roman Numerals I, II, III, and IV.

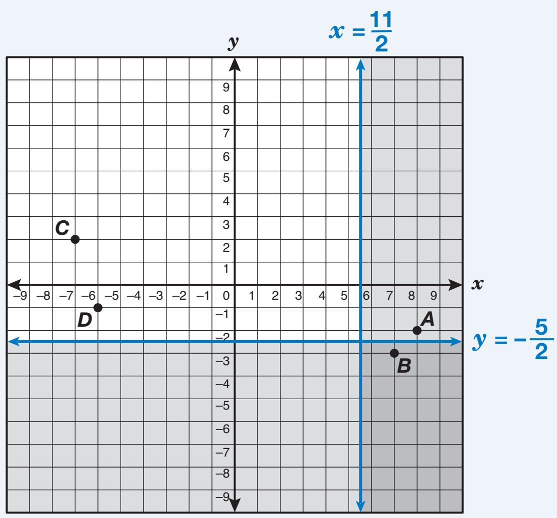


Objective 6

Which of the points on the coordinate grid below satisfies the conditions $x > \frac{11}{2}$ and $y < -\frac{5}{2}$?



- Draw a vertical line through $x = \frac{11}{2}$. All the points to the right of this line have an x-coordinate greater than $\frac{11}{2}$.
- Draw a horizontal line through $y = -\frac{5}{2}$. All the points below this line have a y-coordinate less than $-\frac{5}{2}$.

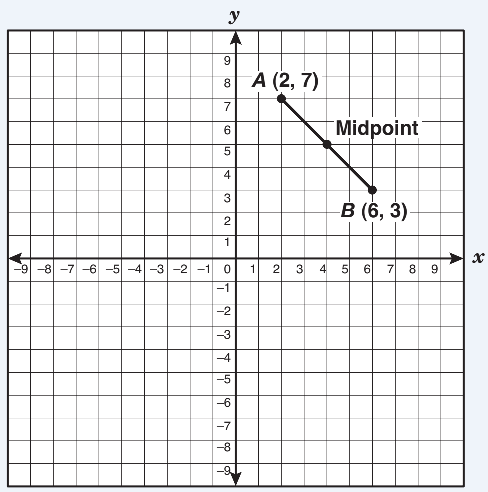


Only point B, with the coordinates $(7, -3)$, satisfies both conditions.

How Do You Find the Midpoint of a Line Segment?

The **midpoint** of a line segment is the point that lies halfway between the segment's endpoints and divides it into two congruent parts. You can find the coordinates of the midpoint of a segment if you know the coordinates of its endpoints. Since the midpoint is halfway between the endpoints, its coordinates are the average of the coordinates of the endpoints.

Find the midpoint of $\overline{AB}$.



- To find the x-coordinate of the midpoint, find the x-value that is halfway between 2 and 6.

  $$(2 + 6) \div 2 = 4$$

  The x-coordinate of the midpoint is 4.

- To find the y-coordinate of the midpoint, find the y-value that is halfway between 7 and 3.

  $$(7 + 3) \div 2 = 5$$

  The y-coordinate of the midpoint is 5.

The midpoint of $\overline{AB}$ is (4, 5).

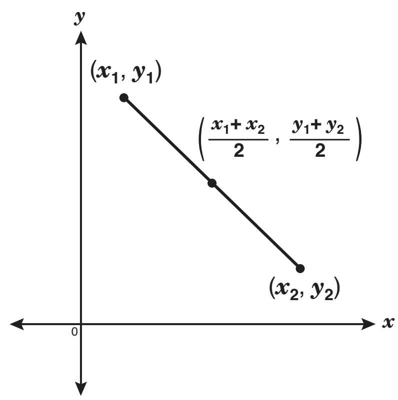
Objective 6

You can also find the midpoint of a line segment by using the following formula.

Midpoint Formula

For any two points (x_1, y_1) and (x_2, y_2), the coordinates of the midpoint of the line segment they determine are given by the formula below.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Use the midpoint formula to find the midpoint, M, of $\overline{RS}$ with endpoints R (3, 2) and S (1, −4).

Replace the variables in the midpoint formula with values from the coordinates of the two given points.

$$R\ (3, 2)\colon x_1 = 3 \text{ and } y_1 = 2$$

$$S\ (1, -4)\colon x_2 = 1 \text{ and } y_2 = -4$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

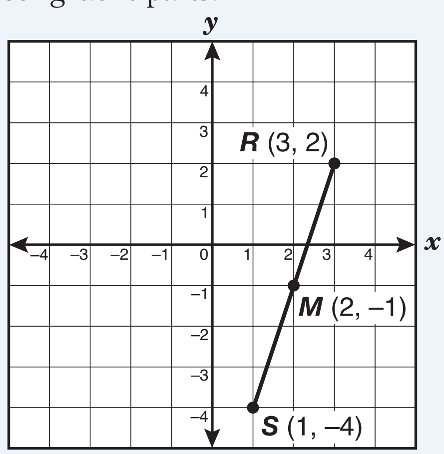
$$M = \left(\frac{3 + 1}{2}, \frac{2 + (-4)}{2}\right)$$

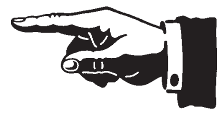
The coordinates of the midpoint are the average of the x- and y-coordinates.

$$M = \left(\frac{4}{2}, \frac{-2}{2}\right)$$

$$M = (2, -1)$$

The midpoint of $\overline{RS}$ is the point M (2, −1). Notice that point M lies on the segment connecting point R and point S and divides the segment into two congruent parts.



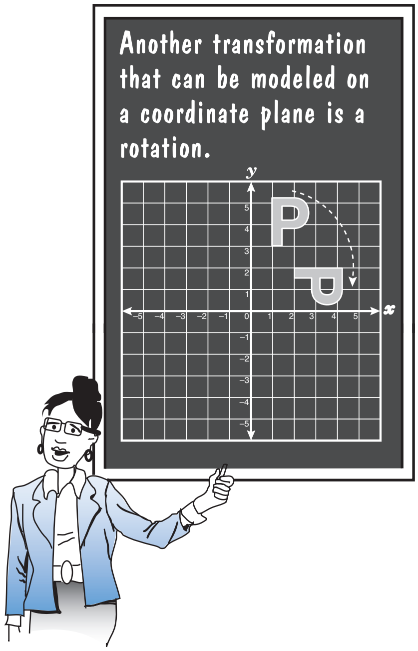


Do you see that . . .

Objective 6

How Can You Show Transformations on a Coordinate Plane?

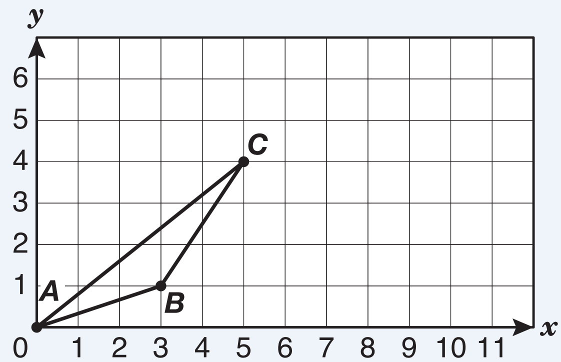
Translations, reflections, and dilations are transformations that can be modeled on a coordinate plane. A figure has been translated or reflected if it has been moved without changing its shape or size. A figure has been dilated if its size has been changed proportionally.



Translations

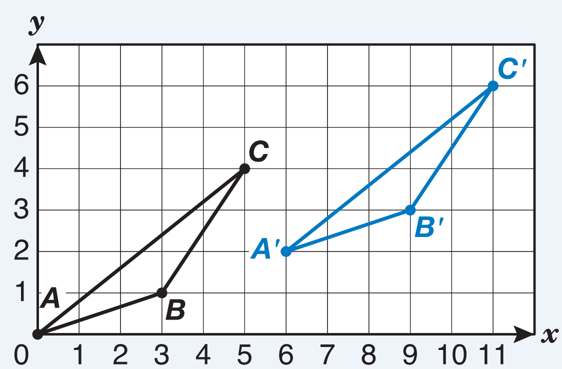
A **translation** of a figure is a movement of the figure along a line. It can be described by stating how many units to the left or right the figure is moved and how many units up or down it is moved. A figure and its translated image are always congruent.

If $\triangle ABC$ is translated 6 units to the right and 2 units up, what are the coordinates of the vertices of the translated triangle $A'B'C'$?



The vertices of $\triangle ABC$ are A (0, 0), B (3, 1), and C (5, 4).

If the triangle is translated 6 units to the right, then 6 must be added to the x-coordinate of each vertex. If the triangle is translated 2 units up, then 2 must be added to the y-coordinate of each vertex.



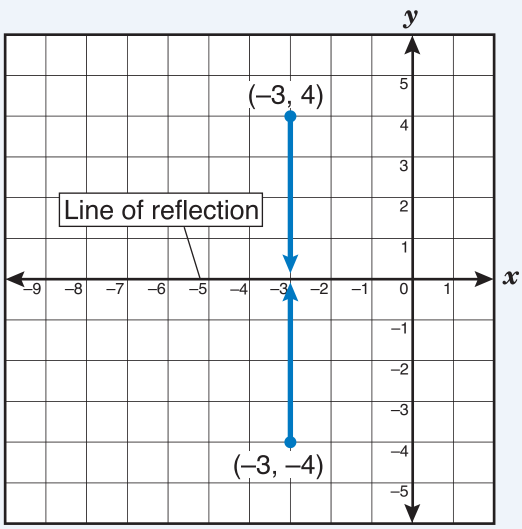
The vertices of the translated figure are A' (6, 2), B' (9, 3), and C' (11, 6).

Reflections

A **reflection** of a figure is the mirror image of the figure across a line. The line is called the **line of reflection**. The new figure is a reflection of the original figure, with the line of reflection serving as the mirror. A figure and its reflected image are always congruent.

Each point of the reflected image is the same distance from the line of reflection as the corresponding point of the original figure, but on the opposite side of the line of reflection.

If the point $(-3, 4)$ is reflected across the x-axis, what will be the coordinates of its reflection?

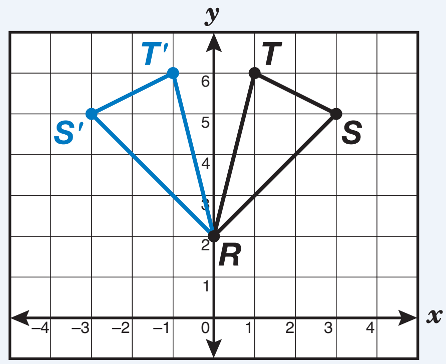


- The x-coordinate of the point will be unchanged because the point is being reflected across the x-axis. The reflected point will have an x-coordinate of -3.
- The y-coordinate of the point is 4 units above the x-axis, so the y-coordinate of the reflected point will be 4 units below the x-axis. The reflected point will have a y-coordinate of -4.

The coordinates of the reflected point will be $(-3, -4)$. The point $(-3, 4)$ and its image $(-3, -4)$ are equally distant from the line of reflection, the x-axis.

Objective 6

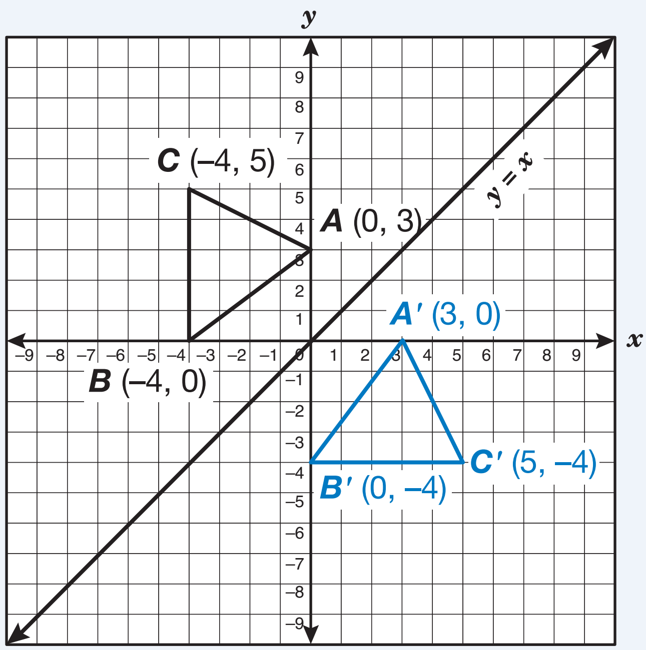
If $\triangle RST$ is reflected across the y-axis, what are the coordinates of its reflection, $\triangle RS'T'$?



- The vertices of $\triangle RST$ are R (0, 2), S (3, 5), and T (1, 6).
- Point R is on the y-axis, so it is a common vertex for the triangles.
- Point S is 3 units to the right of the y-axis, so S' is 3 units to the left of the y-axis. The coordinates of S' are (−3, 5).
- Point T is 1 unit to the right of the y-axis, so T' is 1 unit to the left of the y-axis. The coordinates of T' are (−1, 6).

The vertices of $\triangle RS'T'$ are R (0, 2), S' (−3, 5), and T' (−1, 6).

$\triangle ABC$ is shown on the graph below. If $\triangle ABC$ is reflected across the line $y = x$, what are the coordinates of A', B', and C'?

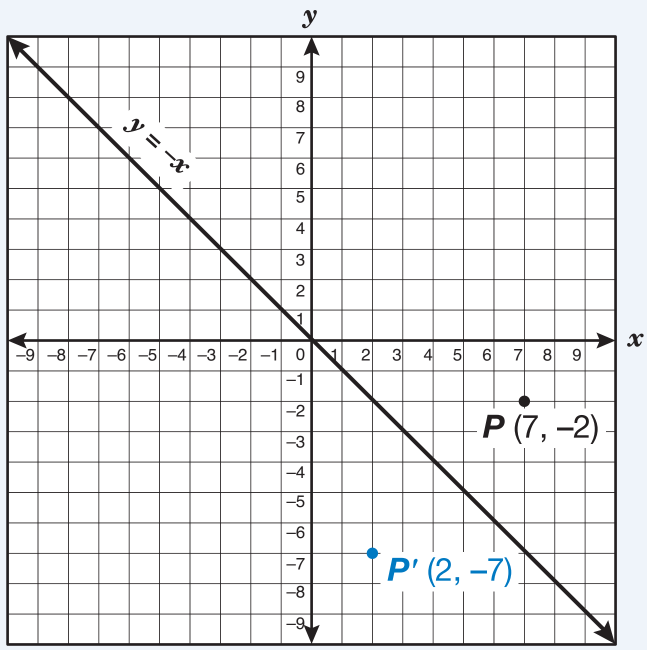


- Notice that for reflections across $y = x$, both coordinates will change. For any point with coordinates (x, y), the image after a reflection across $y = x$ will have the coordinates (y, x). That is, the order of the coordinates will be reversed.
- When A (0, 3) is reflected across $y = x$, we reverse the coordinate values to get A'. The coordinates of A' are (3, 0).
- When B (−4, 0) is reflected across $y = x$, we reverse the coordinate values to get B'. The coordinates of B' are (0, −4).
- When C (−4, 5) is reflected across $y = x$, we reverse the coordinate values to get C'. The coordinates of C' are (5, −4).

When $\triangle ABC$ is reflected across $y = x$, the coordinates of A' are (3, 0); B', (0, −4); and C', (5, −4).

Objective 6

If point P (7, -2) is reflected across the line $y = -x$, what are the coordinates of P'?



- Notice that for reflections across $y = -x$, both coordinates will change. For any point with coordinates (x, y), the image after a reflection across $y = -x$ will have the coordinates $(-y, -x)$. That is, the order of the coordinates will be reversed, and the signs change.
- When P (7, -2) is reflected across $y = -x$, we reverse the coordinate values and change the signs to get P'. The coordinates of P' are (2, -7).

When P (7, -2) is reflected across $y = -x$, the coordinates of P' are (2, -7).

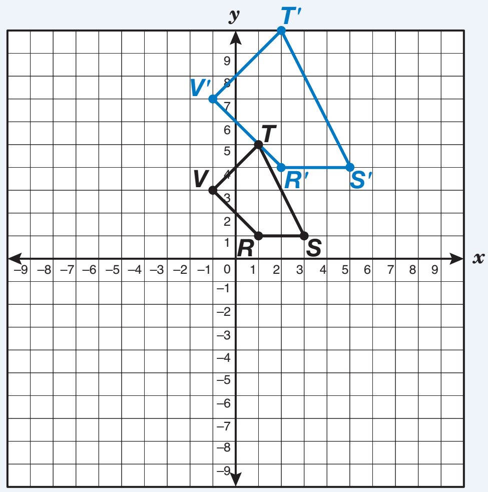
Dilations

A **dilation** is a proportional enlargement or reduction of a figure through a point called the center of dilation. The size of the enlargement or reduction is called the **scale factor** of the dilation.

- If the dilated image is larger than the original figure, then the scale factor > 1. This is called an **enlargement**.
- If the dilated image is smaller than the original figure, then the scale factor < 1. This is called a **reduction**.

A figure and its dilated image are always similar.

What scale factor was used to transform quadrilateral *RSTV* to quadrilateral $R'S'T'V'$?



To find the scale factor, compare the lengths of a pair of corresponding sides.

- Of the line segments that make up the quadrilaterals, the ones whose lengths are easiest to find are $\overline{RS}$ and $\overline{R'S'}$, because they are horizontal.
- The length of $\overline{RS}$ is the difference between the *x*-coordinates of points *R* and *S*.

$$RS = 3 - 1 = 2$$

- The length of $\overline{R'S'}$ is the difference between the *x*-coordinates of points R' and S'.

$$R'S' = 5 - 2 = 3$$

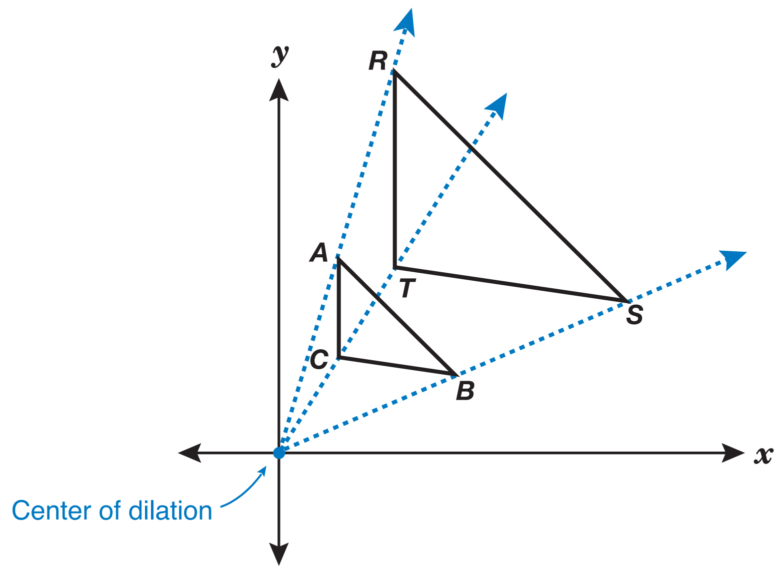
- You can tell from the graph that the figure has been enlarged, so the scale factor is greater than 1. Use this fact to be certain you state the ratio correctly.
- The scale factor is the ratio of their lengths.

$$\frac{R'S'}{RS} = \frac{3}{2} = 1.5$$

The scale factor used to dilate quadrilateral *RSTV* to quadrilateral $R'S'T'V'$ is 1.5. Each side of the dilated quadrilateral is 1.5 times the length of the corresponding side of the original quadrilateral.

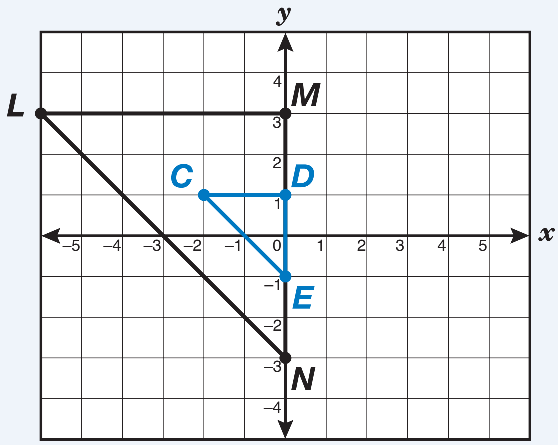
Objective 6

Another way to view a dilation is as a projection through a center of dilation.



Looking at a dilation in this way can help you see it as an enlargement or a reduction. $\triangle ABC$ has been dilated to form $\triangle RST$.

$\triangle LMN$ was dilated to form $\triangle CDE$ with (0, 0) as the center of dilation. What scale factor was used to dilate $\triangle LMN$?



To find the scale factor, compare the lengths of a pair of corresponding sides of the two triangles.

- Use the lengths of $\overline{CD}$ and $\overline{LM}$ to find the scale factor, since both segments are horizontal.
- The length of $\overline{CD}$ is the difference between the *x*-coordinates of points *C* and *D*.

$$CD = 0 - (-2) = 2$$

- The length of $\overline{LM}$ is the difference between the *x*-coordinates of points *L* and *M*.

$$LM = 0 - (-6) = 6$$

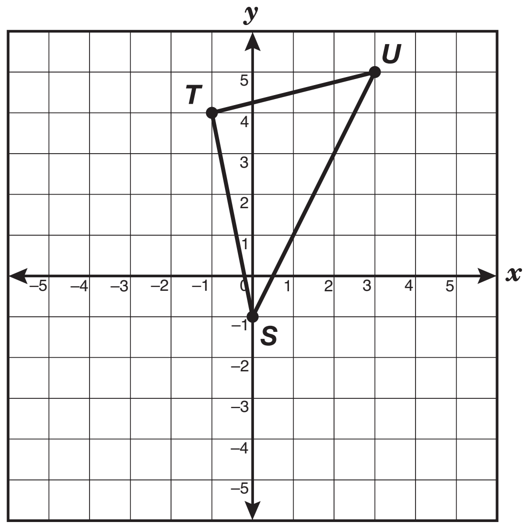
- From the graph you can tell that $\triangle LMN$ has been reduced, so the scale factor is less than 1. Use this fact to be certain you state the ratio correctly.
- The scale factor is the ratio of their lengths.

$$\frac{CD}{LM} = \frac{2}{6} = \frac{1}{3}$$

The scale factor used to dilate $\triangle LMN$ to $\triangle CDE$ is $\frac{1}{3}$. Each side of the dilated triangle is $\frac{1}{3}$ the length of the corresponding side of the original triangle.

Try It

$\triangle STU$ has vertices S (0, −1), T (−1, 4), and U (3, 5). Find the coordinates of the vertices of its reflection across the x-axis.



Because this is a reflection across the x-axis, the _____ -coordinates do not change.

The vertex S (0, −1) is 1 unit ____________ the x-axis.

S' must be 1 unit ____________ the x-axis.

The coordinates of S' are (_____ , _____).

The vertex T (−1, 4) is ___________ units above the x-axis.

T' must be 4 units ___________ the x-axis.

The coordinates of T' are (_____ , _____).

The vertex U (3, 5) is ___________ units above the x-axis.

U' must be 5 units ___________ the x-axis.

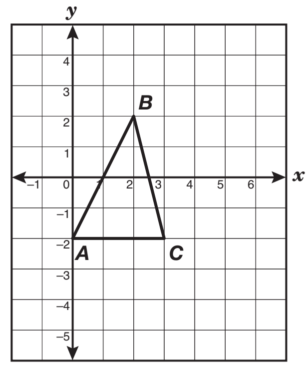
The coordinates of U' are (_____ , _____).

Because this is a reflection across the x-axis, the x-coordinates do not change. The vertex S (0, −1) is 1 unit below the x-axis. S' must be 1 unit above the x-axis. The coordinates of S' are (0, 1). The vertex T (−1, 4) is 4 units above the x-axis. T' must be 4 units below the x-axis. The coordinates of T' are (−1, −4). The vertex U (3, 5) is 5 units above the x-axis. U' must be 5 units below the x-axis. The coordinates of U' are (3, −5).

Now practice what you've learned.

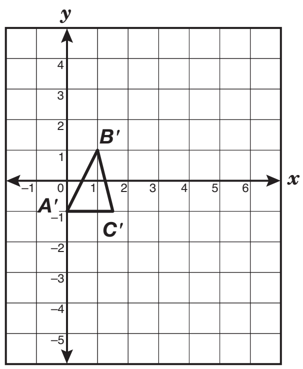
Question 59

The triangle in the graph below will be dilated by a scale factor of 2, using the point (0, 0) as the center of dilation.

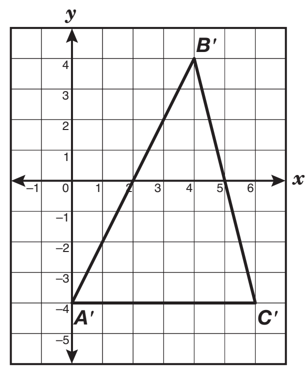


Which graph shows this dilation?

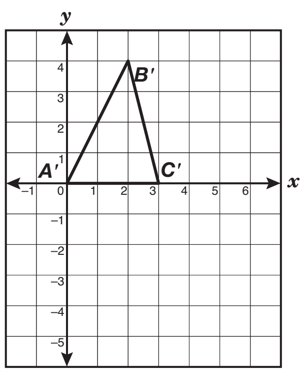
A



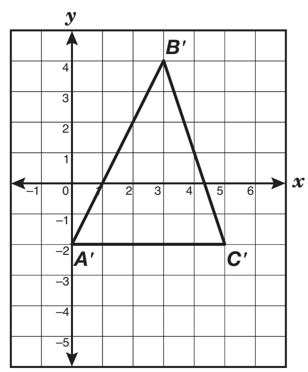
C



B



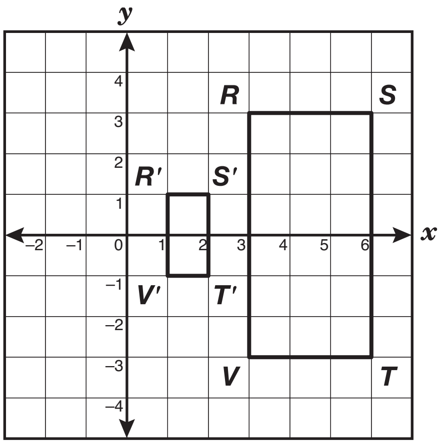
D



Answer Key: page 245

Question 60

Rectangle $R'S'T'V'$ is a dilation of rectangle $RSTV$. What is the scale factor of the dilation?



A $\frac{1}{3}$

B $\frac{1}{2}$

C 2

D 3

Answer Key: page 245

Question 61

A triangle has side lengths 1, 1.5, and 2 units. Which of the following could be the lengths of the sides of a triangle that was formed by dilating the given triangle?

A 1, 3, and 4 units

B 2, 4, and 6 units

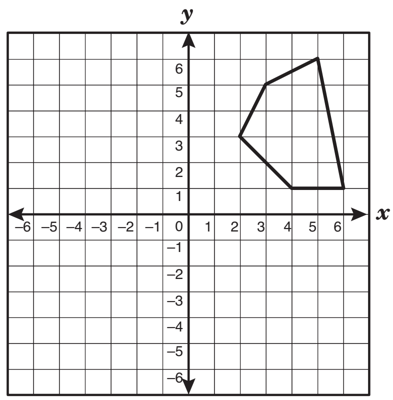
C 4, 4.5, and 5 units

D 4, 6, and 8 units

Answer Key: page 245

Question 62

Which of the following sets of points would be three of the vertices of the pentagon below reflected across the y-axis?



A $(-5, 6), (-4, 1), (-3, 5)$

B $(5, -6), (4, -1), (3, -5)$

C $(6, 5), (1, 4), (5, 3)$

D $(-5, -6), (-4, -1), (-3, -5)$

Answer Key: page 245

Question 63

The quadrilateral with vertices R (1, 1), S (1, 4), T (5, 8), and V (7, −2) is translated 2 units to the right and 3 units down. Which of the following are the coordinates of two of the vertices of the translated quadrilateral?

A (3, 4), (7, 5)

B (−1, −2), (3, 5)

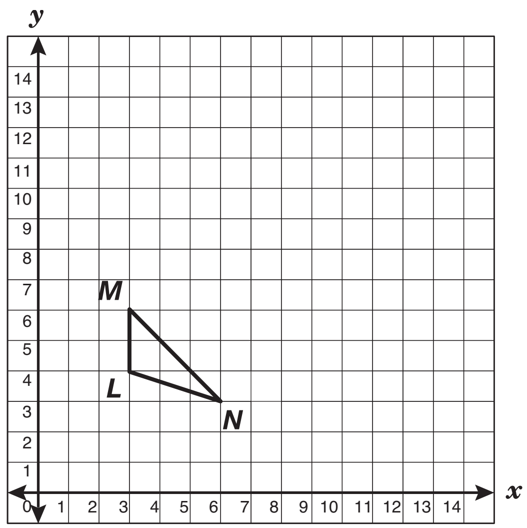
C (7, 11), (3, 4)

D (3, −2), (7, 5)

Answer Key: page 245

Question 64

Triangle *LMN* is dilated by a scale factor of 2 with (0, 0) as the center of dilation. What will be the coordinates of *L*′, *M*′, and *N*′ of the dilated triangle?



A *L*′ (5, 7), *M*′ (5, 10), and *N*′ (11, 7)

B *L*′ (3, 8), *M*′ (3, 12), and *N*′ (6, 6)

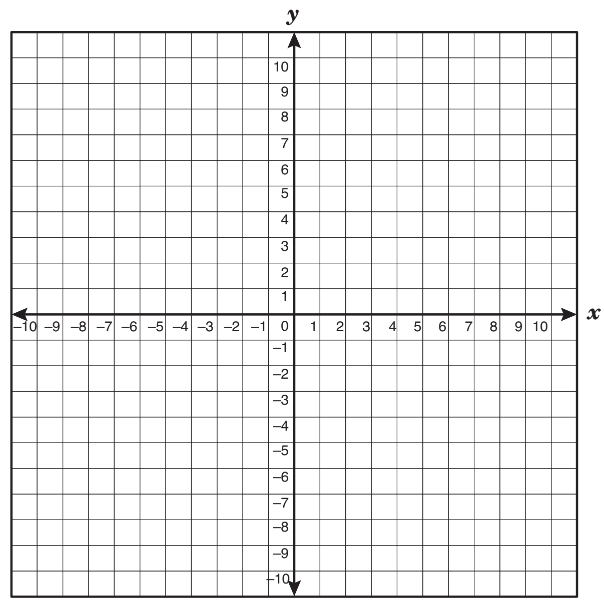
C *L*′ (6, 8), *M*′ (6, 12), and *N*′ (12, 6)

D *L*′ (5, 6), *M*′ (5, 8), and *N*′ (8, 6)

Answer Key: page 245

Question 65

The endpoints of $\overline{RS}$ are *R* (−3, −7) and *S* (3, −5). What are the coordinates of the midpoint of $\overline{RS}$?



A (6, 2)

B (6, 12)

C (0, −12)

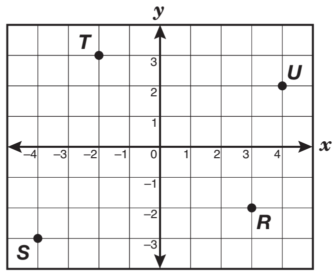
D (0, −6)

Answer Key: page 246

Objective 6

Question 66

For which of the points on the graph is $-\frac{7}{2} < x < \frac{3}{2}$?



A Point R

B Point S

C Point T

D Point U

Answer Key: page 246

Question 67

The coordinates of a given point are $(m, 2n)$. What are the coordinates of the point when it is translated 2 units to the right?

A $(m + 2, 2n + 2)$

B $(m, 2n + 2)$

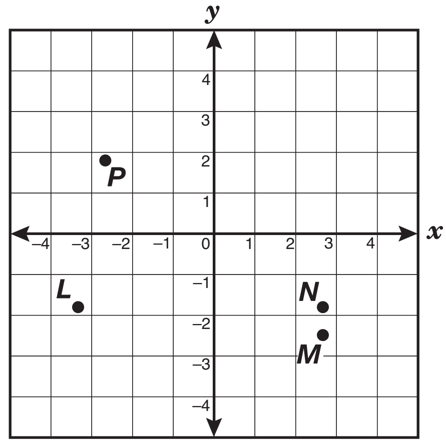
C $(2m, 4n)$

D $(m + 2, 2n)$

Answer Key: page 246

Question 68

Which point best represents $\left(\frac{8}{3}, -\frac{9}{5}\right)$?



A Point L

B Point M

C Point N

D Point P

Answer Key: page 246

Objective 7

The student will demonstrate an understanding of two- and three-dimensional representations of geometric relationships and shapes.

For this objective you should be able to use geometry to model and describe the physical world.

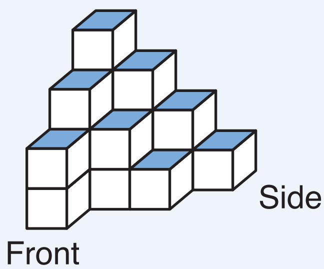
How Do You Recognize a Three-Dimensional Figure from Different Perspectives?

Given a drawing of a three-dimensional figure, you should be able to recognize other drawings that represent the same figure from a different perspective.

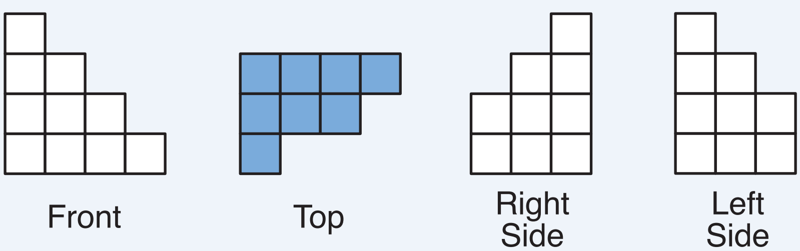
A three-dimensional figure can be represented by drawing the figure from three different views: front, top, and side.

To recognize the three-dimensional figure from different perspectives, you must visualize what the three-dimensional figure would look like if you were seeing it from the front, from above, or from one side.

The three-dimensional figure below is made up of many small cubes. Can you visualize what it would look like from the front, from above, and from a side?



These are the front, top, and side views of this three-dimensional figure.

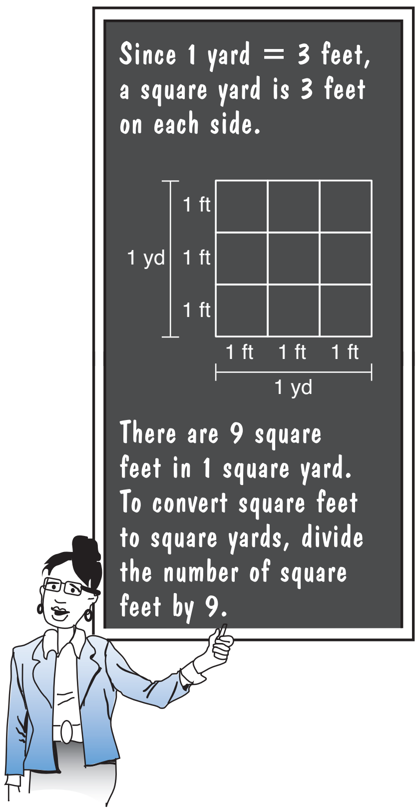


Objective 7

What Kinds of Problems Can You Solve with Geometry?

The laws of geometry govern the physical world around us. You can solve many types of problems using geometry, including problems involving these geometric concepts:

- the area or perimeter of figures;
- the measures of the sides or angles of polygons;
- the surface area and volume of three-dimensional figures;
- the ratios of the sides of similar figures; and
- the relationship between the sides of a right triangle.



Joan is making a tablecloth with a lace border. The tablecloth fabric costs $4.25 per square yard. The lace border costs $0.35 per foot. What is the approximate total cost of the fabric and lace border for a rectangular tablecloth that is 3 feet wide and 5 feet long?

To answer this question, you need to know the area of the tablecloth to find the cost of the fabric, and you need to know its perimeter to find the cost of the lace border.

- Use the formula for the area of a rectangle. The dimensions are given in feet.

$$A = lw$$

$$A = 5 \cdot 3 = 15 \text{ ft}^2$$

The area of the tablecloth is 15 square feet. You want to know the area in square yards.

$$15 \text{ ft}^2 \div 9 \text{ ft}^2 \text{ per yd}^2 \approx 1.67 \text{ yd}^2$$

Joan needs about 1.67 square yards of fabric.

- Fabric costs $4.25 per square yard.

$$1.67 \text{ yd}^2 \cdot \$4.25 \text{ per yd}^2 \approx \$7.10 \text{ for fabric}$$

- Use the perimeter formula to find the perimeter of the tablecloth.

$$P = 2(l + w)$$

$$P = 2(5 + 3) = 16 \text{ ft}$$

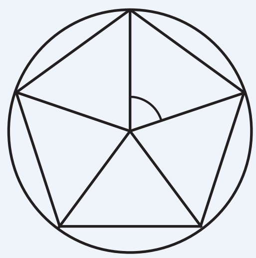
The perimeter of the tablecloth is 16 feet.

- Lace border costs $0.35 per foot.

$$16 \text{ ft} \cdot \$0.35 \text{ per ft} = \$5.60 \text{ for lace border}$$

The approximate cost of these materials is $7.10 + $5.60 = $12.70.

A jewelry designer is working with a design that is a regular pentagon inscribed in a circle. He needs to cut a small stone to fit into each of the triangles shown below. What is the measure of the angle indicated?



- Think about what you know.

  The angles at the center of a pentagon have a sum of 360°, and they are congruent because the pentagon is a regular 5-sided polygon.

- Use what you know to write an equation. Let x represent the measure of the angle.

$$x = 360 \div 5$$

$$x = 72$$

The measure of the angle indicated is 72°.

An architect is constructing a scale model of a building with a rectangular base. The actual building will be 300 feet tall, but the scale model is 18 inches tall. The dimensions of the building's base are 200 feet by 150 feet. What are the dimensions of the base of the scale model?

Use a proportion to find each of the dimensions of the base of the scale model.

Length	Width
$\frac{18}{300} = \frac{l}{200}$	$\frac{18}{300} = \frac{w}{150}$
$300l = 18 \cdot 200$	$300w = 18 \cdot 150$
$300l = 3600$	$300w = 2700$
$l = 12$ in.	$w = 9$ in.

The dimensions of the model's base are 12 inches by 9 inches.

Try It

The dining area of a restaurant includes a patio 25 feet wide by 40 feet long. On an architect's scale drawing of the restaurant, the width of the patio is 5 inches. What is the length of the patio in the scale drawing?

Use a proportion to find the length of the patio in the scale drawing.

Find the ratios of corresponding measurements.

Width: $\frac{5}{\square}$ Length: $\frac{l}{\square}$

Write a proportion and solve it.

$$\frac{5}{\square} = \frac{l}{40}$$

$$\_\_\_\_\_ l = 5 \cdot \_\_\_\_\_$$

$$25l = \_\_\_\_\_$$

$$l = \_\_\_\_\_$$

The length of the patio in the scale drawing is _______ inches.

Width: $\frac{5}{25}$ Length: $\frac{l}{40}$

$$\frac{5}{25} = \frac{l}{40}$$

$$25l = 5 \cdot 40$$

$$25l = 200$$

$$l = 8$$

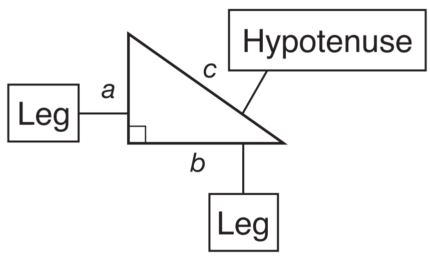
The length of the patio in the scale drawing is 8 inches.

What Is the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the sides of a right triangle. This special relationship applies only to right triangles.

The sides of a right triangle have special names.

- The **hypotenuse** of a right triangle is the longest side of the triangle. The hypotenuse is always opposite the right angle in the triangle. In the diagram below, the length of the hypotenuse is represented by c.
- The **legs** of the right triangle are the two sides that form the right angle. In the diagram below, the lengths of the legs are represented by a and b.

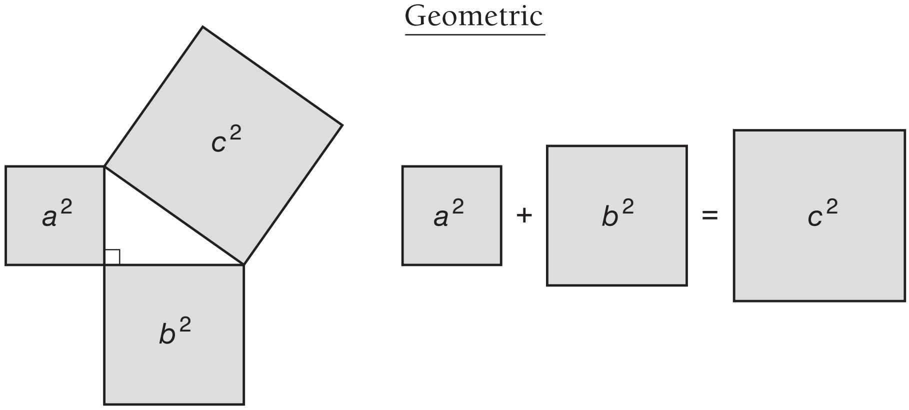


The Pythagorean Theorem can be stated algebraically or verbally.

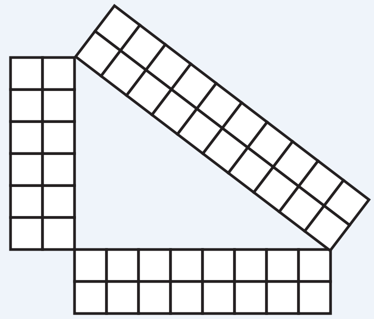
Algebraic	Verbal
In any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$.	In any right triangle the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

The Pythagorean Theorem can also be interpreted with a geometric model.

Geometric

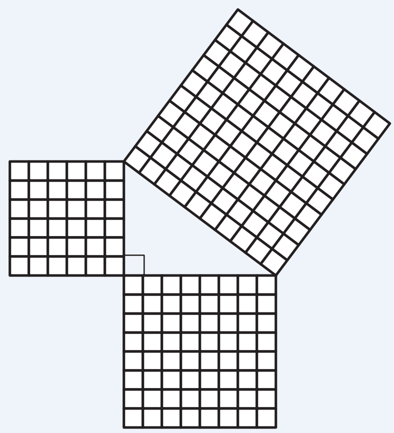


Does the model below demonstrate the Pythagorean Theorem?



A triangle is a right triangle if its sides satisfy the Pythagorean Theorem. A geometric model of the Pythagorean Theorem uses squares, not rectangles, to show that the sum of the areas formed by the legs is equal to the area formed by the hypotenuse. The model above does not demonstrate the Pythagorean Theorem.

Look at another model.



The square formed by the hypotenuse is 10 • 10 units. It has an area of 10 • 10 = 100 square units.

The square formed by one leg is 6 • 6 units. It has an area of 6 • 6 = 36 square units.

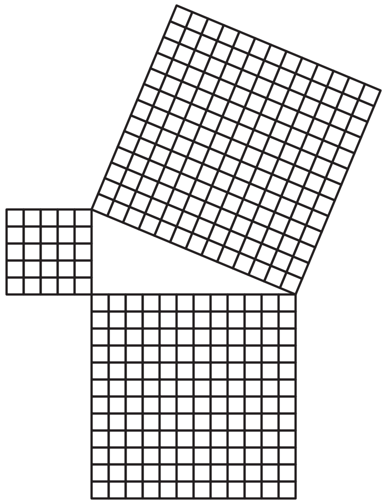
The square formed by the other leg is 8 • 8 units. It has an area of 8 • 8 = 64 square units.

Since 100 = 36 + 64, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

The second model does show that the sides of the triangle satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$. The sides form a right triangle.

Try It

The model below shows how three squares could be joined at their vertices to form a triangle. Do the squares form a right triangle?



The longest side is _______ units long, so it is the hypotenuse.

The legs are _______ units and _______ units long.

The square formed by the hypotenuse is _______ by _______ units.

It has an area of _______ • _______ = _______ square units.

The square formed by the shorter leg is _______ by _______ units.

It has an area of _______ • _______ = _______ square units.

The square formed by the longer leg is _______ by _______ units.

It has an area of _______ • _______ = _______ square units.

Since _______ = _______ + _______, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs.

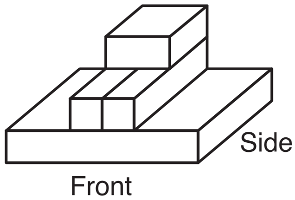
Yes, the squares form a right triangle.

The longest side is 13 units long, so it is the hypotenuse. The legs are 5 units and 12 units long. The square formed by the hypotenuse is 13 by 13 units. It has an area of 13 • 13 = 169 square units. The square formed by the shorter leg is 5 by 5 units. It has an area of 5 • 5 = 25 square units. The square formed by the longer leg is 12 by 12 units. It has an area of 12 • 12 = 144 square units. Since 169 = 25 + 144, the area of the square formed by the hypotenuse is equal to the sum of the areas of the two squares formed by the legs. Yes, the squares form a right triangle.

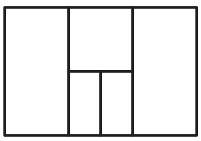
Now practice what you've learned.

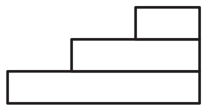
Question 69

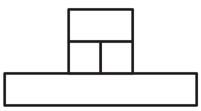
The object below is built from blocks.

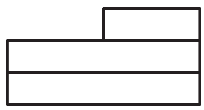


Which is not a top, front, or side view of the object?

A 

C 

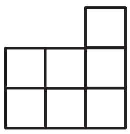
B 

D 

Answer Key: page 246

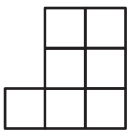
Question 70

The front view of a three-dimensional structure built with 9 identical cubes is shown below.

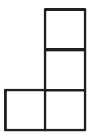


Which of the following could not represent the right-side view of this structure?

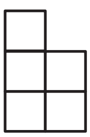
A



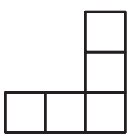
B



C



D



Answer Key: page 246

Question 71

An interior decorator painted two rectangular panels. One panel is 10 feet by 20 feet, and the other is 4 feet by 15 feet. The can of paint she used covers at most 400 square feet. She then used all the paint that remained in the can to completely paint a third rectangular panel. Which of the following is a reasonable estimate of the dimensions of the third panel?

A 12 ft by 20 ft

B 15 ft by 15 ft

C 10 ft by 16 ft

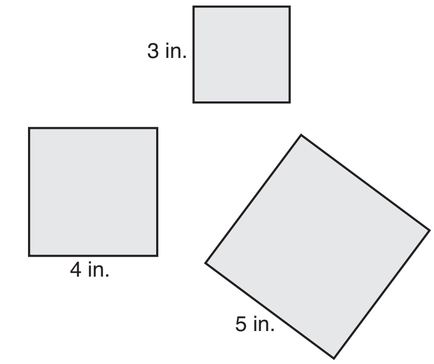
D 10 ft by 12 ft

Answer Key: page 246

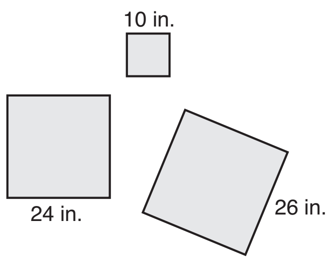
Question 72

Each set of squares below can be joined at their vertices to form a triangle. Which set of squares could not be used to form the sides of a right triangle?

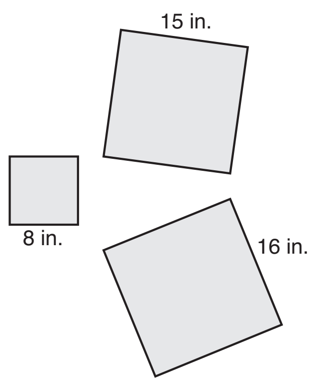
A



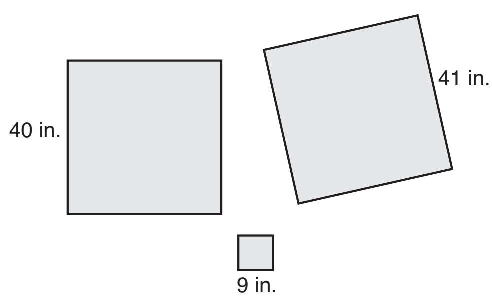
B



C



D

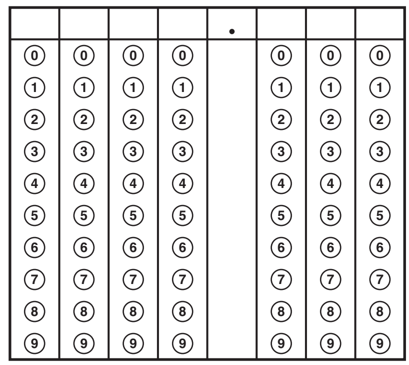


Answer Key: page 247

Question 73

An archaeologist is making a scale drawing of the foundation of an ancient building. The foundation is a rectangle that measures 18 feet by 45 feet. If the shorter dimension of the drawing is 4 inches, what is the longer dimension in inches?

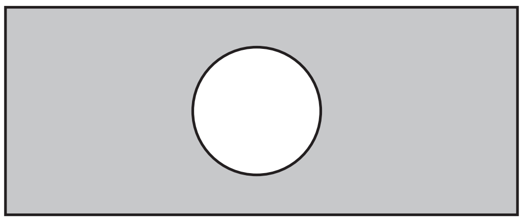
Record your answer and fill in the bubbles. Be sure to use the correct place value.



Answer Key: page 247

Question 74

Ed is installing a new bathroom sink countertop. The rectangular countertop is 5 feet 4 inches long by 2 feet 2 inches wide. He plans to tile the countertop with square tiles that are 2 inches on each side. The circular sink has a diameter of 16 inches.



What is the minimum number of tiles Ed will need to cover the countertop area, not including the sink?

A 732

B 366

C 410

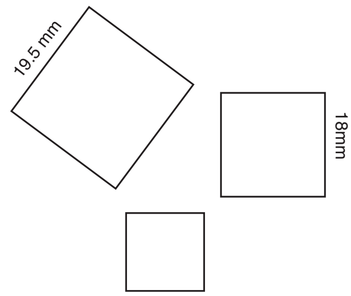
D 404

Answer Key: page 247

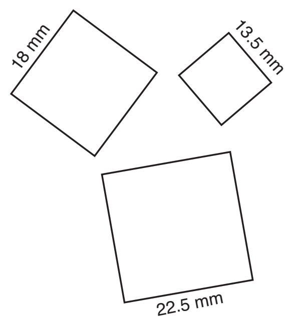
Question 75

Using the dimensions of the squares shown below, determine which set of squares can be joined at their vertices to form a right triangle.

A



B



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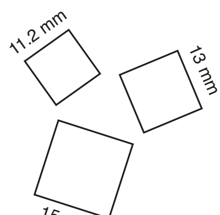

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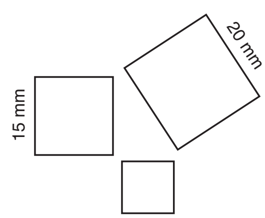


Answer Key: page 247

Objective 8

The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

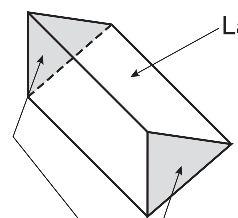
For this objective you should be able to

- use procedures to determine measures of three-dimensional figures;
- use indirect measurement to solve problems; and
- describe how changes in dimensions affect linear, area, and volume measurements.

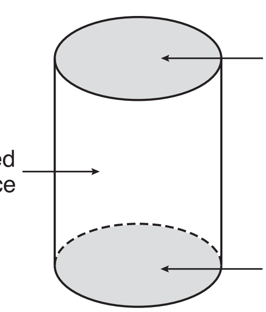
How Do You Find the Surface Area of Three-Dimensional Figures?

You can use models or formulas to find the surface area of prisms, cylinders, and other three-dimensional figures.

- A **prism** is a three-dimensional figure with two bases. The bases are congruent polygons. The other faces of the prism are rectangular and are called lateral faces. The prism is named by the shape of its bases. For example, a triangular prism has two triangles as its bases.



- A **cylinder** is a three-dimensional figure with two congruent circular bases and a curved surface.

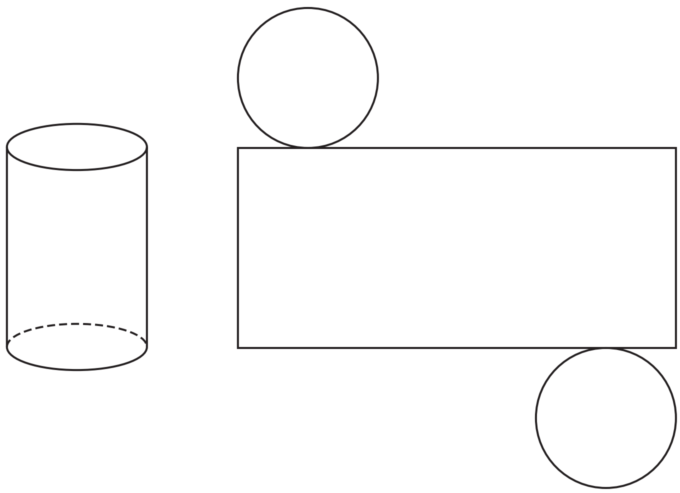


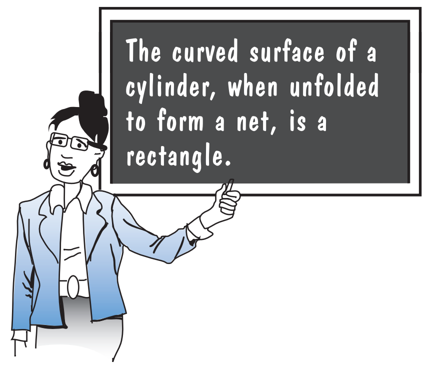
Like the area of a plane figure, the surface area of a three-dimensional figure is measured in square units.

- The **total surface area** of a three-dimensional figure is equal to the sum of the areas of all its surfaces.
- The **lateral surface area** of a three-dimensional figure is equal to the sum of the areas of all its faces and curved surfaces but does not include the area of the figure's bases.

One way to find the surface area of a three-dimensional figure is to use a net of the figure. A **net** of a three-dimensional figure is a two-dimensional drawing that shows what the figure would look like when opened up and unfolded with all its surfaces laid out flat. Use the net to find the area of each surface.

For example, the net for a cylinder is shown below. It is composed of two circles for the bases and a rectangle for the curved surface.



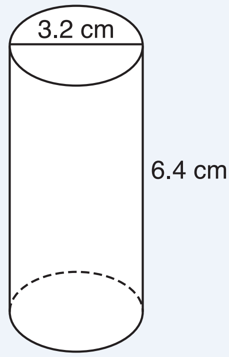


You can also find the surface area of a three-dimensional figure by using a formula. Substitute the appropriate dimensions of the figure into the formula and calculate its surface area. The formulas for total surface area and lateral surface area of several three-dimensional figures are included in the Mathematics Chart.

- The **total surface area** of a cylinder is equal to the sum of the areas of the two circular bases plus the area of the curved surface of the cylinder.
- The **lateral surface area** of a cylinder is equal to the area of the rectangle of the curved surface of the cylinder.

Objective 8

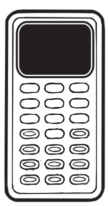
Find the lateral surface area and the total surface area of the cylinder shown below to the nearest tenth of a square centimeter.



Use the formula for the lateral surface area of a cylinder, $S = 2\pi rh$.

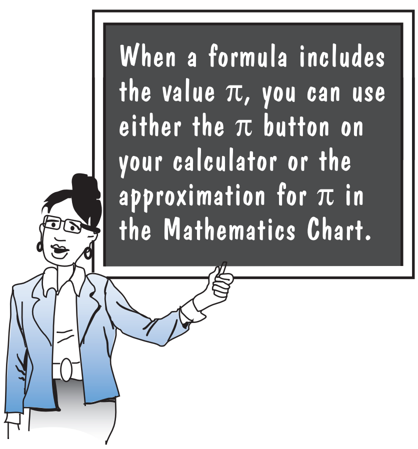
- The diameter of the cylinder shown on the diagram is 3.2 centimeters. The radius, r, is $\frac{1}{2}$ the diameter. Since $\frac{1}{2} \cdot 3.2 = 1.6$, the radius of the cylinder is 1.6 cm.
- The height, h, shown on the diagram is 6.4 centimeters.
- Substitute the values of r and h into the formula.

$$S = 2\pi rh$$
$$S = 2(\pi)(1.6)(6.4)$$
$$S \approx 64.3398$$



The lateral surface area is approximately 64.3 cm^2.

The formula for the total surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$.



- The area of the lateral surface, $2\pi rh \approx 64.3398$ cm^2, was found above.
- Find the area of the two circular bases.

  Substitute the value of the radius, $r = 1.6$, into the second part of the formula, $2\pi r^2$.

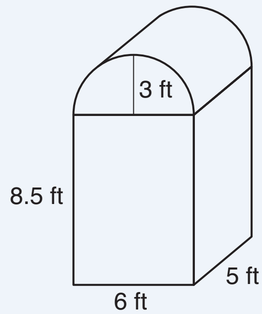
$$2\pi r^2 = 2(\pi)(1.6)^2 \approx 16.0850 \text{ cm}^2$$

- To calculate the total surface area of the cylinder, add the lateral surface area and the area of the two bases.

$$S \approx 64.3398 + 16.0850 \approx 80.4248$$

The total surface area of the cylinder is approximately 80.4 cm^2.

A storage shed has rectangular sides and a roof in the shape of a half-cylinder. Both the sides and the roof of the shed are to be painted. The dimensions of the shed are shown below.



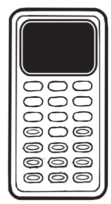
Find the total area of the surfaces to be painted to the nearest square foot.

- Calculate the area of the rectangular sides of the storage shed.
  There are two sides with dimensions 6 ft by 8.5 ft.
  Each of these sides has an area of $6 \cdot 8.5 = 51$ ft^2.
  There are two sides with dimensions 5 ft by 8.5 ft.
  Each of these sides has an area of $5 \cdot 8.5 = 42.5$ ft^2.
  The four sides have a total area of $2(51) + 2(42.5) = 187$ ft^2.

- Calculate the surface area of the roof.
  The roof is a half-cylinder with radius 3 ft and height 5 ft.
  The formula for the surface area of a cylinder is $S = 2\pi rh + 2\pi r^2$.
  Substitute the values for r and h into the formula.

$$S = 2\pi rh + 2\pi r^2$$
$$S = 2\pi \cdot 3 \cdot 5 + 2\pi \cdot 3^2$$
$$S = 30\pi + 18\pi$$
$$S = 48\pi$$
$$S \approx 150.80 \text{ ft}^2$$

  The area of the cylinder is about 150.80 ft^2. Since the roof is only half a cylinder, divide the surface area by 2. The area of the roof is approximately 75.40 ft^2.



- Add the area of the roof to the total area of the sides of the building to find the total area to be painted.

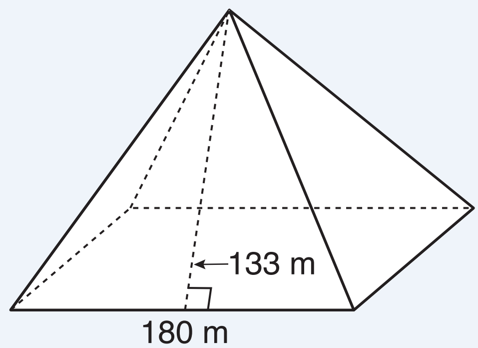
$$187 \text{ ft}^2 + 75.40 \text{ ft}^2 \approx 262.40 \text{ ft}^2$$

To the nearest square foot, the area to be painted is 262 ft^2.

Objective 8

The Pyramid Arena in Memphis, TN, has a slant height of 133 meters, and its square base has a side length of 180 meters. The external surface of the pyramid (excluding the floor) is covered with stainless steel.

Approximately how much stainless steel covers the lateral surface area of the Pyramid?



Use the formula for the lateral surface area of a pyramid: $S = \frac{1}{2}Pl$.

The perimeter of the base of the pyramid is $4 \cdot 180 = 720$.

$$S = \frac{1}{2}(720)(133)$$

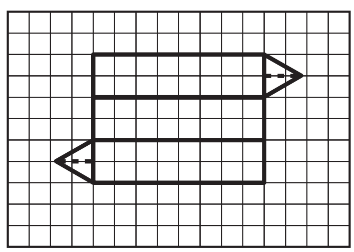
$$S = (360)(133)$$

$$S = 47{,}880$$

The lateral surface area of the pyramid is 47,880 square feet.

Try It

The net of a triangular prism is shown on the coordinate grid below.



The height of each triangle (indicated by the dotted lines) is approximately 1.7 units. Use the grid to find the other dimensions of the prism and its total surface area to the nearest square unit.

The surface area of the prism is equal to the ______________ of the areas of all its faces.

Find the area of a triangular face.

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot$ ________ $\cdot$ ________

$A =$ ________

Each triangular face has an area of ________ square units.

The prism has ________ triangular faces.

Since 2 • ________ = ________, their combined area is ________ square units.

Find the area of a rectangular face.

$A = lw$

$A =$ ________ $\cdot$ ________

$A =$ ________

Each rectangular face has an area of ________ square units.

The prism has ________ rectangular faces.

Since 3 • ________ = ________, their combined area is ________ square units.

The total surface area of the triangular prism is

________ + ________ = ________ square units.

The surface area of the prism is equal to the sum of the areas of all its faces.

$A = \frac{1}{2}bh = \frac{1}{2} \cdot 2 \cdot 1.7 = 1.7$

Each triangular face has an area of 1.7 square units. The prism has 2 triangular faces. Since 2 • 1.7 = 3.4, their combined area is 3.4 square units.

$A = lw = 8 \cdot 2 = 16$

Each rectangular face has an area of 16 square units. The prism has 3 rectangular faces. Since 3 • 16 = 48, their combined area is 48 square units.

The total surface area of the triangular prism is 3.4 + 48 = 51.4 square units.

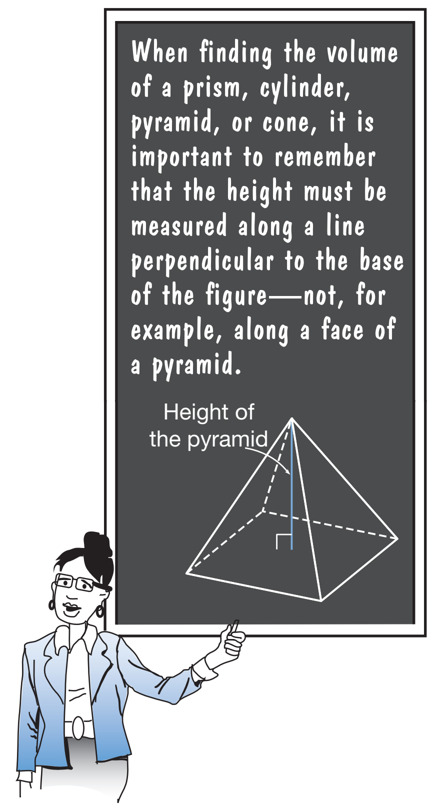
What Is the Volume of a Three-Dimensional Figure?

The **volume** of a three-dimensional figure is a measure of the space it occupies. Volume is measured in cubic units.

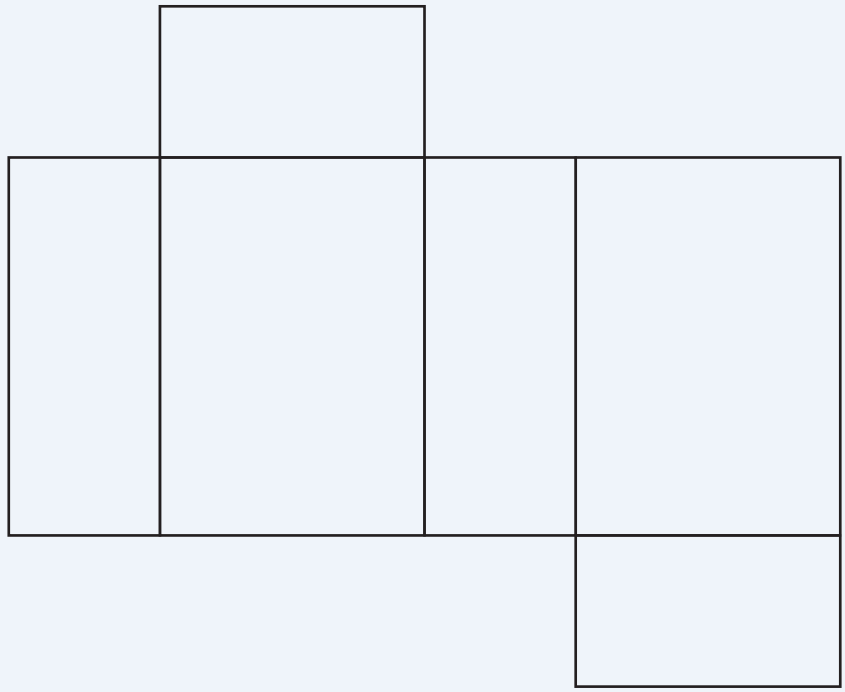
You can use formulas or models to find the volume of three-dimensional figures. The formulas for calculating the volume of several three-dimensional figures are in the Mathematics Chart.

When using a formula to find the volume of a three-dimensional figure, follow these guidelines:

- Identify the three-dimensional figure you are working with. This will help you select the correct volume formula.
- Use models to help visualize the three-dimensional figure and to assign the variables in the volume formula. A model can also be used to find the dimensions of a figure.
- Substitute the appropriate dimensions of the figure for the corresponding variables in the volume formula.
- Calculate the volume. State your answer in cubic units.



The net of a rectangular prism is shown below. Use the ruler provided on the Mathematics Chart to measure the dimensions of the figure to the nearest tenth of a centimeter. Use these dimensions to find the volume of the rectangular prism.



Use the formula for the volume of a prism, $V = Bh$, in which B represents the area of the base of the prism and h represents the prism's height.

- Find the area of the base, B. The base is a rectangle, so its area is equal to its length times its width.

  Measure the length and width using the centimeter ruler. The length is 5.0 cm, and the width is 3.5 cm. Calculate the area of the base using $A = lw$.

  $$B = 5.0 \cdot 3.5$$

  $$B = 17.5 \text{ cm}^2$$

- Measure the height of the prism: $h = 2.0$ cm.
- Substitute the area of the base, B, and the height, h, into the formula for the volume of a prism, $V = Bh$.

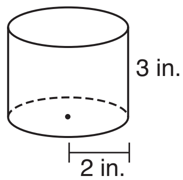
  $$V = 17.5 \cdot 2.0$$

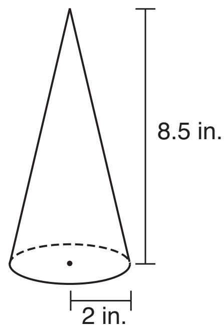
  $$V = 35 \text{ cm}^3$$

The prism has a volume of 35 cubic centimeters.

Try It

Which three-dimensional figure has the greater volume: a cylinder 3 inches high with a radius of 2 inches or a cone of the same radius that is 8.5 inches high?





Cylinder

The formula for the volume of a cylinder is $V = Bh$. Find B, the area of the base of the cylinder.

The base of the cylinder is a ______________.

Use the formula $A = \pi r^2$.

Substitute ______ for r.

$A = \pi r^2$

$A = \pi \cdot$ ______

$A \approx$ ______

$B \approx$ ______ in.^{2}

Substitute 12.57 for B and ______ for h in the formula for the volume of a cylinder.

$V = Bh$

$V \approx 12.57 \cdot$ ______

$V \approx$ ______

The volume of the cylinder is about ______ cubic inches.

Cone

The formula for the volume of a cone is $V = \frac{1}{3}Bh$. Find B, the area of the __________ of the cone.

The base of the cone is a ______________.

Use the formula $A = \pi r^2$.

Substitute ______ for r.

$A = \pi r^2$

$A = \pi \cdot$ ______

$A \approx$ ______

$B \approx$ ______ in.^{2}

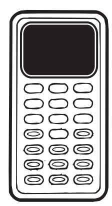
Substitute 12.57 for B and ______ for h in the formula for the volume of a cone.

$V = \frac{1}{3}Bh$

$V \approx \frac{1}{3} \cdot 12.57 \cdot$ ______

$V \approx$ ______

The volume of the cone is about ______ cubic inches.



The ______________ has the greater volume.

Objective 8

Cylinder

The base of the cylinder is a circle. Substitute 2 for r.

$$A = \pi r^2$$
$$A = \pi \cdot 2^2$$
$$A \approx 12.57$$
$$B \approx 12.57 \text{ in.}^2$$

Substitute 12.57 for B and 3 for h in the formula for the volume of a cylinder.

$$V = Bh$$
$$V \approx 12.57 \cdot 3$$
$$V \approx 37.71$$

The volume of the cylinder is about 37.71 cubic inches.

Cone

Find B, the area of the base of the cone. The base of the cone is a circle. Substitute 2 for r.

$$A = \pi r^2$$
$$A = \pi \cdot 2^2$$
$$A \approx 12.57$$
$$B \approx 12.57 \text{ in.}^2$$

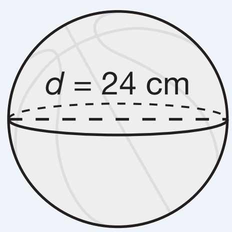
Substitute 12.57 for B and 8.5 for h in the formula for the volume of a cone.

$$V = \frac{1}{3}Bh$$
$$V \approx \frac{1}{3} \cdot 12.57 \cdot 8.5$$
$$V \approx 35.615$$

The volume of the cone is about 35.615 cubic inches.

The cylinder has the greater volume.

Find the volume to the nearest cubic centimeter of the sphere shown below.



Use the formula for the volume of a sphere.

$$V = \frac{4}{3}\pi r^3$$

In this formula, r represents the radius of the sphere. Since the diameter of the sphere is given, first divide by 2 to get the radius.

$$d = 2r$$
$$24 = 2r$$
$$\frac{24}{2} = \frac{2r}{2}$$
$$r = 12$$

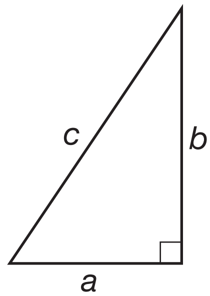
Substitute 12 cm for r into the formula.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(12^3) \\ &= \frac{4}{3}\pi(1728) \\ &\approx 7238.229 \text{ cm}^3 \end{aligned}$$

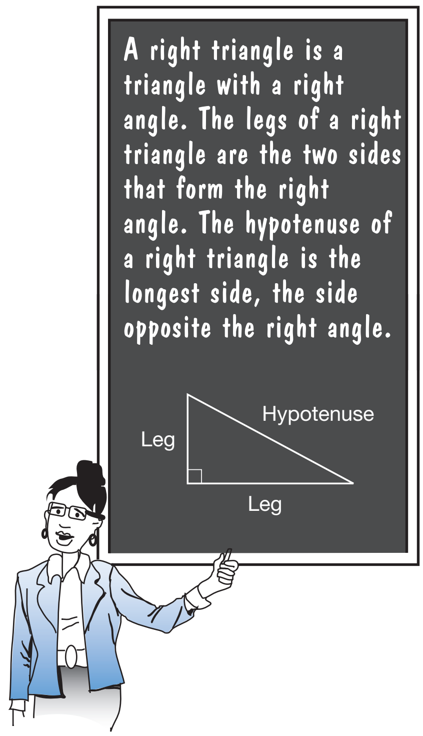
Rounding to the nearest cubic centimeter, the volume of the sphere is approximately 7238 cubic centimeters.

How Can You Solve Problems Using the Pythagorean Theorem?

The **Pythagorean Theorem** is a relationship among the lengths of the three sides of a right triangle. The Pythagorean Theorem applies only to right triangles.



- In any right triangle with leg lengths a and b and hypotenuse length c, $a^2 + b^2 = c^2$.
- If the side lengths of any triangle satisfy the equation $a^2 + b^2 = c^2$, then the triangle is a right triangle, and c is its hypotenuse.



Any set of three whole numbers that satisfy the Pythagorean Theorem is called a **Pythagorean triple**. The set of numbers {5, 12, 13} forms a Pythagorean triple because these numbers satisfy the Pythagorean Theorem. To show this, substitute 13 for c in the formula—since 13 is the greatest number—and substitute 5 and 12 for a and b.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + 12^2 &= 13^2 \\ 25 + 144 &= 169 \\ 169 &= 169 \end{aligned}$$

A triangle with side lengths 5, 12, and 13 units is a right triangle.

Any multiple of a Pythagorean triple is also a Pythagorean triple. Since the set of numbers {5, 12, 13} is a Pythagorean triple, the triple formed by multiplying each number in the set by 2, {10, 24, 26}, is also a Pythagorean triple. A triangle with side lengths 10, 24, and 26 units is also a right triangle.

Objective 8

Would a triangle with side lengths 3 inches, 4 inches, and 5 inches form a right triangle?

Determine whether the side lengths satisfy the Pythagorean Theorem. Since 5 is the greatest length, it would be the length of the triangle's hypotenuse. Substitute 5 for *c* in the formula. Substitute 3 and 4 for *a* and *b*, the two legs.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 \stackrel{?}{=} 5^2$$
$$9 + 16 \stackrel{?}{=} 25$$
$$25 = 25$$

Since the side lengths satisfy the Pythagorean Theorem, the triangle is a right triangle.

This example also shows that the set {3, 4, 5} forms a Pythagorean triple.

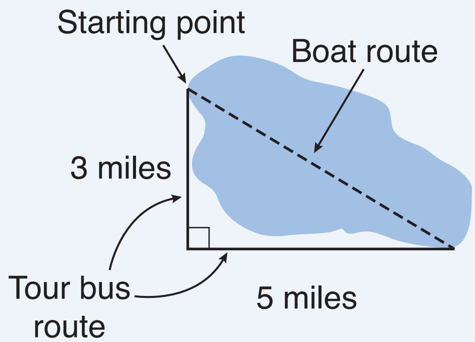
A right triangle has a side length of 24 meters and a hypotenuse of 25 meters. Find the length of the third side.

Substitute 25, the hypotenuse, for *c* and 24, the length of one side, for *b*.

$$a^2 + b^2 = c^2$$
$$a^2 + 24^2 = 25^2$$
$$a^2 + 576 = 625$$
$$-576 = -576$$
$$a^2 = 49$$
$$a = \sqrt{49}$$
$$a = 7$$

The length of the third side is 7 meters.

On a tour around a lake, visitors ride a tour bus 3 miles south and 5 miles east. Then they ride a boat across the lake back to the starting point. Their journey forms a right triangle.



Approximately how many miles is the trip across the lake?

The bus route and the boat route form a right triangle. The two parts of the journey traveled by bus form the legs of the right triangle. Their lengths are given as 3 miles and 5 miles. The boat's return path across the lake forms the hypotenuse, c.

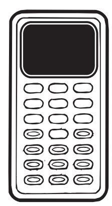
Use the Pythagorean Theorem to find the length of the boat's path across the lake.

- Substitute 3 and 5 for the legs, a and b.

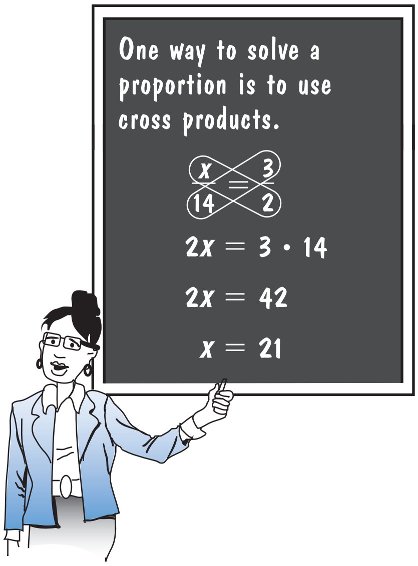
$$a^2 + b^2 = c^2$$
$$3^2 + 5^2 = c^2$$
$$9 + 25 = c^2$$
$$34 = c^2$$
$$\sqrt{34} = c$$

- Find a decimal approximation of $\sqrt{34}$.

$$\sqrt{34} \approx 5.831$$



Rounded to the nearest tenth, the trip across the lake is approximately 5.8 miles.

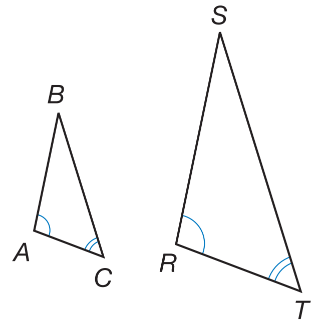


How Can You Use Proportional Relationships to Solve Problems?

You can use proportional relationships to find missing side lengths in similar figures. To solve problems that involve similar figures, follow these guidelines:

- Identify which figures are similar and which sides correspond. Similar figures have the same shape, but not necessarily the same size. The lengths of the corresponding sides of similar figures are proportional.

  Triangle ABC is similar to triangle RST.

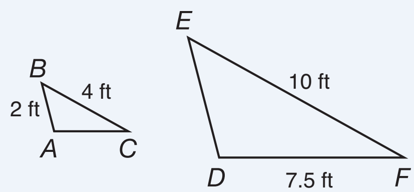


$$\triangle ABC \sim \triangle RST$$

$$\frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$$

- Write a proportion and solve it.
- Answer the question asked.

The triangles in the drawing below are similar.



Find the length of $\overline{AC}$.

- Write a proportion comparing the ratio of the unknown side and its corresponding side to the ratio of a pair of corresponding sides whose lengths are known.

  $\overline{AC}$ corresponds to $\overline{DF}$. The length of $\overline{AC}$ is unknown; $DF = 7.5$ ft.

  $\overline{BC}$ and $\overline{EF}$ are a pair of corresponding sides with known lengths; $BC = 4$ ft, and $EF = 10$ ft.

$$\frac{BC}{EF} = \frac{AC}{DF}$$

- Substitute the known values.

$$\frac{4}{10} = \frac{AC}{7.5}$$

- Use cross products to solve.

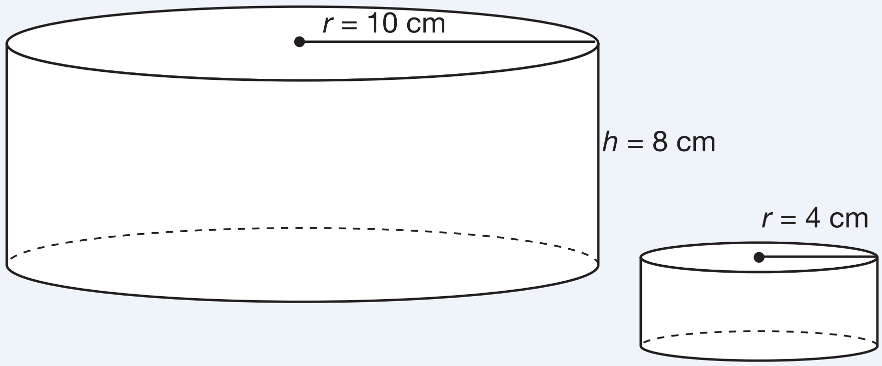
$$4(7.5) = 10 \cdot AC$$

$$30 = 10 \cdot AC$$

$$3 = AC$$

The length of $\overline{AC}$ is 3 feet.

Two similar cylindrical cans are shown below.



What is the volume of the smaller can in terms of π?

The radius of the smaller can corresponds to the radius of the larger can, and the height of the smaller can corresponds to the height of the larger can. The ratios of these corresponding dimensions are equal.

$$\frac{\text{smaller radius}}{\text{larger radius}} = \frac{\text{smaller height}}{\text{larger height}}$$

Substitute the measurements given in the diagram. Let h represent the height of the smaller can.

$$\frac{4}{10} = \frac{h}{8}$$

Use cross products to solve for the height of the smaller can.

$$10h = 4 \cdot 8$$

$$10h = 32$$

$$h = 3.2$$

The height of the smaller can is 3.2 cm. To find the volume of the smaller can, we now use $V = Bh$.

$$V = \pi r^2 h$$

$$V = \pi(4)^2(3.2)$$

$$V = \pi(16)(3.2)$$

$$V = 51.2\pi$$

The volume of the smaller can is 51.2π cubic centimeters.

Try It

Two regular hexagons are inscribed in circles. The smaller hexagon has a side length of 12 centimeters, and the larger one has a side length of 60 centimeters. If the radius of the larger circle is 52 centimeters, what is the radius of the smaller circle?

Since the hexagons are similar figures, the ratios of the lengths of their corresponding sides will be proportional.

$$\frac{\text{larger}}{\text{smaller}} \qquad \frac{\square}{12} = \frac{\square}{r}$$

Use cross products to solve.

$$\_\_\_\_\_\_ \, r = 12 \cdot \_\_\_\_\_\_$$

$$\_\_\_\_\_\_ \, r = \_\_\_\_\_\_$$

$$r = \_\_\_\_\_\_ \text{ cm}$$

The radius of the smaller circle is _______ centimeters.

$$\frac{60}{12} = \frac{52}{r}$$

$$60r = 12 \cdot 52$$

$$60r = 624$$

$$r = 10.4 \text{ cm}$$

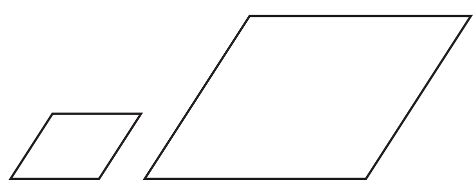
The radius of the smaller circle is 10.4 centimeters.

See Objective 6, page 148, for more information about dilations.

How Is the Perimeter of a Figure Affected When Its Dimensions Are Changed Proportionally?

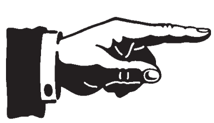
When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The perimeter of the dilated figure will change by the same scale factor.

To **dilate** a figure means to enlarge or reduce it by a given scale factor. For example, the quadrilateral on the left has been dilated (enlarged) by a scale factor of 2.5 to form the quadrilateral on the right.



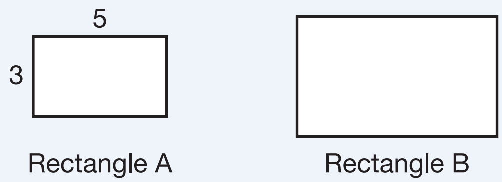
The perimeter of the larger quadrilateral is 2.5 times the perimeter of the smaller quadrilateral.

Do you see that . . .



If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their perimeters will be in the ratio $\frac{a}{b}$.

In the drawing below, rectangle B is a dilation of rectangle A by a scale factor of 1.5.



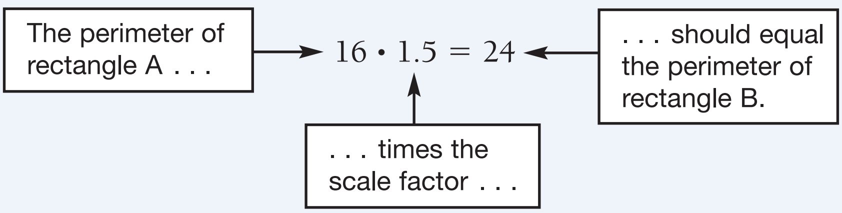
What effect should this dilation have on the perimeter of rectangle B?

The perimeter of rectangle B should increase by the same factor, 1.5.

- To prove this, first find the perimeter of rectangle A.

$$P = 2(l + w) = 2(5 + 3) = 2(8) = 16 \text{ units}$$

- Use the scale factor to find the perimeter of rectangle B.



- Use the formula to find the perimeter of rectangle B.

  The dimensions of rectangle B equal the dimensions of rectangle A multiplied by the scale factor, 1.5.

  Find the length and width of rectangle B.

$$l = 5 \cdot 1.5 = 7.5$$

$$w = 3 \cdot 1.5 = 4.5$$

  Find the perimeter.

$$P = 2(l + w)$$

$$P = 2(7.5 + 4.5)$$

$$P = 2(12)$$

$$P = 24 \text{ units}$$

- Compare the two perimeters.

$$\frac{24}{16} = \frac{1.5}{1}$$

The perimeter of rectangle B increased by a factor of 1.5, the same factor by which its dimensions increased.

Try It

The perimeter of a triangle is 24.8 meters. If the triangle is enlarged by a scale factor of 3.4, what will be the perimeter of the larger triangle?

The perimeter of the triangle should increase by a scale factor of ______.

$$P_{\text{larger}} = P_{\text{smaller}} \cdot \underline{\hspace{2cm}}$$

Substitute the known values.

$$P_{\text{larger}} = \underline{\hspace{2cm}} \cdot \underline{\hspace{2cm}}$$

$$P_{\text{larger}} = \underline{\hspace{2cm}} \text{ m}$$

The perimeter of the larger triangle will be ______ meters.

The perimeter of the triangle should increase by a scale factor of 3.4.

$$P_{\text{larger}} = P_{\text{smaller}} \cdot 3.4$$

$$P_{\text{larger}} = 24.8 \cdot 3.4$$

$$P_{\text{larger}} = 84.32 \text{ m}$$

The perimeter of the larger triangle will be 84.32 meters.

How Is the Area of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The area of the dilated figure will change by the square of the scale factor.

Do you see that . . .

If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$.

A 5-by-7-inch photograph is enlarged by a scale factor of 4. How is the area of the photograph affected?

The area of the photograph should increase by the square of the scale factor: $4^2 = 16$. The area should increase by a factor of 16.

- To prove this, first find the area of the original photograph.

$$A = lw$$
$$A = 7 \cdot 5$$
$$A = 35 \text{ in.}^2$$

- Use the scale factor to find the area of the enlarged photograph.

The area of the original photograph . . . → $35 \cdot (4^2) = 35 \cdot 16 = 560$ ← . . . should equal the area of the enlarged photo.

↑ . . . times the square of the scale factor . . .

- Use the formula to find the area of the enlarged photo. The dimensions of the enlarged photo are the dimensions of the original photo multiplied by the scale factor, 4.

  Find the length and width of the enlarged photo.

$$l = 7 \cdot 4 = 28$$
$$w = 5 \cdot 4 = 20$$

  Find the area.

$$A = lw$$
$$A = 28 \cdot 20$$
$$A = 560 \text{ in.}^2$$

- The ratio of the two areas is $\frac{560}{35}$. When reduced, this ratio is $\frac{16}{1}$, or $\left(\frac{4}{1}\right)^2$.

The area of the enlarged photograph increased by a factor of 16.

Try It

A certain bakery sells two sizes of round cakes. The radius of the small cake is $\frac{1}{2}$ the radius of the large cake. If the top of the large cake has an area of 200 in.^{2}, what is the area of the top of the small cake?

The top of the small cake is a dilation of the top of the large cake by a scale factor of $\frac{\square}{\square}$.

The area of the dilated figure will change by the __________ of the scale factor.

The area of the top of the __________ cake is equal to the area of the top of the __________ cake multiplied by $\left(\frac{\square}{\square}\right)^2$.

The area of the top of the small cake can be calculated as follows.

$$A = 200 \cdot \left(\frac{\square}{\square}\right)^2$$

$$A = 200 \cdot \frac{\square}{\square}$$

$$A = \text{\_\_\_\_\_\_\_\_\_\_ in.}^2$$

The area of the top of the small cake is __________ square inches.

The top of the small cake is a dilation of the top of the large cake by a scale factor of $\frac{1}{2}$. The area of the dilated figure will change by the square of the scale factor. The area of the top of the small cake is equal to the area of the top of the large cake multiplied by $\left(\frac{1}{2}\right)^2$.

$$A = 200 \cdot \left(\frac{1}{2}\right)^2$$

$$A = 200 \cdot \frac{1}{4}$$

$$A = 50 \text{ in.}^2$$

The area of the top of the small cake is 50 square inches.

How Is the Volume of a Figure Affected When Its Dimensions Are Changed Proportionally?

When the dimensions of a figure are changed proportionally, the figure is dilated by a scale factor. The volume of the dilated figure will change by the cube of the scale factor.

Do you see that . . .

If the dimensions of two similar three-dimensional figures are in the ratio $\frac{a}{b}$, then their volumes will be in the ratio $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$.

A rectangular prism with a volume of 60 cubic units is dilated by a scale factor of 3. What is the volume of the dilated prism?

- The volume of the original prism is 60 cubic units.
- Find the volume of the dilated prism.

The volume of the original prism . . . → $60 \cdot (3^3) = 60 \cdot 27 = 1620$ ← . . . should equal the volume of the dilated prism.

↑ . . . times the cube of the scale factor . . .

The volume of the dilated prism is 1620 cubic units.

Try It

A breakfast-cereal manufacturer is using a scale factor of 2.5 to increase the size of one of its cereal boxes. If the volume of the original cereal box was 240 in.^{3}, what is the volume of the enlarged box?

If the dimensions of the box are increased by a scale factor of _____ , then the volume of the box will increase by the __________ of the scale factor.

$V =$ ________ • (________)^3

$V =$ ________ • ________

$V =$ ________ in.^{3}

The volume of the enlarged box is ________ cubic inches.

If the dimensions of the box are increased by a scale factor of 2.5, then the volume of the box will increase by the cube of the scale factor.

$V = 240 \cdot (2.5)^3$

$V = 240 \cdot 15.625$

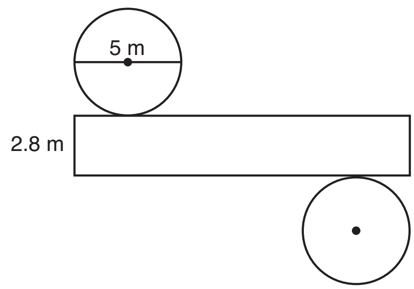
$V = 3750$ in.^{3}

The volume of the enlarged box is 3750 cubic inches.

Now practice what you've learned.

Question 76

The net below can be folded to form a cylinder.



What is the approximate total surface area of the cylinder?

A 64 square meters

B 83 square meters

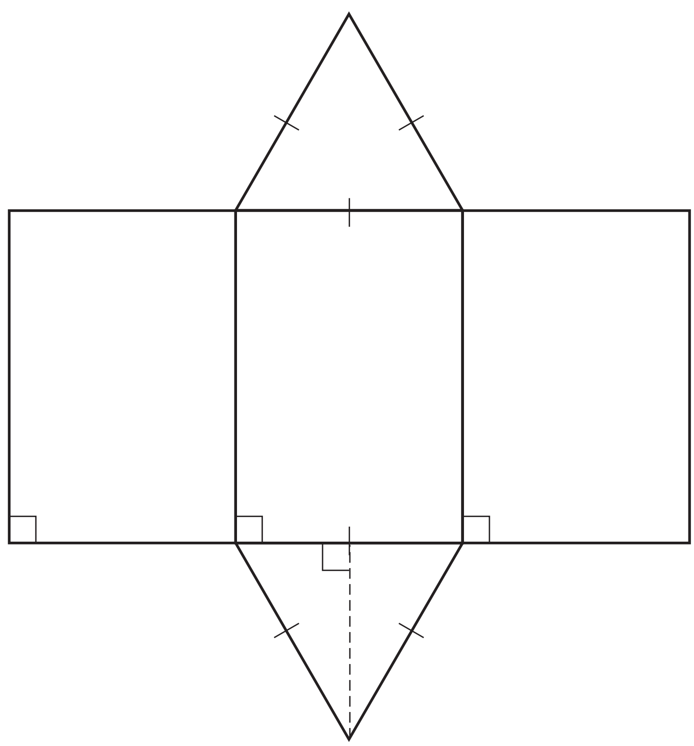
C 246 square meters

D 286 square meters

Answer Key: page 247

Question 77

The net of a triangular prism is shown below. Use the ruler on the Mathematics Chart to measure the dimensions of the prism to the nearest tenth of a centimeter.



Which is closest to the total surface area of the prism?

A 37 cm^2

B 23 cm^2

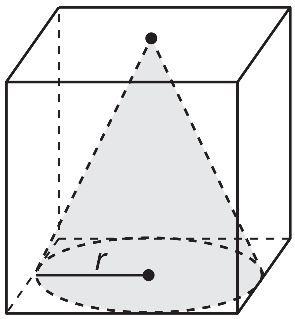
C 47 cm^2

D 40 cm^2

Answer Key: page 248

Question 78

A cone comes tightly packaged in a cubical box, as shown below.



If the cone has radius r, which expression best represents the volume, V, of the box?

A $V = (2r)^3$

B $V = 3(2r^3)$

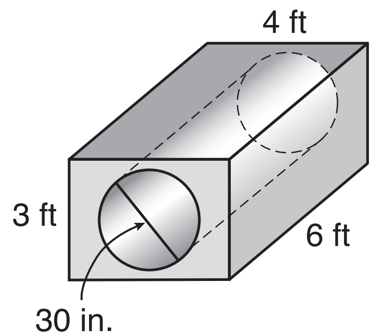
C $V = 2r^3$

D $V = r^3$

Answer Key: page 248

Question 79

A pipe in the shape of a cylinder with a 30-inch diameter is to go through a passageway shaped like a rectangular prism. The passageway is 3 ft high, 4 ft wide, and 6 ft long. The space around the pipe is to be filled with insulating material.



What is the volume, to the nearest cubic foot, of the space to be filled with insulating material?

A 72 ft^3

B 43 ft^3

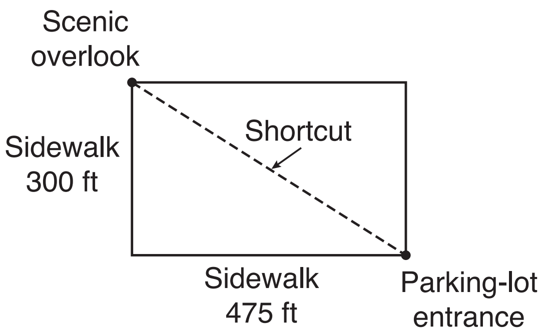
C 30 ft^3

D 29 ft^3

Answer Key: page 248

Question 80

Jillian walks from the parking-lot entrance to the scenic overlook by following a sidewalk along the edge of the rectangular park. She walks back to the parking lot by taking a shortcut through the park. The drawing below shows her journey.



To the nearest foot, how much shorter was her trip back to the parking lot than her walk to the scenic overlook?

A 417 ft

B 213 ft

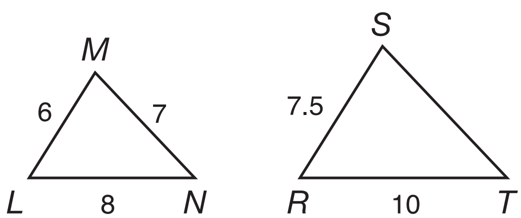
C 562 ft

D 261 ft

Answer Key: page 248

Question 81

$\triangle LMN$ and $\triangle RST$ are similar.



What is the length of $\overline{ST}$?

A 6.4 units

B 7.75 units

C 13.3 units

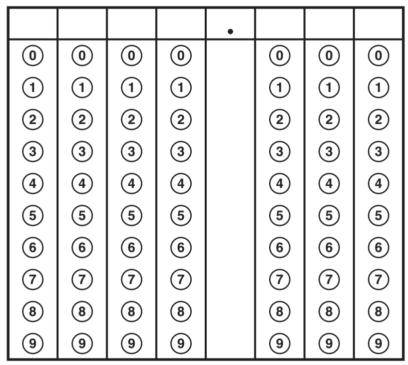
D 8.75 units

Answer Key: page 249

Question 82

An art store sells two sizes of rectangular poster boards. The smaller poster board has a width of 18 inches and a height of 24 inches. The larger poster board is similar to the smaller one and has a height of 28 inches. What is the width in inches of the larger poster board?

Record your answer and fill in the bubbles. Be sure to use the correct place value.



Answer Key: page 249

Question 83

The ratio of the diameter of a larger circle to the diameter of a smaller circle is $\frac{3}{2}$. Which number represents the ratio of the area of the larger circle to the area of the smaller circle?

A $\frac{3}{2}$

B $\frac{2}{3}$

C $\frac{4}{9}$

D $\frac{9}{4}$

Answer Key: page 249

Question 84

Triangle *MNO* has a perimeter of 45 centimeters. Triangle *MNO* is dilated by a factor of $\frac{2}{5}$ to produce triangle *PQR*. What is the perimeter of triangle *PQR*?

A 18 cm
B 9 cm
C 10 cm
D 15 cm

Answer Key: page 249

Question 85

A cylindrical tank has a volume of 300 gallons. A similar tank next to it has dimensions that are 3 times as large. What is the volume of the larger tank?

A 5400 gal
B 2700 gal
C 900 gal
D 8100 gal

Answer Key: page 249

Question 86

A box shaped like a rectangular prism has a volume of 162 cubic inches. A smaller box has dimensions that are $\frac{2}{3}$ the dimensions of the larger box. What is the volume of the smaller box?

A 108 in.^{3}
B 48 in.^{3}
C 72 in.^{3}
D 27 in.^{3}

Answer Key: page 249

Question 87

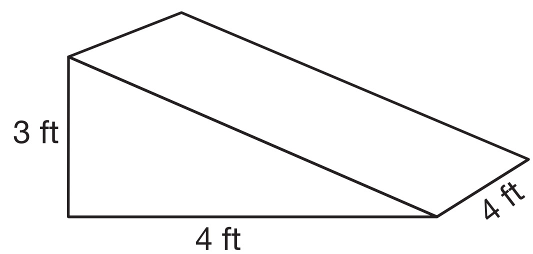
The diameter of a globe is 12 inches. Which of the following is closest to the volume of this globe?

A 75 cubic inches
B 151 cubic inches
C 905 cubic inches
D 7238 cubic inches

Answer Key: page 250

Question 88

A skateboard ramp in the shape of a right triangular prism is constructed out of wood, as shown below.



What is the total surface area of the ramp?

A 60 square feet
B 120 square feet
C 240 square feet
D 480 square feet

Answer Key: page 250

Objective 9

The student will demonstrate an understanding of percents, proportional relationships, probability, and statistics in application problems.

For this objective you should be able to

- identify proportional relationships in problem situations and solve problems;
- apply concepts of theoretical and experimental probability to make predictions;
- use statistical procedures to describe data; and
- evaluate predictions and conclusions based on statistical data.

How Do You Solve Problems Involving Proportional Relationships?

A **ratio** is a comparison of two quantities. A **proportion** is a statement that two ratios are equal. There are many real-life problems that involve proportional relationships. For example, you use proportions when converting units of measurement. You also use proportions to solve problems involving percents and rates.

To solve problems that involve proportional relationships, follow these guidelines:

- Identify the ratios to be compared. Be certain to compare the corresponding quantities, in the same order.
- Write a proportion, an equation in which the two ratios are set equal to each other.
- Solve the proportion. Use the fact that the cross products in a proportion are equal.

On a game show Mr. Williams answered 12 out of 15 questions correctly. What percent of the questions did Mr. Williams answer incorrectly?

- Mr. Williams answered 12 questions correctly. Therefore, he answered $15 - 12 = 3$ questions incorrectly.
- Write a ratio that compares the number of questions answered incorrectly to the total number of questions: $\frac{3}{15}$.
- To find the percent of questions answered incorrectly, find the part of 100, or the ratio $\frac{n}{100}$, that is equivalent to $\frac{3}{15}$.
- Write a proportion.

$$\frac{n}{100} = \frac{3}{15}$$

- To solve for n, use cross products and then divide by 15.

$$15n = 3 \cdot 100$$
$$15n = 300$$
$$n = 20$$

Mr. Williams answered 20% of the questions incorrectly.

Try It

At this time last year, Alfredo had $145 in his savings account. Today he has $152.98. If his savings continue to grow at the same rate, how much money will he have in his account at this time next year?

To find the rate at which Alfredo's savings increased, divide the number of dollars by which his savings increased from last year to this year by $________.

Alfredo's savings increased by

$152.98 − ________ = ________.

The rate by which his savings account grew is

________ ÷ 145 ≈ ________.

0.055 = ________%

His savings should increase by ________% over the next year.

Use a proportion to find 5.5% of his current balance, $152.98.

$$\frac{5.5}{\square} = \frac{x}{\square}$$

________x = 5.5(________)

________x = ________

x = ________

Alfredo's savings account at this time next year should have

$152.98 + ________ = ________.

To find the rate at which Alfredo's savings increased, divide the number of dollars by which his savings increased from last year to this year by $145. Alfredo's savings increased by $152.98 − $145.00 = $7.98. The rate by which his savings account grew is 7.98 ÷ 145 ≈ 0.055.

$$0.055 = 5.5\%$$

His savings should increase by 5.5% over the next year.

$$\frac{5.5}{100} = \frac{x}{152.98}$$

$$100x = 5.5(152.98)$$

$$100x = 841.39$$

$$x = 8.41$$

Alfredo's savings account at this time next year should have $152.98 + $8.41 = $161.39.

What Is Probability?

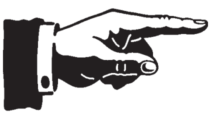
Probability is a measure of how likely an event is to occur. The probability of an event occurring is the ratio of the number of favorable outcomes to the number of all possible outcomes. In a probability experiment, favorable outcomes are the outcomes that you are interested in.

The probability, P, of an event occurring must be from 0 to 1.

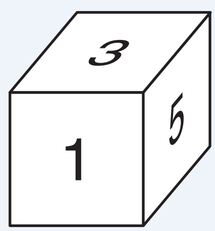
- If an event is impossible, its probability is 0.
- If an event is certain to occur, its probability is 1.

For example, the probability of drawing a blue marble from a bag containing 5 red marbles and 2 blue marbles is the ratio of the number of favorable outcomes, 2 blue marbles, to the number of possible outcomes, 7 marbles. The probability of drawing a blue marble is $\frac{2}{7}$. This is often written as $P(\text{blue}) = \frac{2}{7}$.

Do you see that . . .



A number cube has faces numbered 1 to 6. What is the probability of rolling an even number?



The sample space for this experiment is {1, 2, 3, 4, 5, 6}. There are a total of 6 possible outcomes.

A favorable outcome for this experiment is rolling a 2, 4, or 6. There are 3 favorable outcomes for this experiment.

The probability of rolling an even number is the ratio of the number of favorable outcomes to the number of possible outcomes: 3 out of 6, or $\frac{3}{6} = \frac{1}{2}$.

How Do You Find the Probability of Compound Events?

An event made up of a sequence of simple events is called a **compound event**. For example, flipping a coin and then rolling a number cube is a compound event.

One way to find the probability of a compound event is to multiply the probabilities of simple, mutually exclusive events that make up the compound event.

If $P(A)$ represents the probability of event A and $P(B)$ represents the probability of event B, then the probability of the compound event (A and B) can be represented algebraically.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

For example, the probability of a coin landing heads up on one toss is $P(H) = \frac{1}{2}$. The probability of a coin landing heads up on two tosses can be found as follows:

$$P(H \text{ and } H) = P(H) \cdot P(H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

When finding the probability of a compound event, you should first determine whether the simple events included are dependent or independent events.

- If the outcome of the first event affects the possible outcomes for the second event, the events are called **dependent events**.

  Suppose you draw 2 marbles, one at a time, from a bag with 4 white marbles and 1 red marble in it. You draw the second marble without replacing the first one you drew. Does the outcome of the first draw affect the likelihood of drawing a red marble on the second draw? Yes, because there were 5 marbles in the bag on the first draw, but only 4 on the second draw. The events are dependent.

- If the outcome of the first event does not affect the possible outcomes for the second event, the events are called **independent events**.

  Suppose you spin a spinner with the numbers 1 through 4 written on it, and then you spin it again. Does the outcome of the first spin affect the likelihood of spinning a 3 on the second spin? No. The events are independent.

A probability experiment consists of rolling a number cube with faces numbered 1 through 6 and then tossing a coin. What is the probability of rolling a 3 on the number cube and tossing tails on the coin?

- There are 6 possible outcomes for rolling the number cube: 1, 2, 3, 4, 5, and 6. There is only one favorable outcome, rolling a 3. The probability of rolling a 3 on the number cube is $\frac{1}{6}$.

$$P(3) = \frac{1}{6}$$

Objective 9

- There are two possible outcomes for tossing the coin: heads (H) and tails (T). There is only one favorable outcome, T. The probability of tossing tails on the coin is $\frac{1}{2}$.

$$P(T) = \frac{1}{2}$$

- Find the probability of rolling a 3 on the number cube and tossing tails on the coin.

$$\begin{aligned} P(3 \text{ and } T) &= P(3) \cdot P(T) \\ &= \frac{1}{6} \cdot \frac{1}{2} \\ &= \frac{1}{12} \end{aligned}$$

Another way to find the probability of this compound event is to look at the sample space for this experiment and identify the favorable outcomes. This method works only if the outcomes are all equally likely. Use a table to list all the possible outcomes. The one favorable outcome is shaded.

Sample Space

Number Cube	Coin
1	Heads
1	Tails
2	Heads
2	Tails
3	Heads
3	**Tails**
4	Heads
4	Tails
5	Heads
5	Tails
6	Heads
6	Tails

Do you see that . . .

There are 12 possible outcomes, but only 1 of them is favorable. The probability of rolling a 3 on the number cube and tossing tails on the coin is $\frac{1}{12}$.

This matches the result obtained using the rule for the probability of a compound event.

$$P(3 \text{ and } T) = P(3) \cdot P(T) = \frac{1}{12}$$

Objective 9

A jar contains 4 red balls, 5 green balls, and 3 black balls. A ball is drawn, and then a second ball is drawn without replacing the first one. What is the probability of drawing two red balls in a row?

There are 12 balls in the jar on the first draw, and 11 balls in the jar on the second draw. The two events are dependent. The outcome of the first ball drawn affects the possible outcome of the second ball drawn.

- Find the probability of drawing a red ball on the first draw. There are 12 possible outcomes for the first draw. There are 4 favorable outcomes, 4 red balls.

$$P(\text{red}_{\text{first}}) = \frac{4}{12} = \frac{1}{3}$$

- Find the probability of drawing a red ball on the second draw. There are now only 11 balls in the jar, so there are 11 possible outcomes for the second draw. Assume the first ball drawn was red. There are 3 favorable outcomes, 3 red balls left in the jar.

$$P(\text{red}_{\text{second}}) = \frac{3}{11}$$

- Find the compound probability.

$$P(\text{red}_{\text{first}} \text{ and } \text{red}_{\text{second}}) = P(\text{red}_{\text{first}}) \cdot P(\text{red}_{\text{second}})$$
$$= \frac{1}{3} \cdot \frac{3}{11} = \frac{3}{33} = \frac{1}{11}$$

The probability of drawing two red balls in a row is $\frac{1}{11}$.

What Is the Difference Between Theoretical and Experimental Probability?

The **theoretical probability** of an event occurring is the ratio comparing the number of ways the favorable outcomes should occur to the number of all possible outcomes. If you toss a coin, theoretically the coin should land on heads $\frac{1}{2}$ of the time.

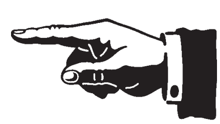
$$P(H) = \frac{1}{2} = 0.5$$

The **experimental probability** of an event occurring is the ratio comparing the actual number of times the favorable outcome occurs in a series of repeated trials to the total number of trials. If you toss a coin 100 times, it is possible that the coin will land heads up 48 times and tails up 52 times. The experimental probability of the coin landing heads up in this situation would be $\frac{48}{100}$.

$$P(H) = \frac{48}{100} = 0.48$$

The two types of probabilities, theoretical and experimental, are not always equal. In this case the theoretical probability is 0.5, but the experimental probability is 0.48.

For a given situation the experimental probability is usually close to, but slightly different from, the theoretical probability. The greater the number of trials, the closer the experimental probability should be to the theoretical probability.



Do you see that . . .

Objective 9

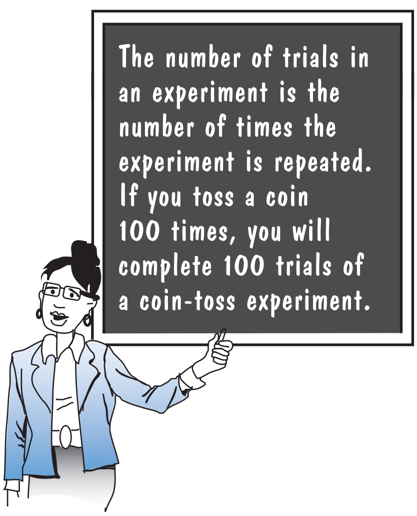
A spinner in a game has 4 equal-sized regions: red, green, blue, and yellow. A group of players makes 40 trial spins and creates the table below.

Color	Frequency
Red	9
Green	12
Blue	10
Yellow	9

How does the theoretical probability of landing on green compare to the experimental results?

- Since the colored regions are of equal size, the spinner is equally likely to land on any one of the colored regions. There are four possible outcomes for this experiment: red, green, blue, and yellow. There is one favorable outcome: green. The theoretical probability of landing on the green is $\frac{1}{4}$, or $P(\text{green}) = 0.25$.
- The experimental probability of landing on green is the ratio of the number of times the spinner landed on green in the experiment to the total number of spins. The spinner landed on green 12 times out of 40 spins. The experimental probability of landing on green is $\frac{12}{40} = \frac{3}{10}$, or $P(\text{green}) = 0.3$.

The experimental probability, 0.3, and the theoretical probability, 0.25, are close to each other but not equal.



How Do You Use Probability to Make Predictions and Decisions?

You can use either theoretical or experimental probabilities to make predictions. If you know the probability of an event occurring and you also know the total number of trials, then you can predict the likely number of favorable outcomes.

- Write a ratio that represents the probability of an event occurring.
- Write a ratio that compares the number of favorable outcomes to the number of trials.
- Write a proportion.
- Solve the proportion.

A manufacturer tests 125 lightbulbs to determine the number of hours the lightbulbs last. The results are shown in the table below.

Outcome (hours)	Frequency
950–974	15
975–999	20
1,000–1,024	50
1,025–1,049	40

Based on the experimental results, how many lightbulbs from a shipment of 50,000 lightbulbs should be expected to last at least 1,000 hours?

- Find the experimental probability that a lightbulb will last at least 1,000 hours.

  Based on the table, a total of 50 + 40 = 90 lightbulbs lasted at least 1,000 hours. There were a total of 125 trials. The experimental probability that a lightbulb will last at least 1,000 hours is $\frac{90}{125} = \frac{18}{25}$.

- Write a proportion.

$$\frac{18}{25} = \frac{x}{50{,}000}$$

- Solve by using cross products and dividing by 25.

$$25x = 18 \cdot 50{,}000$$
$$25x = 900{,}000$$
$$x = 36{,}000$$

Therefore, 36,000 lightbulbs from the shipment of 50,000 lightbulbs should be expected to last at least 1,000 hours.

Try It

Robert has a bag with 90 jelly beans, which come in 4 different colors. The table below shows the color of 25 jelly beans he randomly selected from the bag without replacement.

Selected Jelly Beans

Color	Number of Jelly Beans
Red	6
Blue	8
Green	4
Yellow	7

Based on the information shown in the table, which is the best prediction of the number of jelly beans remaining in the bag that are not blue?

Of the _______ jelly beans that were randomly selected, _______ were not blue. So the experimental probability of selecting a jelly bean that was not blue is _______. Since there were originally _______ jelly beans in the bag, and _______ were selected and not replaced, there are now _______ jelly beans in the bag. Let n represent the number of jelly beans that are _______.

Write a proportion and solve.

$$\frac{n}{\square} = \frac{\square}{25}$$

$$25n = \_\_\_\_ \cdot \_\_\_\_$$

$$25n = \_\_\_\_$$

$$\frac{25n}{\square} = \frac{1105}{\square}$$

$$n = \_\_\_\_$$

Because the decimal part of a jelly bean makes no sense in this context, the best prediction of the number of jelly beans remaining in the bag that are not blue is _______.

Of the 25 jelly beans that were randomly selected, 17 were not blue. So the experimental probability of selecting a jelly bean that was not blue is $\frac{17}{25}$.

Since there were originally 90 jelly beans in the bag, and 25 were selected and not replaced, there are now 65 jelly beans in the bag. Let n represent the number of jelly beans that are not blue. Write a proportion and solve.

$$\frac{n}{\boxed{65}} = \frac{\boxed{17}}{25}$$

$$25n = 65 \cdot 17$$

$$25n = 1105$$

$$\frac{25n}{\boxed{25}} = \frac{1105}{\boxed{25}}$$

$$n = 44.2$$

Because the decimal part of a jelly bean makes no sense in this context, the best prediction of the number of jelly beans remaining in the bag that are not blue is 44.

How Do You Use Mode, Median, Mean, and Range to Describe Data?

There are many ways to describe the characteristics of a set of data. The mode, median, and mean are all called **measures of central tendency**.

Mode	The **mode** of a set of data describes which value occurs most frequently. If two or more numbers occur the same number of times and more often than all the other numbers in the set, those numbers are all modes for the data set. If each of the numbers in a set occurs the same number of times, the set of data has no mode.	Use the mode to show which value or values in a set of data occur most often. For the set {1, 4, 9, 3, 1, 6} the mode is 1 because it occurs most frequently. The set {1, 4, 3, 3, 1, 6} has two modes, 1 and 3, because they both occur twice and most frequently.
Median	The **median** of a set of data describes the middle value when the set is ordered from greatest to least or from least to greatest. If there are an even number of values, the median is the average of the two middle values. Half the values are greater than the median, and half the values are less than the median. The median is a good measure of central tendency to use when a set of data has an outlier, a number that is very different in value from the other numbers in the set.	Use the median to show which number in a set of data is in the middle when the numbers are listed in order. For the set {1, 4, 9, 3, 6} the median is 4 because it is in the middle when the numbers are listed in order: {1, 3, 4, 6, 9}. For the set {1, 4, 9, 3, 1, 6} the median is $\frac{3+4}{2} = 3.5$ because 3 and 4 are in the middle when the numbers are listed in order: {1, 1, 3, 4, 6, 9}. Their values must be averaged to find the median.
Mean	The **mean** of a set of data describes the average of the numbers. To find the mean, add all the numbers and then divide by the number of items in the set. The mean of a set of data can be greatly affected if one of the numbers is an outlier, a number that is very different in value from the other numbers in the set. The mean is a good measure of central tendency to use when a set of data does not have any outliers.	Use the mean to show the numerical average of a set of data. For the set {1, 4, 9, 3, 1, 6} the mean is the sum, 24, divided by the number of items, 6. The mean is $24 \div 6 = 4$.
Range	The **range** of a set of data describes how big a spread there is from the largest value in the set to the smallest value.	Use the range to show how much the numbers vary. For the set {1, 4, 9, 3, 1, 6} the range is $9 - 1 = 8$.

To decide which of these measures to use to describe a set of data, look at the numbers and ask yourself, *What am I trying to show about the data?*

Objective 9

Do you see that . . .

Each night for one week, a restaurant manager recorded the number of customers who came for dinner. The results are shown in the table below.

Night	Customers
Sunday	58
Monday	57
Tuesday	60
Wednesday	55
Thursday	65
Friday	66
Saturday	149

Which measure of the data would best describe the number of customers who eat in the restaurant on a typical night?

- The range is the difference between the largest value and the smallest: $149 - 55 = 94$. The range does not describe the number of customers who eat in the restaurant on a typical night.
- Each of the numbers in the set of data occurs only once. The set of data has no mode.
- The mean number of customers is approximately 73. However, the number of customers on Saturday night, 149, is much higher than the number of customers on any other night. This data point is an outlier.

  In this case, the mean does not give a very good representation of the number of customers who eat in the restaurant on a typical night; it is too high.
- The median number of customers is found by listing the number of customers in order and finding the middle value. Listed in order, the numbers are {55, 57, 58, 60, 65, 66, 149}. The middle number is 60. The median, 60, is not affected by the outlier.

In this case, the median best describes the number of customers who eat in the restaurant on a typical night.

A science club sold candy bars to raise money. The table shows the number of candy bars sold during the first five days of the sale.

Day	Number
1	103
2	115
3	122
4	117
5	117

On the sixth day of the sale, the club sold only 56 candy bars. How does the significantly smaller number of candy bars sold on the sixth day affect the different measures of the data?

Range

The range is the difference between the largest value and the smallest.

- For the first 5 days, the range is $122 - 103 = 19$.
- With the sixth day included, the range is $122 - 56 = 66$.

The range of values increases significantly because the sixth value, an outlier, is well outside the previous range.

Median

The median is found by listing the number of candy bars sold in order and picking the middle value.

- For the first 5 days: 103, 115, 117, 117, 122

$$\text{median} = 117$$

- With the sixth day included: 56, 103, 115, 117, 117, 122

$$\text{median} = (115 + 117) \div 2 = 116$$

The median value is not significantly affected by the outlier.

Mode

The mode of a set of numbers tells which value occurs most frequently.

- For the first 5 days, the mode is 117.
- With the sixth day included, the mode is still 117.

The mode is not affected by the outlier.

Mean

The mean is the average of the values.

- For the first 5 days:

$$\frac{103 + 115 + 122 + 117 + 117}{5} = 114.8$$

- With the sixth day added, the mean is:

$$\frac{103 + 115 + 122 + 117 + 117 + 56}{6} = 105$$

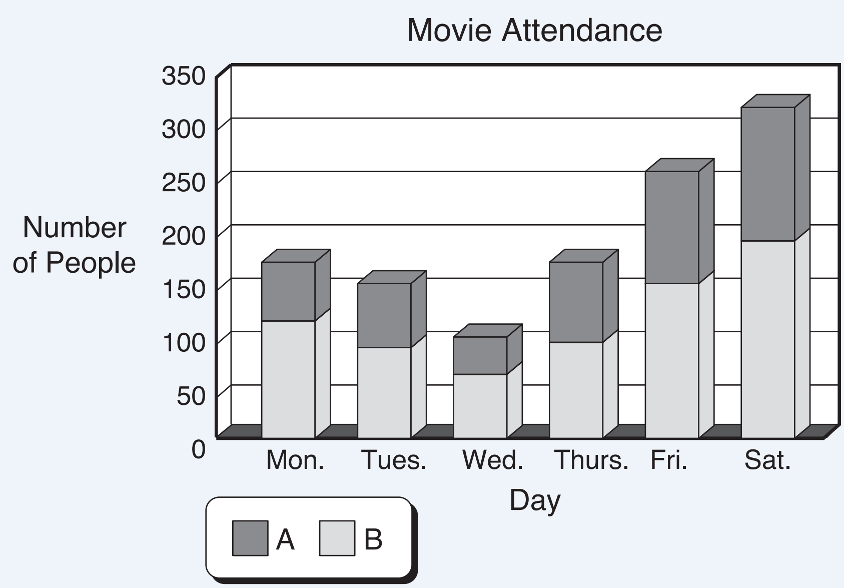
Since 56 is much smaller than the other values, it significantly lowers the mean.

How Do You Use Graphs to Represent Data?

There are many ways to represent data graphically. Bar graphs, histograms, and circle graphs are three types of graphs used to display data. Other types of graphs include line plots, stem and leaf plots, and box and whisker plots. Graphical representations of data often make it easier to see relationships in the data. However, if the conclusions drawn from a graph are to be valid, you must read and interpret the data from the graph accurately.

A **bar graph** uses bars of different heights or lengths to show the relationships between different groups or categories of data.

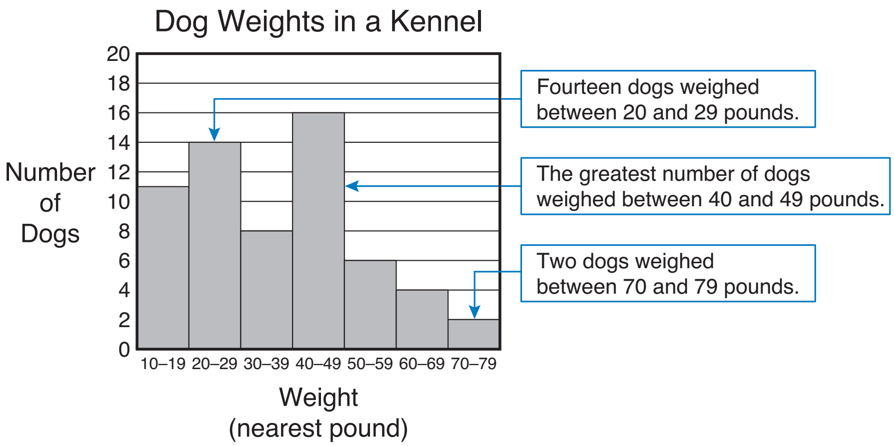
The bar graph below shows the number of people in a town who attended one of two different movies, A or B, on each of six days last week. What conclusions can you draw about the number of people attending each of the movies each day?



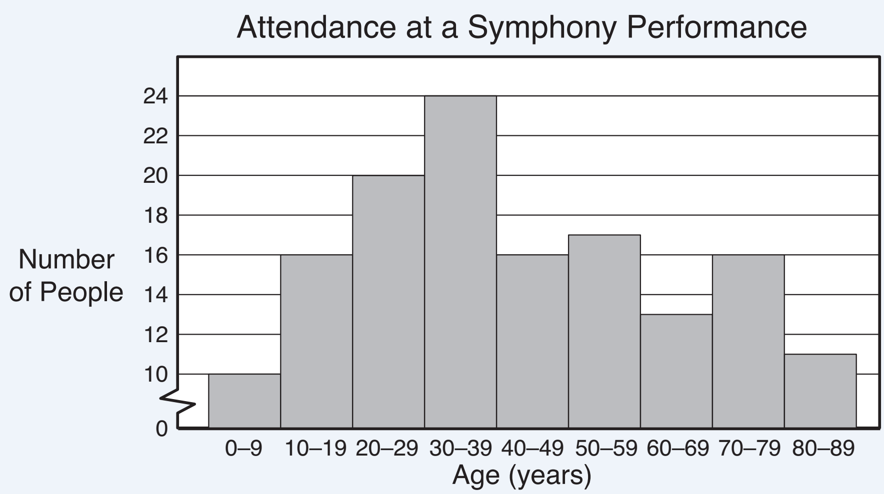
- The graph shows that more people attended each of the two movies on Saturday than on any other day.
- For the week, more people attended Movie B than Movie A.
- The least number of people attended either movie on Wednesday.
- The combined attendance at both movies on Monday and Thursday was the same.

A **histogram** is a special kind of bar graph that shows the number of data points that fall within specific intervals of values. The intervals into which the data's range is divided should be equal. If the intervals are not equal, the graph could be misleading and result in invalid conclusions.

Look at the histogram below showing the number of dogs of various weights in a kennel.



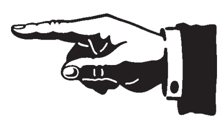
The histogram shows the ages of people attending a symphony performance. Their ages are divided into equal 10-year intervals.



What conclusions can you draw from the graph about attendance at the performance?

The broken line on the vertical axis means that attendance values from 0 through 9 are not shown in the graph. Using the broken line allows the graph to have shorter bars. Because of this, the lengths of the bars should not be used to make direct comparisons. Instead, read the values from the graph and compare them.

- A total of 24 people between the ages of 30 and 39 attended the performance. This age group had the greatest number of people in attendance.



Do you see that . . .

- A total of 20 people between the ages of 20 and 29 attended the performance, and 10 people between the ages of 0 and 9 attended. Twice as many people between the ages of 20 and 29 attended the performance as people between the ages of 0 and 9.

  If you had compared the bar lengths for these two data intervals you could have reached an invalid conclusion. The bar for one is about five times as large as for the other, yet the numerical value is only twice as large.

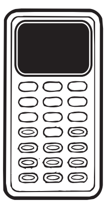
A **circle graph** represents a set of data by showing the relative size of the parts that make up the whole. The circle represents the whole, or the sum of all the data elements. Each section of the circle represents a part of the whole. The number of degrees in the central angle of the section should be proportional to the number of degrees in a circle, 360°.

Suppose you were constructing a circle graph that compared the number of school-sponsored sports teams on which juniors in your school play. Their participation data are shown in the table below.

Juniors Playing Sports

# of Sports	# of Students
0	80
1	50
2	40
3 or more	30

What size central angle should be used for the sector of the circle used to represent the students who play no sports?



One way to solve this problem is to first find the percent of the circle that should be used to represent the students who play no sports.

- The table represents a total of 200 students. Since 80 of them play no sports, the fraction of students who play no sports is $\frac{80}{200}$.
- Convert $\frac{80}{200}$ to a percent.

$$\frac{80}{200} = \frac{40}{100} = 40\%$$

- So, 40% of the students play no sports. Therefore, 40% of the circle should be used to represent students who play no sports.

Next, find what central angle should be used for the sector of the circle used to represent students who play no sports. Since a circle has 360°, find 40% of 360°.

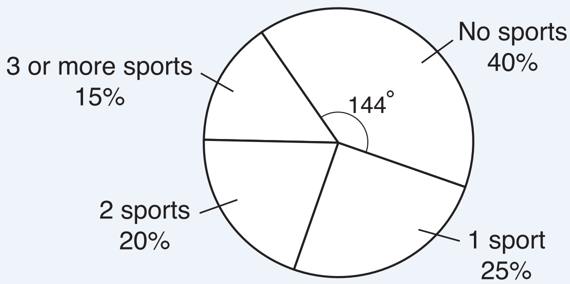
$$\frac{40}{100} = \frac{n}{360}$$

$$100n = 14{,}400$$

$$n = 144$$

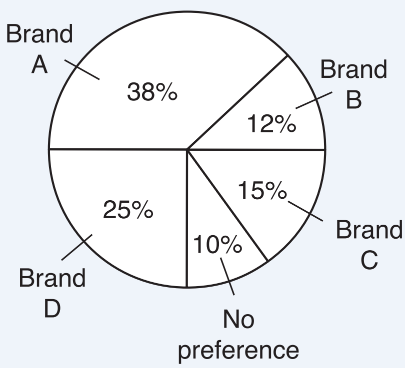
A central angle of 144° should be used for the sector of the circle used to represent the students who play no sports.

Juniors Playing Sports



Customers at a grocery store were asked which brand of soft drink they prefer. The results of the survey are shown in the circle graph below.

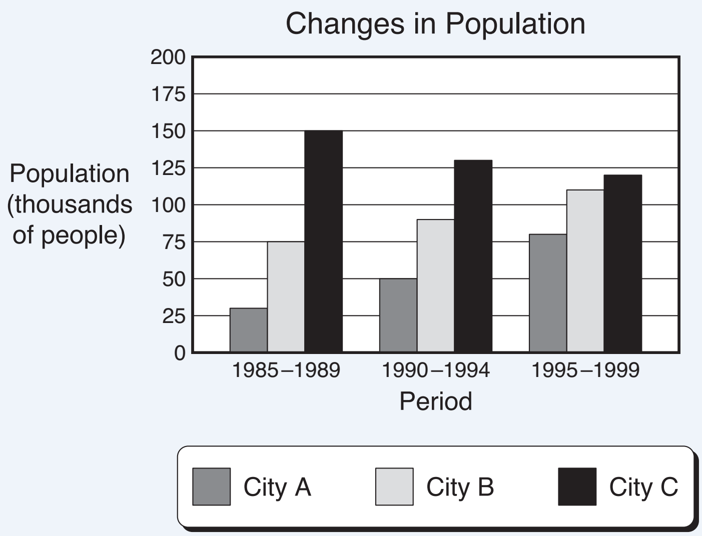
Soft-Drink Preferences



What conclusions can you draw about the soft-drink preferences of the customers at the store?

- The graph shows that the most preferred brand is Brand A, 38%.
- The graph shows that 25%, or $\frac{1}{4}$, of the customers surveyed preferred Brand D.
- The smallest fraction of customers, 10%, had no preference in soft drinks.

The graph below shows the average populations of three cities during three five-year intervals.



For which city was the increase in population the greatest between 1985 and 1999?

- The population of City C decreased between 1985 and 1999.
- The population of City B increased between 1985 and 1999. Find the increase in population. The lowest average population, approximately 75,000, was during the period 1985–1989, and the highest average population, approximately 110,000, was during the period 1995–1999. The approximate increase in population was 110,000 − 75,000 = 35,000 people.
- The population of City A increased between 1985 and 1999. Find the increase in population. The lowest average population, approximately 30,000, was during the period 1985–1989, and the highest average population, approximately 80,000, was during the period 1995–1999. The approximate increase in population was 80,000 − 30,000 = 50,000 people.

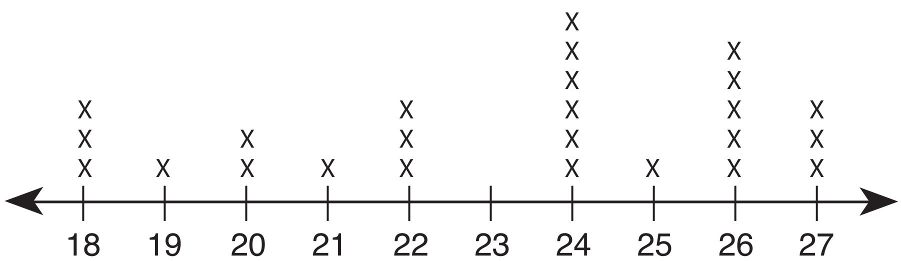
The increase in population between 1985 and 1999 was the greatest for City A.

A **line plot** represents a set of data by showing how often a piece of data appears in that set. It consists of a number line that includes the values of the data set. An X is placed above the corresponding value each time that value appears in the data set.

For example, consider the data set below:

{18, 18, 18, 19, 20, 20, 21, 22, 22, 22, 24, 24, 24, 24, 24, 24, 25, 26, 26, 26, 26, 26, 27, 27, 27}

This data set could be organized in the line plot below.



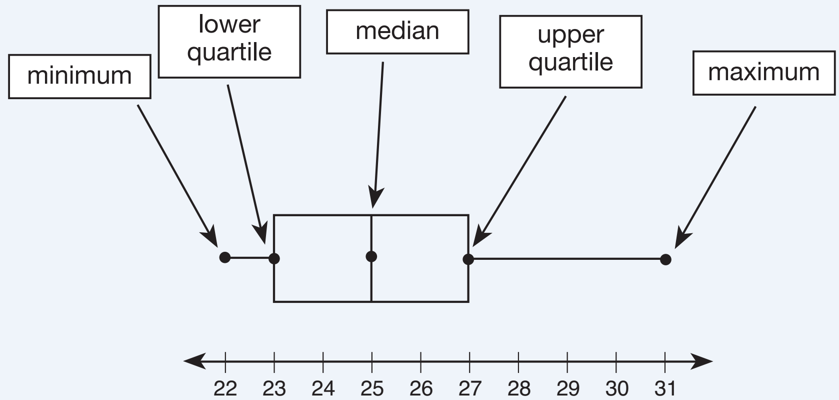
Notice there were three values of 18 in the data set, and there are three Xs above 18 in the line plot. Similarly, there is one value of 25 in the data set, and there is one X above 25 in the line plot.

A **box and whisker plot** can be used to describe the distribution of a data set (that is, how near or far the data points are from each other). It also gives specific information about certain values related to the data set.

The table below represents the age of college students in a particular class.

Age	Frequency
22	2
23	6
24	5
25	4
26	3
27	2
28	2
29	2
30	0
31	1

The box and whisker plot below represents the data set shown in the table above.



As you can see from the box and whisker plot, the data appear to be closer together for the lower values and more spread out for the higher values.

The **lower quartile** can be thought of as the "middle value of the first half" of the data set. One-fourth (one quarter) of the data set will be at or below the lower quartile. Likewise, the **upper quartile** is the "middle value of the second half" of the data set. Three-fourths of the data will be at or below the upper quartile. In this sense, quartiles are very much like medians.

Specifically, in the data set shown above, there are 27 values. When the data set is ordered, the median is the number in the 14th place (in this case, 25). The data set is now split into two halves. The lower half of the data has places 1 through 13, and the upper half has places 15 through 27.

As it was mentioned earlier, the lower quartile is the middle value of the lower half of data (places 1 through 13). The middle value would then be in the 7th place, which is 23. Likewise, the upper quartile is the middle value of the upper half of data (places 15 through 27). The middle value of the upper half of data would be in the 21st place, which is 27.

What data can be interpreted from this graph?

The youngest age for a member of this class was 22, and the oldest age was 31. The median age was 25. The lower quartile is 23, and the upper quartile is 27.

Now practice what you've learned.

Question 89

Rhonda estimated it would take 12 hours to complete her research project. If this represents only 80% of the number of hours it actually took her to complete the project, how many hours did Rhonda spend on the project?

A 9.6 h

B 15 h

C 3 h

D 960 h

Answer Key: page 250

Question 90

A factory worker can manufacture 35 electronic switches in 1.5 hours. At this rate, how many hours will it take him to manufacture 210 switches?

A 6 h

B 9 h

C 4900 h

D 140 h

Answer Key: page 250

Question 91

Jonathan draws two tickets from a box to select the door-prize winners at a party. The tickets are numbered from 1 to 25. What is the probability that both of the tickets drawn will have numbers less than 5?

A $\frac{1}{50}$

B $\frac{2}{75}$

C $\frac{12}{625}$

D $\frac{1}{5}$

Answer Key: page 250

Question 92

Reggie is a professional baseball player. He has the following batting record.

Type of Hit	Number
Singles	210
Doubles	20
Triples	1
Home runs	6
No hits	574

Based on this record, what is the probability that Reggie will get a hit during his next time at bat?

A 0.413

B 0.186

C 0.292

D 0.366

Answer Key: page 250

Question 93

A horticulturist selected a sample of seeds from a crop. She planted the seeds, and after 3 months she measured the height of each plant to the nearest eighth of an inch. The results are shown in the table below.

Height (inches)	Number of Plants
$0–1\frac{7}{8}$	9
$2–3\frac{7}{8}$	16
$4–5\frac{7}{8}$	26
6 or more	24

Based on the results in the table, how many seeds out of 600 could the horticulturist expect to reach a height of at least 2 inches in 3 months?

A 450

B 550

C 528

D 384

Answer Key: page 251

Question 94

Jean is a member of the school's bowling club. At the last practice session, each team member bowled 3 games and recorded his or her score on a master list. Which measure of the data should Jean use if she wants to identify the score that has as many scores below it as above it?

A Mean

B Mode

C Median

D Range

Answer Key: page 251

Question 95

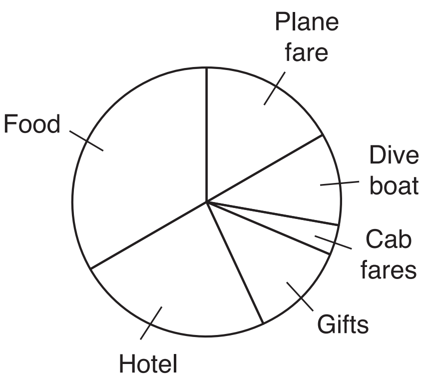
The table below represents Trish's expenses for a scuba-diving vacation in the Caribbean.

Trish's Scuba Vacation

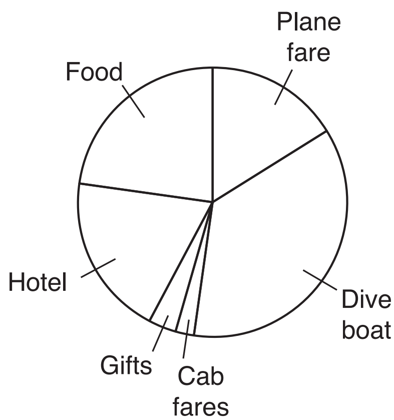
Expense	Amount (dollars)
Plane fare	600
Food	200
Hotel	360
Cab fares	40
Gifts	50
Dive boat	250

Which graph matches the information in the table?

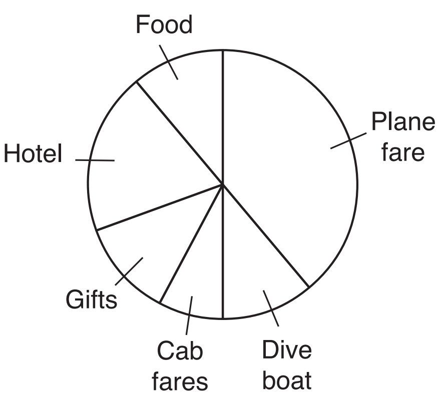
A



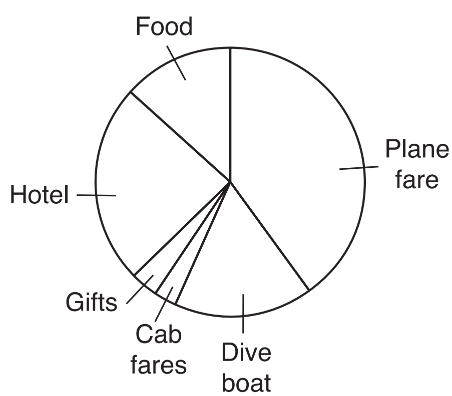
C



B



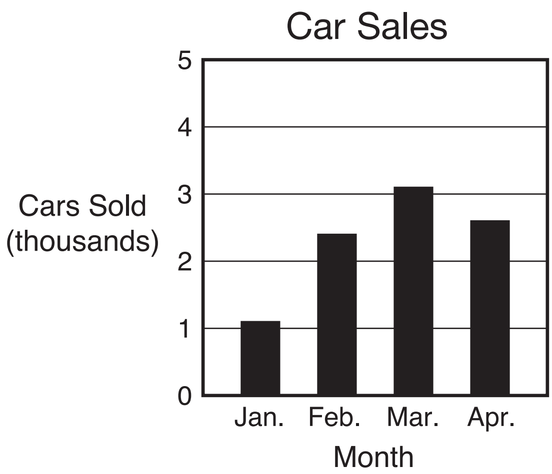
D



Answer Key: page 251

Question 96

The graph below represents car sales at a dealership for the first four months of the year.



Which of the tables below represents the data in the graph?

A

Month	Cars Sold
January	900
February	2400
March	2900
April	2400

B

Month	Cars Sold
January	1100
February	2400
March	3100
April	2600

C

Month	Cars Sold
January	1500
February	2900
March	3100
April	2400

D

Month	Cars Sold
January	1100
February	1900
March	2900
April	2400

Answer Key: page 251

Question 97

The total number of hours Ava spent studying each week for the first 8 weeks of school are shown in the table below.

Week	Hours Studying
1	4
2	7
3	4
4	7
5	7
6	6
7	7
8	7

Which measure should Ava use to show the number of hours she most frequently spends studying in a school week?

A Mean

B Median

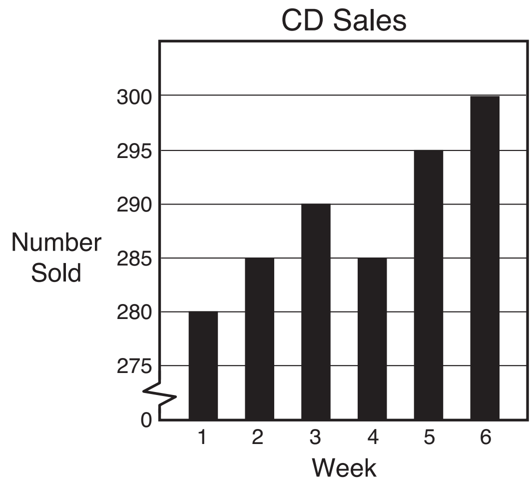
C Mode

D Range

Answer Key: page 251

Question 98

The graph shows CD sales at a music store for 6 consecutive weeks.



Based on the data in the graph, which of the following conclusions is true?

A Sales increased at a constant rate each week over the six-week period.

B The store sold an average of approximately 289 CDs per week over the six-week period.

C Three times as many CDs were sold during the sixth week as during the first week.

D The store sold more than 1800 CDs during the six-week period.

Answer Key: page 251

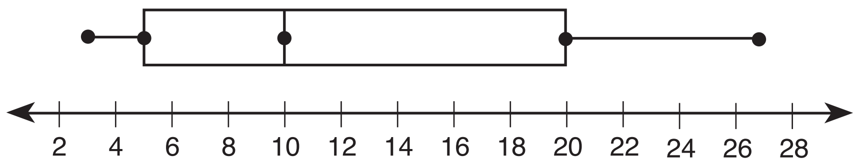
Question 99

A set of data is shown below.

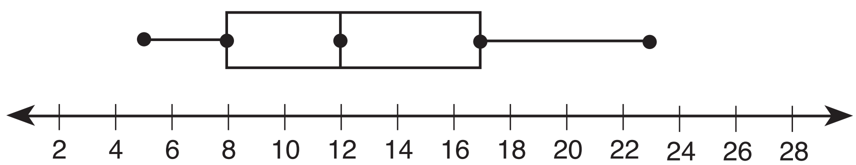
12, 5, 3, 17, 27, 23, 8, 5

Which of the following box and whisker plots best represents these data?

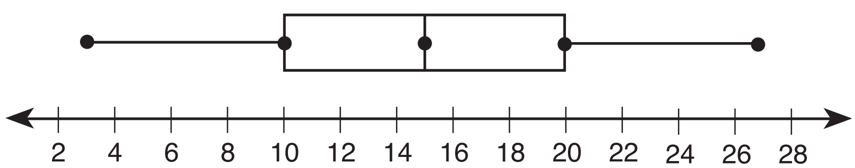
A



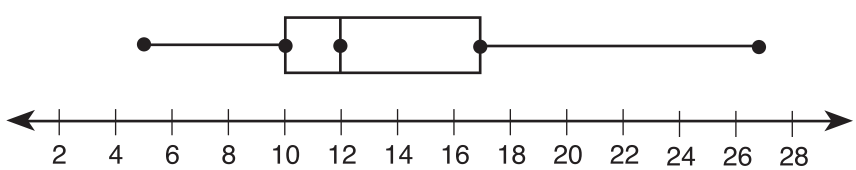
B



C



D



Answer Key: page 251

Objective 10

The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.

For this objective you should be able to

- apply mathematics to everyday experiences and activities;
- communicate about mathematics; and
- use logical reasoning.

How Do You Apply Math to Everyday Experiences?

Suppose you want to compute the likelihood of winning an election based on the results of a random survey. Or suppose you need to estimate the volume of a container based on its dimensions. Finding the solution to problems such as these often requires the use of math.

Solving problems involves more than just arithmetic; logical reasoning and careful planning also play very important roles. The steps in problem solving include understanding the problem, making a plan, carrying out the plan, and evaluating the solution to determine whether it is reasonable.

The Bradley family is planning a summer vacation. They expect to drive a total of 250 to 300 miles. Their car gets 20 miles per gallon of gasoline, and gas is expected to cost from \$2.95 per gallon to \$3.15 per gallon. Based on this information, what is the least amount the Bradley family should expect to spend on fuel for their vacation? What is the greatest amount?

- What information is given?

  length of trip: 250 to 300 miles

  gas mileage: 20 mpg

  cost of gas: \$2.95/gal to \$3.15/gal

- What do you need to find?

  You need to find the number of gallons of gas the Bradleys could use and, based on those figures, the total cost of the gasoline.

- Find the number of gallons of gas that could be used—minimum and maximum amounts.

  Divide the number of miles to be traveled by the rate the gas will be used, 20 mpg.

  Calculate first using the minimum figure for the length of the trip, 250 miles, and then using the maximum, 300 miles.

- Find the minimum and maximum possible cost of the gasoline.

  Multiply the cost of gasoline by the number of gallons to be used.

  Calculate first using the minimum cost of the gasoline, $2.95/gal, and then using the maximum, $3.15/gal.

- Find the number of gallons of gas that will be used.

Minimum Calculation	Maximum Calculation
250 miles	300 miles
250 miles ÷ 20 mpg = 12.5 gallons	300 miles ÷ 20 mpg = 15 gallons

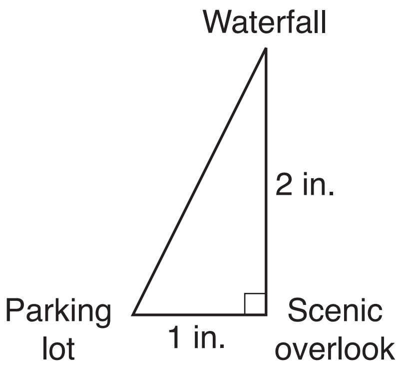
- Find the cost of the gas that will be used.

Minimum Calculation	Maximum Calculation
12.5 gallons at $2.95/gal	15 gallons at $3.15/gal
12.5 gallons • $2.95/gal = $36.88	15 gallons • $3.15/gal = $47.25

The least amount the Bradley family should expect to spend on gasoline for their vacation is $36.88, and the greatest amount is $47.25.

Try It

A group of hikers at a state park follow the trail shown on the map below. They hike from the parking lot to the waterfall and then to a scenic overlook. From the overlook the group hikes back to the parking lot.



If 1 inch on the map equals 2 miles, to the nearest tenth of a mile how far did the group hike?

What information are you given in the problem?

The distance from the waterfall to the overlook is ______ inches on the map.

The distance from the overlook to the parking lot is

______ inch on the map.

The scale used on the map is ______ inch = ______ miles.

You need to find the distance in miles from the parking lot to the waterfall.

The hikers' path forms a ________________________ triangle.

The distance from the parking lot to the waterfall is the

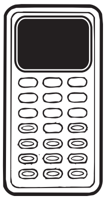
________________________ of the triangle.

Find this distance on the map using the Pythagorean Theorem.

_____________ this distance to the two known distances on the map.

Use the map's scale to find the actual distance the group hiked.

See Objective 8, page 177, for more information about the Pythagorean Theorem.



Substitute the values for a and b into the Pythagorean Theorem and solve for c.

______ + ______ = c^2

______ + ______ = c^2

______ = c^2

$\sqrt{\_\_\_\_\_\_} = c$

$c \approx$ ______ inches

On the map the total distance of the hike is

______ + ______ + ______ $\approx$ ______ inches.

Use the scale to convert the distance to miles. The distance in miles is ______ • ______ $\approx$ ______ $\approx$ ______ miles.

The question asks for the total distance to the nearest ________________________ of a mile.

The total distance the group hiked is ______ miles.

The distance from the waterfall to the overlook is 2 inches on the map. The distance from the overlook to the parking lot is 1 inch on the map. The scale used on the map is 1 inch = 2 miles.

The hikers' path forms a right triangle. The distance from the parking lot to the waterfall is the hypotenuse of the triangle. Add this distance to the two known distances on the map.

$$a^2 + b^2 = c^2$$

$$1^2 + 2^2 = c^2$$

$$5 = c^2$$

$$\sqrt{5} = c$$

$$c \approx 2.24 \text{ inches}$$

On the map the total distance of the hike is $2.24 + 1 + 2 \approx 5.24$ inches.

Use the scale to convert the distance to miles. The distance in miles is $5.24 \cdot 2 \approx 10.48 \approx 10.5$ miles. The question asks for the total distance to the nearest tenth of a mile. The total distance the group hiked is 10.5 miles.

What Is a Problem-Solving Strategy?

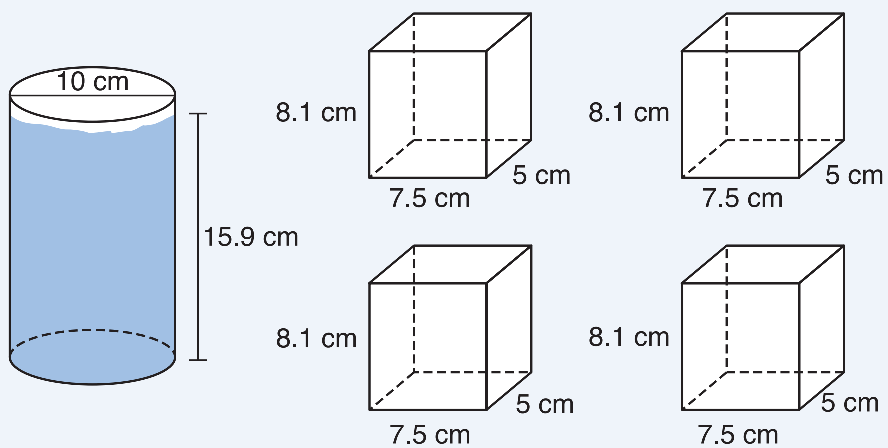
A **problem-solving strategy** is a plan for solving a problem. Different strategies work better for different types of problems. Sometimes you can use more than one strategy to solve a problem. As you practice solving problems, you will discover which strategies you prefer and which work best in various situations.

Some problem-solving strategies include

- drawing a picture;
- looking for a pattern;
- guessing and checking;
- acting it out;
- making a table;
- working a simpler problem; and
- working backward.

Alfonso is pouring liquid gelatin from a container into four small gelatin molds. The liquid gelatin is in a cylindrical container with a diameter of 10 centimeters. It fills the container to a level of 15.9 cm. Each of the gelatin molds is a rectangular prism that is 5 cm by 7.5 cm by 8.1 cm. Will Alfonso be able to transfer all of the gelatin from the cylindrical container into the four molds? If not, how many cubic centimeters of gelatin will remain after he fills the molds?

Draw a picture of the containers and label the dimensions.



- The dimensions of the cylinder and the dimensions of the prisms are given in the problem.
- You need to find the volume of the cylinder and the combined volume of the four rectangular prisms.

- Find the volume of the cylinder. Use the formula for the area of a circle, $A = \pi r^2$, to find the area of the circular base. The diameter is 10 cm, so the radius, r, is 5 cm. Use the area of the base to calculate the volume of the gelatin using the formula $V = Bh$.

$$A = \pi r^2$$
$$A = \pi \cdot 5^2$$
$$A = \pi \cdot 25$$
$$A \approx 78.5 \text{ cm}^2$$

$$V = Bh$$
$$V \approx 78.5 \cdot 15.9$$
$$V \approx 1248.15 \text{ cm}^3$$

The volume of the gelatin is approximately 1248 cm^3.

- Find the combined volume of the four rectangular prisms. The area of the rectangular base equals its length times its width.

$$B = 5 \cdot 7.5 = 37.5 \text{ cm}^2$$
$$V = Bh$$
$$V = 37.5 \cdot 8.1$$
$$V = 303.75 \text{ cm}^3$$

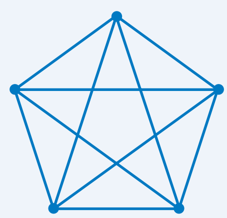
The total volume of the four smaller containers is $4 \cdot 303.75 = 1215 \text{ cm}^3$.

Since $1215 < 1248$, the liquid gelatin will not fit in the four containers. There will be about 33 cubic centimeters $(1248 - 1215)$ of liquid gelatin remaining.

At a business meeting each person in the room shakes hands with every other person. If there are 5 people at the meeting, how many handshakes take place?

One way to solve this problem is to draw a picture.

Draw 5 points representing the 5 people.



To model the handshakes, draw a line segment connecting each point to the four other points. Each point represents one person.

Count the number of line segments. There are 10 line segments.

At the meeting, ten handshakes take place.

Try It

A store sells boxes of candy wrapped in decorative paper. Each box is in the shape of a rectangular prism with the following dimensions: length $6\frac{5}{8}$ inches, width $4\frac{1}{4}$ inches, and height $2\frac{15}{16}$ inches. Wrapping a box requires about 20% more paper than the box's surface area. Approximately how many square feet of paper would be needed to wrap 200 boxes of candy?

Estimate the answer by rounding the dimensions of the box to the nearest inch.

$6\frac{5}{8}$ inches ≈ ______ inches

$4\frac{1}{4}$ inches ≈ ______ inches

$2\frac{15}{16}$ inches ≈ ______ inches

Find the surface area of one candy box.

Each surface is shaped like a ______________________.

Find the surface area by finding the ______________________ of the areas of all the surfaces.

S = 2(______ • ______) + 2(______ • ______) + 2(______ • ______)

= ______ square inches

Multiply by ______ to find the combined surface area of all the boxes.

______ in.^{2} • ______ = ___________ in.^{2}

There are 12 inches in 1 foot and 144 square inches in 1 square foot.

Divide by ______ to convert the surface area to square feet.

___________ in.^{2} ÷ 144 ≈ ___________ ≈ 170 ft^2

It takes 20% more paper to wrap a box than its surface area. Find 20% of ______.

20% of ______ = 0.20(______) = ______ ft^2

Find the total number of square feet of paper needed to wrap the candy boxes.

______ ft^2 + ______ ft^2 = ______ ft^2

The store would need approximately ______ square feet of paper to wrap 200 candy boxes.

$$6\frac{5}{8} \text{ inches} \approx 7 \text{ inches}$$

$$4\frac{1}{4} \text{ inches} \approx 4 \text{ inches}$$

$$2\frac{15}{16} \text{ inches} \approx 3 \text{ inches}$$

Each surface is shaped like a rectangle. Find the surface area by finding the sum of the areas of all the surfaces.

$$S = 2(7 \cdot 4) + 2(4 \cdot 3) + 2(7 \cdot 3) = 122 \text{ square inches}$$

Multiply by 200 to find the combined surface area of all the boxes.

$$122 \text{ in.}^2 \cdot 200 = 24{,}400 \text{ in.}^2$$

Divide by 144 to convert the surface area to square feet.

$$24{,}400 \text{ in.}^2 \div 144 \approx 169.44 \approx 170 \text{ ft}^2$$

It takes 20% more paper to wrap a box than its surface area. Find 20% of 170.

$$20\% \text{ of } 170 = 0.20(170) = 34 \text{ ft}^2$$

$$170 \text{ ft}^2 + 34 \text{ ft}^2 = 204 \text{ ft}^2$$

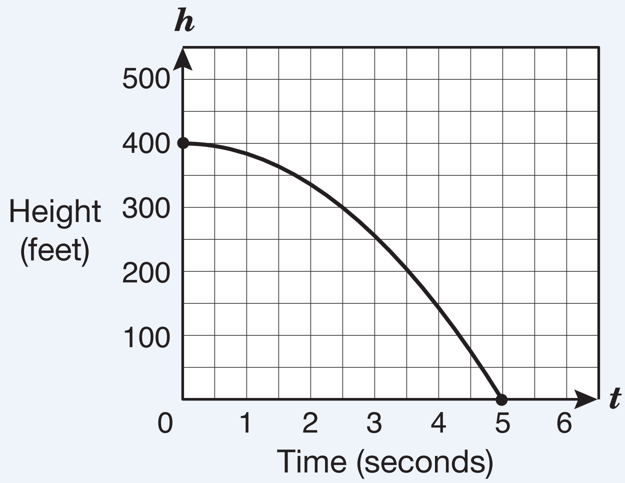
The store would need approximately 204 square feet of paper to wrap 200 candy boxes.

How Do You Communicate About Mathematics?

It is important to be able to rewrite a problem using mathematical language and symbols. The words in the problem will give clues about the operations that you will need in order to solve the problem. In some problems it may be necessary to use algebraic symbols to represent quantities and then use equations to express the relationships between the quantities. In other problems you may need to represent the given information using a table or graph.

See Objective 5, page 110, for more information about quadratic functions.

The function $h = -16t^2 + 400$ represents the height in feet, h, of an object dropped from a height of 400 feet in terms of t, the number of seconds it falls. The following graph represents this relationship.



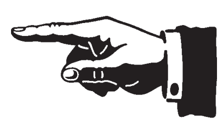
Why is the graph of this relationship restricted to Quadrant I?

All points in Quadrants II, III, and IV have at least one negative coordinate. If this graph were extended into those quadrants, then either an h or t value would be negative, or both.

In this problem the variable t can only be positive because it represents the number of seconds during which the object falls. The number of seconds begins at 0. It ends at some positive value, the number of seconds when the object hits the ground. The number of seconds cannot be negative.

In this problem the variable h can only have positive values because it represents the height of the object above the ground. Height begins at 400 feet and ends at 0 feet, the height of the object when it hits the ground. The object is never below the ground, so h is never negative.

Since neither t nor h can be negative, this graph cannot be drawn in Quadrants II, III, or IV.



Do you see that . . .

Try It

The following table summarizes the number of points scored per game by a player on the Central High School boys' basketball team for the first 10 games of the season.

Basketball Scoring Record

Game	Points Scored
1	12
2	18
3	26
4	15
5	8
6	12
7	20
8	14
9	13
10	18

If the scoring record above is used to predict the number of points he is likely to score at his next game, what is the probability that he will score more points than his mean score for the first 10 games?

His mean score to date is the total number of points he has scored divided by the number of games he has played.

$$(12 + 18 + 26 + 15 + 8 + 12 + 20 + 14 + 13 + 18) \div \_\_\_\_\_\_$$
$$= \_\_\_\_\_\_ \div \_\_\_\_\_\_$$
$$= \_\_\_\_\_\_$$

To find the probability of scoring above his mean score, find the number of favorable outcomes, the number of games in which he

scored ________________ his mean.

In ______ out of the last 10 games, he scored over 15.6 points.

The probability that he will score more than 15.6 points is $\frac{\square}{10}$.

(12 + 18 + 26 + 15 + 8 + 12 + 20 + 14 + 13 + 18) ÷ 10 = 156 ÷ 10 = 15.6

To find the probability of scoring above his mean score, find the number of favorable outcomes, the number of games in which he scored above his mean. In 4 out of the last 10 games, he scored over 15.6 points. The probability that he will score more than 15.6 points is $\frac{4}{10}$.

How Do You Use Logical Reasoning as a Problem-Solving Tool?

You can use logical reasoning to find patterns in a set of data. You can then use those patterns to draw conclusions about the data.

Finding patterns involves identifying characteristics that objects or numbers have in common. Look for the pattern in different ways. A sequence of geometric objects may have some property in common. For example, they may all be rectangular prisms or all be dilations of the same object.

The following table shows a series of dilations of a rectangle.

Dilation	Length (inches)	Width (inches)
1	120	50
2	108	45
3	86.4	36
4	60.48	25.2

By what scale factor would the next rectangle in the series be dilated?

To find the pattern, you must first find the scale factor used in each dilation. To do so, find the ratio of corresponding sides.

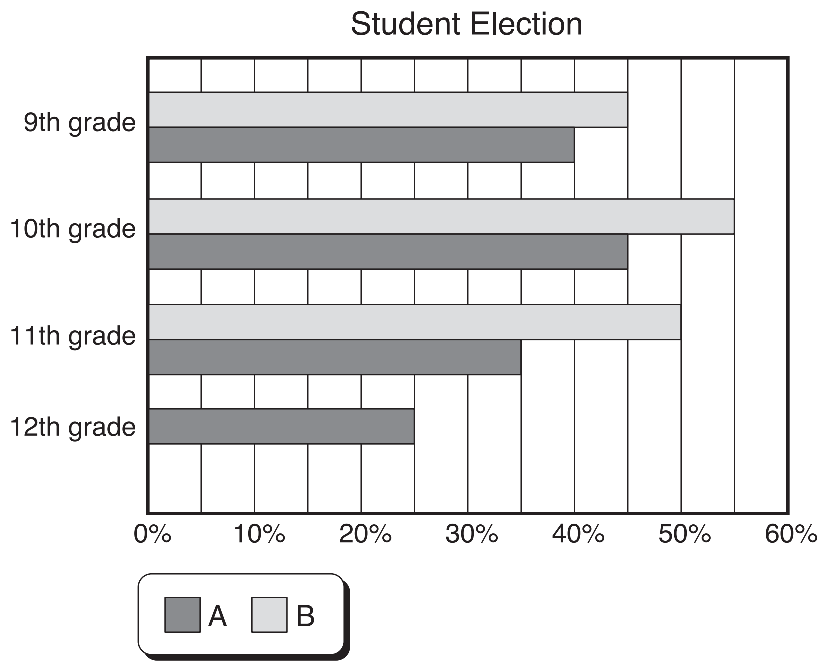
Dilation	Length (inches)	Width (inches)	Scale Factor
1	120	50	
2	108	45	$\frac{108}{120} = \frac{45}{50} = 0.9$
3	86.4	36	$\frac{86.4}{108} = \frac{36}{45} = 0.8$
4	60.48	25.2	$\frac{60.48}{86.4} = \frac{25.2}{36} = 0.7$

The scale factors are decreasing by 0.1 at each step. If the pattern continues, the scale factor for the next dilation should be $0.7 - 0.1 = 0.6$.

See Objective 6, page 148, for more information about dilations.

Try It

The graph below summarizes the percent of students from each grade who voted for Candidate A and Candidate B, the top two candidates in a school election.



If the voting pattern established by the 9th, 10th, and 11th graders continues, how many votes should Candidate B expect to get from the 320 twelfth graders who voted?

Look for a pattern in the data.

The percent of 9th-grade students voting for Candidate B was ______% more than the percent voting for Candidate A.

The percent of 10th-grade students voting for Candidate B was ______% more than the percent voting for Candidate A.

The percent of 11th-grade students voting for Candidate B was ______% more than the percent voting for Candidate A.

If the pattern continues, the percent of 12th-grade students voting for Candidate B should be ______% more than the percent voting for Candidate A.

If ______% of the 12th graders voted for Candidate A, then

______% + 20% = ______% should vote for Candidate B.

Find ______% of 320 students. Write a proportion.

$$\frac{\square}{100} = \frac{x}{\square}$$

$$100x = \_\_\_\_\_\_ \cdot \_\_\_\_\_\_$$

$$100x = \_\_\_\_\_\_$$

$$x = \_\_\_\_\_\_$$

Candidate B should get ______ votes from the 12th graders.

The percent of 9th-grade students voting for Candidate B was 5% more than the percent voting for Candidate A. The percent of 10th-grade students voting for Candidate B was 10% more than the percent voting for Candidate A. The percent of 11th-grade students voting for Candidate B was 15% more than the percent voting for Candidate A.

If the pattern continues, the percent of 12th-grade students voting for Candidate B should be 20% more than the percent voting for Candidate A. If 25% of the 12th graders voted for Candidate A, then 25% + 20% = 45% should vote for Candidate B. Find 45% of 320 students.

$$\frac{45}{100} = \frac{x}{320}$$

$$100x = 45 \cdot 320$$

$$100x = 14{,}440$$

$$x = 144$$

Candidate B should get 144 votes from the 12th graders.

The solution to a problem can be justified by identifying the mathematical properties or relationships that produced the answer. You should have a reason for drawing a conclusion, and you should be able to explain that reason.

The table below shows the number of people who owned homes in Williamstown for several different years.

Home Ownership in Williamstown

Year	Number
2004	22,500
2005	24,750
2006	27,225
2007	28,300

Did the number of people who owned homes increase by the same percent each year during this period?

First check whether the number of people who owned homes increased each year. Then check whether the percent increase is the same for each year.

- To find the percent increase from 2004 to 2005, subtract the number of people owning homes in 2004 from the number owning homes in 2005. Then divide by the number owning homes in 2004.

$$\text{Percent increase} = \frac{24{,}750 - 22{,}500}{22{,}500} = \frac{2{,}250}{22{,}500} = 0.10 = 10\%$$

- To find the percent increase from 2005 to 2006, subtract the number of people owning homes in 2005 from the number owning homes in 2006. Then divide by the number owning homes in 2005.

$$\text{Percent increase} = \frac{27{,}225 - 24{,}750}{24{,}750} = \frac{2{,}475}{24{,}750} = 0.10 = 10\%$$

- To find the percent increase from 2006 to 2007, subtract the number of people owning homes in 2006 from the number owning homes in 2007. Then divide by the number owning homes in 2006.

$$\text{Percent increase} = \frac{28{,}300 - 27{,}225}{27{,}225} = \frac{1{,}075}{27{,}225} \approx 0.0395 \approx 4\%$$

The percent increase from 2006 to 2007 is less than the percent increase for the previous years. The number of people who owned homes did not increase by the same percent each year.

Now practice what you've learned.

Question 100

Vickie is comparing the cost of a DVD player at two different stores. The DVD player costs $119 at Store A and $138 at Store B. Vickie has a 15%-off coupon for Store B, and Store A is having a 10%-off sale. Which statement best describes the difference in price of the DVD player at the two stores after discounts?

A The DVD player costs $10.20 less at Store B than at Store A.

B The DVD player costs $23.05 more at Store B than at Store A.

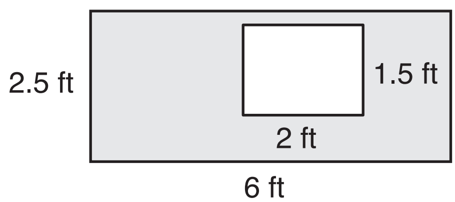
C The DVD player costs $10.20 more at Store B than at Store A.

D The DVD player costs $23.05 less at Store B than at Store A.

Answer Key: page 252

Question 101

Jessica is decorating a section of one wall of her kitchen with ceramic tiles. The area she is decorating is represented by the shaded area in the diagram below. She is using 3-inch square tiles. The tiles are sold in boxes of 100.



How many boxes of tiles will Jessica need to cover that section of the wall?

A 3

B 1

C 4

D 2

Answer Key: page 252

Question 102

A small company that manufactures staplers estimates that c, the cost per day in dollars of producing n staplers, is given by the formula $c = 1.12n + 300$, where $300 represents the fixed costs associated with operating the shop for a day. By about what percent would the cost of producing 250 staplers increase if the company's fixed costs increased by 15%?

A 15%

B 8%

C 18%

D 6%

Answer Key: page 252

Question 103

Jessie and Philippe are on opposite ends of a road that is 5 miles long. Jessie is walking toward Philippe at 4 miles per hour, and Philippe is riding his bike toward Jessie at 6 miles per hour. In how many minutes will they meet each other?

A 120 min

B 50 min

C 30 min

D 2 min

Answer Key: page 253

Question 104

As a candle burns, its height decreases. Matt's science class measured the height of a candle as it burned. They discovered that h, its height in inches, could be represented by the function $h = 12 - 0.1m$, where m equals the number of minutes the candle burns.

Which of the following statements is not true?

A There is a linear relationship between the candle's height and the number of minutes it burns.

B The candle was 12 inches tall when it started burning.

C The height of the candle is directly proportional to the number of minutes it burns.

D The candle burned for at most 120 minutes.

Answer Key: page 253

Question 105

Which problem can be solved using the equation $3x + 40 = 100$?

A Carrie bought a pair of shoes for \$40, a shirt, and a pair of pants that cost twice as much as the shirt. She spent a total of \$100. What was the cost of the shirt?

B The perimeter of a rectangle is 100 units. The length of the rectangle is 40 units more than three times the width. What is the width?

C Alex deposited \$100 at a 3% yearly interest rate. After how many years will he earn \$40 in interest?

D The student council spent \$40 on cups with the school logo. The cups sell for \$3 each. How many cups must the council sell to make a profit of \$100?

Answer Key: page 253

Question 106

Joyce has been given two linear functions, $f(x) = 2(x - 3) + 7$ and $g(x) = \frac{1}{2}x + 7$. She wants to find the values of x for which $f(x) \geq g(x)$.

Which of the following would be a reasonable strategy Joyce could use to solve the problem?

A Write the two functions in slope-intercept form and compare their slopes.

B Write the two functions in slope-intercept form and compare their intercepts.

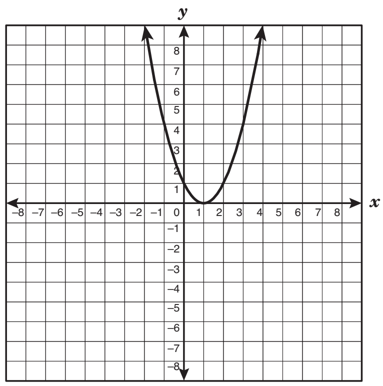
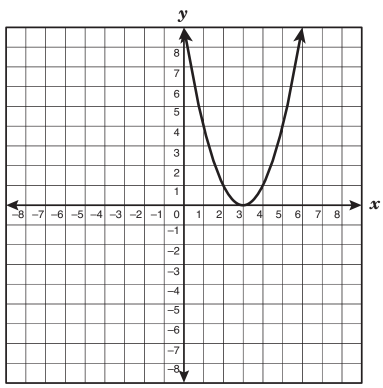
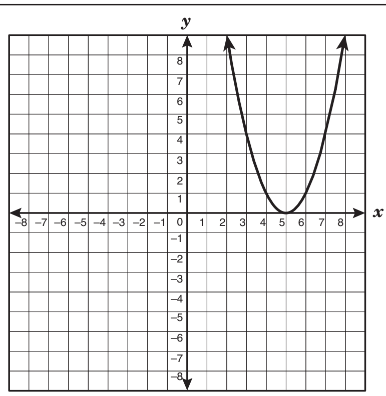
C Solve the inequality $2x + 1 \geq \frac{1}{2}x + 7$.

D Graph the two functions and see which graph has a positive slope.

Answer Key: page 253

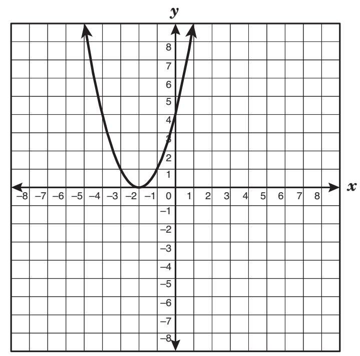
Question 107

The table below shows the graphs of three related quadratic functions.

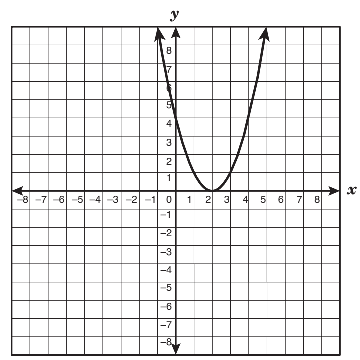
Function	Graph
$y = (x - 1)^2$	
$y = (x - 3)^2$	
$y = (x - 5)^2$	

Which of the following is most likely the graph of $y = (x - 2)^2$?

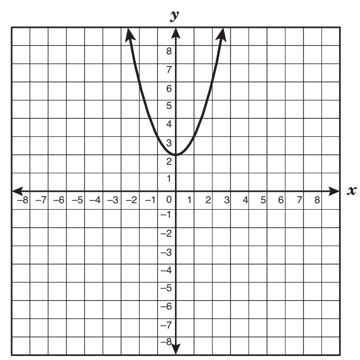
A



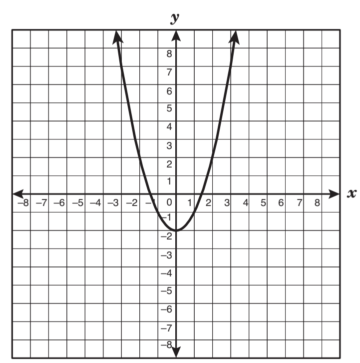
B



C



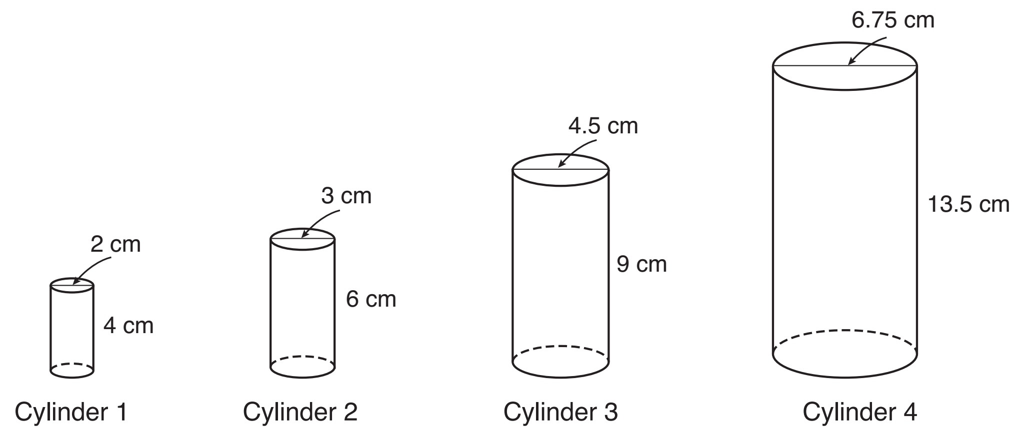
D



Answer Key: page 253

Question 108

Look at the pattern of the cylinders below.



If the pattern continues, which of the following would be a correct step to find the volume of the next cylinder in this series?

A Increase the diameter and height of the previous cylinder by a scale factor of 1.5.

B Increase the diameter and height of the previous cylinder by a scale factor of $(1.5)^2$.

C Increase the diameter and height of the previous cylinder by a scale factor of $(1.5)^3$.

D Increase the diameter and height of the previous cylinder by a scale factor of $\sqrt{1.5}$.

Answer Key: page 254

Question 109

Andrew is keeping a record of the number of people who visit his website each week. His site had twice as many visitors the second week as the first. The third week the site had 22 more visitors than the second week. The fourth week it had 2.5 times as many visitors as the first week. If x represents the number of visitors the site had during the first week, which expression could be used to find the mean number of visitors?

A $x + 2x + (2x + 22) + 2.5x$

B $\frac{x + 2x + (2x + 22) + 2.5x}{4}$

C $x + 2x + (x + 22) + 2.5x$

D $\frac{x + 2x + (2x + 22) + 2.5(x + 22)}{4}$

Answer Key: page 254

Mathematics Answer Key

Objective 1

Question 1 (page 24)

A Incorrect. The total salary she is paid for a week depends on the number of hours she works, which is the independent quantity.

B Incorrect. Her hourly rate of pay, \$5.50, is a constant.

C Correct. The total salary she is paid for a week is the dependent quantity. It depends on the number of hours she works, which is the independent quantity. Her hourly rate of pay, \$5.50, is a constant.

D Incorrect. The number of days worked in a week is not a variable in this problem.

Question 2 (page 24)

C Correct. The number of pretzels in the box, p, is determined by the volume of the box, V. Since p depends on the value of V, it is the dependent variable.

Question 3 (page 24)

C Correct. The ordered pairs (1, 1) and (1, 16) both belong to the relationship represented in C. If this relationship were a function, each first coordinate would be paired with exactly one second coordinate. In this choice the first coordinate, 1, is paired with two different second coordinates, 1 and 16. Choice C is not a function.

Question 4 (page 24)

B Correct.
Check the ordered pairs in the table to see whether they satisfy the rule for the function.

Independent Quantity	Rule $f(x) = 2x^2 + 2x + 1$	Dependent Quantity
0	$f(0) = 2 \cdot 0^2 + 2 \cdot 0 + 1$ $f(0) = 0 + 0 + 1$ $f(0) = 1$	1
1	$f(1) = 2 \cdot 1^2 + 2 \cdot 1 + 1$ $f(1) = 2 + 2 + 1$ $f(1) = 5$	5
2	$f(2) = 2 \cdot 2^2 + 2 \cdot 2 + 1$ $f(2) = 8 + 4 + 1$ $f(2) = 13$	13
3	$f(3) = 2 \cdot 3^2 + 2 \cdot 3 + 1$ $f(3) = 18 + 6 + 1$ $f(3) = 25$	25

The function $f(x) = 2x^2 + 2x + 1$ correctly matches the dependent and independent quantities in the table.

Question 5 (page 25)

D Correct. Jane's profit is equal to the difference between the amount of money she collects and her expenses. The variable x stands for the number of pets she cares for. The amount she collects for caring for x pets is $25x$. Her expenses are \$210. The difference, $25x - 210$, represents her profit. Therefore, the function $f(x) = 25x - 210$ best represents her net profit in terms of x, the number of pets she cares for.

Question 6 (page 25)

A Correct. To find the correct inequality, first state the problem in words using the variable, the constants, and the relationship among them.

The current volume, 75 decibels, plus the amount by which the volume is increased, q, must be less than 120 decibels, the level at which neighbors would complain.

Substitute the appropriate symbols for the quantities in an inequality: $75 + q < 120$.

Question 7 (page 25)

B Correct.
Check the ordered pairs in the table to see whether they satisfy the rule for the function.

Independent Quantity	Rule $f(x) = \frac{2}{5}x - 3$	Dependent Quantity
−10	$f(-10) = \frac{2}{5}(-10) - 3$ $f(-10) = -4 - 3$ $f(-10) = -7$	−7
0	$f(0) = \frac{2}{5}(0) - 3$ $f(0) = 0 - 3$ $f(0) = -3$	−3
10	$f(10) = \frac{2}{5}(10) - 3$ $f(10) = 4 - 3$ $f(10) = 1$	1
20	$f(20) = \frac{2}{5}(20) - 3$ $f(20) = 8 - 3$ $f(20) = 5$	5

The ordered pairs in table B correctly represent the function $f(x) = \frac{2}{5}x - 3$.

Question 8 (page 26)

C Correct.

To verify that a graph represents a function, find the coordinates of several points on the graph and substitute them into the rule for the function.

For example, $(0, -9)$ is a point on the graph in choice C.

If x is replaced with 0 and y is replaced with -9, is the functional rule satisfied? In other words, does $f(x) = -9$ when $x = 0$?

$$f(0) = x^2 - 9$$
$$f(0) = 0 - 9$$
$$f(0) = -9$$

Yes, the point $(0, -9)$ is an ordered pair in the function $f(x) = x^2 - 9$.

In the same way, check any other points on the graph. All the points on the graph in choice C satisfy the function $f(x) = x^2 - 9$.

The graph in choice C correctly represents the function $f(x) = x^2 - 9$.

Question 9 (page 27)

B Correct.

$y = x^2 - 3$ is the only equation that works for all the ordered pairs. For example, the ordered pair $(-2, 1)$ works since $(1) = (-2)^2 - 3$, which gives us $1 = 4 - 3$.

Question 10 (page 27)

D Correct.

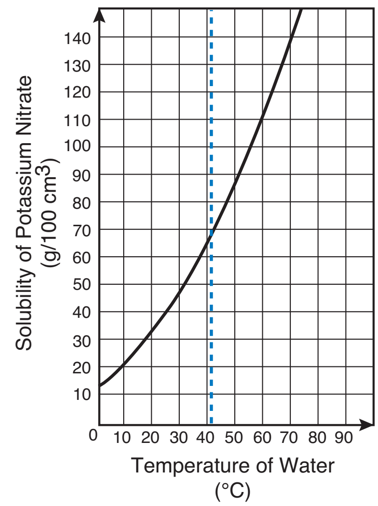
The monthly fee can be determined from how much the total charges increase. According to the table, the total charges increase \$60 every 2 months. This means that the monthly fee must be \$30. We can now use the table to find the registration fee. Since 1 month has total charges of \$170, and the monthly fee is \$30, \$170 − \$30 gives the registration fee of \$140.

Question 11 (page 28)

B Correct. The point on the graph that corresponds to 70 miles per hour is above 300 feet but less than 350 feet. So 325 feet must be the correct answer.

Question 12 (page 28)

D Correct. The horizontal scale of the graph represents temperature. Draw a vertical dashed line at 42°C to see where the line intersects the graph.



Read the corresponding value on the vertical axis, which is about 68 grams.

Question 13 (page 29)

C Correct.

Evaluate each of the formulas for 1,450 ft^2 and compare the results.

Community	Cost of Building a New Home	Evaluated for $f = 1{,}450$
R	$c = 15{,}000 + 80f$	$c = 15{,}000 + 80(1{,}450)$ $c = 15{,}000 + 116{,}000$ $c = 131{,}000$
S	$c = 25{,}000 + 75f$	$c = 25{,}000 + 75(1{,}450)$ $c = 25{,}000 + 108{,}750$ $c = 133{,}750$
T	$c = 60{,}000 + 50f$	$c = 60{,}000 + 50(1{,}450)$ $c = 60{,}000 + 72{,}500$ $c = 132{,}500$
V	$c = 40{,}000 + 65f$	$c = 40{,}000 + 65(1{,}450)$ $c = 40{,}000 + 94{,}250$ $c = 134{,}250$

The least expensive community in which to build a 1,450 ft^2 home would be Community R.

Objective 2

Question 14 (page 57)

A Correct. Because $y = 3x^2 + 4$ can be written in the form $y = ax^2 + bx + c$, it is a quadratic function. The quadratic parent function is $y = x^2$. NOTE: Choice B is the graph of $y = 3x^2 + 4$, but the question asks for the parent function.

Question 15 (page 58)

B Correct. The value of the account, v, depends on the number of years the money is in the bank, t. Therefore, v is the dependent variable. The range is the set of possible values for the dependent variable. The amount of money in the bank begins at $550 and increases each year thereafter. The minimum value for v is $550, but it has no upper limit. The range of the function is any value for v that is equal to or greater than $550, or $v \geq 550$.

Question 16 (page 58)

A Incorrect. The speed would be very slow at the top of the track and would increase as the roller coaster approached the bottom. The speed would then decrease as the roller coaster climbed back up the track. The graph, however, decreases and then increases. The graph does not match the description of the roller coaster's motion.

B Incorrect. The price of a stock, the dependent quantity in choice B, decreases and then increases like the graph, but the stock price decreases to only half its original value, not to 0. The line of the graph falls to 0, so it does not match this description.

C Correct. The bullet starts at a very high speed. It slows as it goes straight up until it reaches a maximum height, when its speed falls to 0. The bullet's speed then increases as it falls back to Earth. The graph decreases and increases in the same fashion. The graph matches this description.

D Incorrect. The race car must start from a speed of 0. Its speed would build to a maximum and stay near that speed for several laps. The speed of the race car would then go back to 0 for the fuel stop. The graph does not match this description.

Question 17 (page 59)

D Correct. This scatterplot does not show a negative trend. The systolic pressure does not decrease as the diastolic pressure increases.

Question 18 (page 60)

A Correct. The number of customers increases by 6 each day, and the amount of sales increases by $30. Continue the sequence until the amount of sales reaches $330. It will take 2 more days because $270 + 30 + 30 = 330$. The number of customers corresponding to this sales amount would be an increase of 6 per day for 2 days, or $30 + 6 + 6 = 42$ customers.

Question 19 (page 60)

D Correct. The volume of a rectangular prism (the refrigerator's shape) is equal to its length times its width times its height. If x represents the depth, or length, of the refrigerator, then $1.75x$ represents its width. The height of the refrigerator is 6 feet.

$$V = lwh$$
$$V = x \cdot 1.75x \cdot 6$$
$$V = (1.75 \cdot 6)(x \cdot x)$$
$$V = 10.5x^2$$

Question 20 (page 60)

C Correct.
Set n is the domain for this function, so we substitute those values in to get the range represented by the set {0, 2, 8, 18, ...}. The expression $2n^2$ is the only expression that works for all the values given. For example, $2(3)^2 = 2(9) = 18$.

Question 21 (page 60)

C Correct. One way to find the relationship between the terms in a sequence and their position in the sequence is to build a table.

Sequence 1, 3, 5, 7, 9, . . .

Position (n)	Value
1	1
2	3
3	5
4	7
5	9

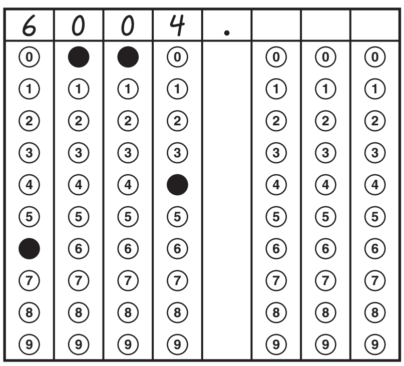
Test the values in the table against the rule in choice C. For example, the rule in choice C says that the third term in the sequence should be $2n - 1 = 2 \cdot 3 - 1$, or 5. When $n = 3$, the rule gives the correct term in the sequence. In the same way, the rule $2n - 1$ predicts each of the other values in the sequence.

Question 22 (page 61)

The correct answer is 6004. Use the functional rule, $P(x) = \frac{1}{2}x - 2$, to find the minimum number of loaves of bread Mr. Jones must sell to produce a profit of \$3000.

$$\frac{1}{2}x - 2 \geq 3000$$
$$\frac{1}{2}x \geq 3002$$
$$x \geq 6004$$

Mr. Jones must sell at least 6004 loaves of bread to have a profit of at least \$3000.



Question 23 (page 61)

C Correct. To find an equivalent expression, simplify the given expression by removing parentheses and combining like terms.

$$2x + 2(3x - 4) + 3(8x - 4) =$$
$$2x + (6x - 8) + (24x - 12) =$$
$$(2x + 6x + 24x) + (-8 - 12) =$$
$$32x - 20$$

Question 24 (page 61)

B Correct. $f(x)$ can be substituted for y.

Objective 3

Question 25 (page 89)

A Incorrect. The formula for the area of a circle, $A = \pi r^2$, involves a squared term, r^2. It is not a linear function.

B Correct. The formula for the perimeter of an equilateral triangle in terms of its side, s, is $P = 3s$. All variables are to the first power, so this is a linear function.

C Incorrect. The surface area of a cube with side length s is given by the formula $A = 6s^2$. The function involves a squared term, s^2. It is not a linear function.

D Incorrect. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$. The function involves the product of two independent variables, r and h, and includes a variable raised to the second power. It is not a linear function.

Question 26 (page 89)

B Correct.
One way to match the equation to its graph is to write the equation $2y - x = 10$ in slope-intercept form, $y = mx + b$.

$$2y - x = 10$$
$$2y = x + 10$$
$$y = \frac{1}{2}x + 5$$

In this form the slope, m, is $\frac{1}{2}$, and the y-intercept, b, is 5. Only the graph in choice B has a slope of $\frac{1}{2}$ and a y-intercept of 5.

Question 27 (page 90)

A Incorrect. The rate of change is the slope of the line graphed.

$$\text{Slope} = \frac{-3 - (-1)}{4 - 1} = -\frac{2}{3}$$

The graph represents a rate of change of $-\frac{2}{3}$.

B Incorrect. To find the slope of the line, write the equation in slope-intercept form, $y = mx + b$. The equation becomes $y = -\frac{2}{3}x + 4$, so the slope is $-\frac{2}{3}$. The equation represents a rate of change of $-\frac{2}{3}$.

C Correct. To find the rate of change represented in the table, compare the difference between any two y-values to the corresponding difference in

x-values. For example, use the two points (1, 4) and (−3, 10).

$$\text{Rate of change} = \frac{10 - 4}{-3 - 1} = \frac{6}{-4} = -\frac{3}{2}$$

The table does not represent a rate of change of $-\frac{2}{3}$.

D Incorrect. The equation is already in slope-intercept form, $y = mx + b$. The value of m is $-\frac{2}{3}$. Therefore, the slope is $-\frac{2}{3}$. The equation represents a rate of change of $-\frac{2}{3}$.

Question 28 (page 90)

B Correct. To find the maximum number of grams of potatoes, look at the point on the graph where the number of grams of beef equals zero, the x-intercept. The graph intersects the x-axis at the point (450, 0). Therefore, the maximum number of grams of potatoes that you could eat to obtain 500 calories is 450 grams.

Question 29 (page 91)

C Correct. The y-intercept is (0, 200). The y-coordinate, 200, represents 200 units on the y-axis. The units on the y-axis are thousands of dollars. The company's initial assets are 200 • \$1,000 = \$200,000.

The slope, or rate of growth, is 50 units in the y direction for every unit in the x direction. Thus, the slope is $\frac{50}{1}$, or 50. The y-axis units are thousands of dollars, and the x-axis units are years. The expected growth rate is \$50,000 per year.

Question 30 (page 91)

B Correct.

A line parallel to $4x - 5y = -2$ will have the same slope. Put $4x - 5y = -2$ into slope-intercept form:

$$\begin{aligned} 4x - 5y &= -2 \\ -4x \qquad & \quad -4x \\ \frac{-5y}{-5} &= \frac{-4x}{-5} - \frac{2}{-5} \\ y &= \frac{4}{5}x + \frac{2}{5} \end{aligned}$$

So the slope of the line is $\frac{4}{5}$. This does not eliminate any choices yet, since all the answers have a slope of $\frac{4}{5}$. Now substitute the point (−3, −8) and the slope $\frac{4}{5}$ into $y = mx + b$ to find the y-intercept of the line.

$$\begin{aligned} (-8) &= \frac{4}{5}(-3) + b \\ -8 &= -\frac{12}{5} + b \\ +\frac{12}{5} \quad & \quad +\frac{12}{5} \\ -\frac{40}{5} + \frac{12}{5} &= b \\ \frac{-28}{5} &= b \end{aligned}$$

Therefore the correct equation is

$$y = \frac{4}{5}x - \frac{28}{5}.$$

Question 31 (page 91)

A Correct.

Both graphs are lines. To determine their relationship, compare the slope of each graph. The first equation, $y = \frac{3}{4}x - 4$, is written in slope-intercept form, $y = mx + b$. Its slope is the value of m, or $\frac{3}{4}$. The second equation has a slope of 1. Since $|1| > |\frac{3}{4}|$, the slope of the second line is greater than that of the first line. Therefore, the second line is steeper than the first line.

Question 32 (page 91)

D Correct.

Use the slope formula to find the slope of the line that passes through the two points (2, −5) and (4, 3).

$$m = \frac{-5 - 3}{2 - 4} = \frac{-8}{-2} = 4$$

The slope of the line is 4.

Substitute 4 for m and the coordinates of the point (2, −5) for x and y in the slope-intercept form of the equation of a line.

$$\begin{aligned} y &= mx + b \\ -5 &= 4 \cdot 2 + b \\ -5 &= 8 + b \\ b &= -13 \end{aligned}$$

Substituting $m = 4$ and $b = -13$ into the slope-intercept form of the equation of a line results in $y = 4x - 13$. The equation $y = 4x - 13$ is equivalent to the equation in choice D.

$$\begin{array}{rl} y = & 4x - 13 \\ -4x = & -4x \\ \hline -4x + y = & -13 \end{array}$$

Question 33 (page 91)

D Correct.
One way to find the two intercepts is to write the equation in standard form, $Ax + By = C$. Then replace x with 0 to determine the y-intercept and replace y with 0 to determine the x-intercept.

The equation $2x = 9 - 3y$ in standard form is $2x + 3y = 9$. Let $y = 0$.

$$2x + 3 \cdot 0 = 9$$
$$2x = 9$$
$$x = \frac{9}{2}$$

The x-intercept is $\frac{9}{2}$. The coordinates of the x-intercept are $(\frac{9}{2}, 0)$.
In the same way, let $x = 0$.

$$2 \cdot 0 + 3y = 9$$
$$3y = 9$$
$$y = 3$$

The y-intercept is 3. The coordinates of the y-intercept are (0, 3).

Question 34 (page 92)

A Correct. The slopes of the lines represent their rate of change, the price per cookie. The slopes of the two lines are equal, so the price per additional cookie remained the same. The first graph contains the point (12, 6), and the second graph contains the point (12, 7). The cost of the first dozen cookies increased from \$6 to \$7.

Question 35 (page 92)

A Correct. The number of miles Sammie walks is directly proportional to the number of minutes she walks. Write a direct-proportion equation.

$$y = kx$$

In this equation, y is the number of miles and x is the number of minutes. She walks 3 miles in 45 minutes. Substitute and solve for k, the proportionality constant.

$$y = kx$$
$$3 = k \cdot 45$$
$$\frac{3}{45} = \frac{k \cdot 45}{45}$$
$$k = \frac{1}{15}$$

The constant of proportionality is $\frac{1}{15}$.
The equation describing this situation is $y = \frac{1}{15}x$. Find the number of miles walked in 2.5 hours. Convert 2.5 hours to minutes so that the units are the same.

2.5 hours • 60 minutes per hour = 150 minutes.
Substitute 150 for x in the equation $y = \frac{1}{15}x$.
Solve the equation for y.

$$y = \frac{1}{15} \cdot 150$$
$$y = 10$$

At this rate Sammie would walk 10 miles in 2.5 hours.

Question 36 (page 93)

B Correct. The function represented by the graph is $f(x) = \frac{1}{3}x + 1$. So $\frac{1}{3}x + 1 = 0$ when $x = -3$.
Notice that this is also the x-coordinate of the x-intercept.

Objective 4

Question 37 (page 107)

C Correct. This is a problem about the perimeter of a rectangle. The width of the rectangle is given as w. The length will be five feet more than the width, so the length can be represented by $w + 5$. The formula for the perimeter of a rectangle is $P = 2w + 2l$. The perimeter of the rectangle can be represented by the equation $P = 2w + 2(w + 5)$.
She can afford 150 feet of fencing. The perimeter must be less than or equal to 150 feet. Use the symbol $\leq$ to write an inequality expressing this relationship.

$$P \leq 150$$
$$2w + 2(w + 5) \leq 150$$

Question 38 (page 107)

C Correct. Let m represent the amount of money she spent on movie rentals, c the cost of her car repairs, and l the cost of her lunches. Since Sharon spent five times as much on car repairs, c, as she did on movie rentals, m, you can write the equation $c = 5m$. Since she spent \$4 less on lunches, l, than on movie rentals, m, you can write the equation $l = m - 4$.

The sum of these expenses is \$80, so $m + c + l = 80$.

Substitute the expressions in terms of m for c and l and solve for m.

$$m + c + l = 80$$
$$m + 5m + m - 4 = 80$$
$$(m + 5m + m) - 4 = 80$$
$$7m - 4 = 80$$
$$7m = 84$$
$$m = 12$$

Sharon spent \$12 on movie rentals. The question asks how much her car repairs cost. Since $c = 5m$, her car expenses were $5 \cdot 12 = \$60$.

Question 39 (page 107)

A Correct. Represent the first number with x and the second number with y.

If the sum of the two numbers is 59, then $x + y = 59$.

If the difference between 2 times the first number, x, and 6 times the second number, y, is -34, then $2x - 6y = -34$.

Solve the system of equations.

$$x + y = 59$$
$$2x - 6y = -34$$

Addition method: Multiply the first equation by 6 to obtain two terms with opposite coefficients, 6 and -6.

$$6x + 6y = 354$$
$$2x - 6y = -34$$

Add the two equations to obtain one equation with one unknown.

$$8x = 320$$
$$x = 40$$

If $x = 40$, then substitute 40 into the first equation to find y.

$$x + y = 59$$
$$40 + y = 59$$
$$y = 19$$

The first number is 40, and the second number is 19.

Substitution method: Solve the first equation for y.

$$y = 59 - x$$

Substitute $(59 - x)$ for y in the second equation.

$$2x - 6y = -34$$
$$2x - 6(59 - x) = -34$$
$$2x - 354 + 6x = -34$$
$$8x - 354 = -34$$
$$8x = 320$$
$$x = 40$$

If $x = 40$, then substitute 40 into the first equation, $x + y = 59$, to find y.

$$y = 19$$

The first number is 40, and the second number is 19.

Question 40 (page 107)

C Correct. If Sid made 125 plain bagels and 25 garlic bagels, it would be represented by the point (25, 125). On the graph, the point (25, 125) is not in the shaded region that represents the solution to this inequality.

Question 41 (page 108)

D Correct. Let w = the width of the kennel. Ira must fence two widths plus the length of the garage. The total number of feet of fencing Ira will use is $w + w + 20 = 2w + 20$. The total cost of materials can be represented by the number of feet of fencing times the cost per foot of the materials, $4(2w + 20)$. If Ira has at most \$120 to spend on the project, then the inequality $4(2w + 20) \leq 120$ can be used to solve the problem.

$$4(2w + 20) \leq 120$$
$$8w + 80 \leq 120$$
$$8w \leq 40$$
$$w \leq 5$$

The kennel can be at most 5 feet wide.

Question 42 (page 108)

B Correct.
This question can be answered by solving the system algebraically, graphically, or by putting the equations into slope-intercept form and interpreting the slopes and intercepts of the lines. We will show only the algebraic method. Multiplying through the first equation by 3 gives us $18x - 6y = 21$. Multiplying through the second equation by 2 yields $-18x + 6y = 10$. Adding these two equations together gives us $0 = 31$, a contradiction. Therefore, there is no solution to this system. Graphically, the two lines would be parallel and would never intersect. They would have the same slope but different y-intercepts.

Question 43 (page 108)

A Correct. The cost of an adult ticket, a, is twice as much as the cost of a child ticket, c. Therefore, $a = 2c$. Sandra bought 3 adult tickets and 5 child tickets. The total cost of the tickets, \$48.40, is equal to 3 times a, the cost of an adult ticket, plus 5 times c, the cost of a child ticket. So $3a + 5c = 48.40$. To find the cost of each ticket, solve the following system of equations.

$$a = 2c$$
$$3a + 5c = 48.40$$

Use the substitution method because the first equation is already solved for a.

Substitute the expression $2c$ for a in the second equation and solve.

$$3a + 5c = 48.40$$
$$3(2c) + 5c = 48.40$$
$$6c + 5c = 48.40$$
$$11c = 48.40$$
$$c = 4.40$$

Since $a = 2c$, the cost of an adult ticket, a, is $2(\$4.40) = \$8.80$. The cost of an adult ticket is \$8.80, and the cost of a child ticket is \$4.40.

Question 44 (page 108)

A Correct. The store's profit is given in the equation $p = 0.25(s - 3000)$. Substitute the greatest and least projected values for the monthly sales, s, to find the expected range of values for the profit, p.

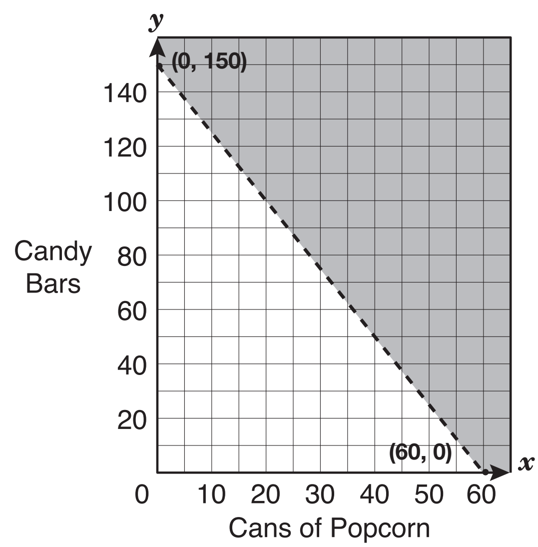
Solve both inequalities to find the possible range of values for p.

$p = 0.25(s - 3000)$	$p = 0.25(s - 3000)$
$p \geq 0.25(5000 - 3000)$	$p \leq 0.25(7000 - 3000)$
$p \geq 0.25(2000)$	$p \leq 0.25(4000)$
$p \geq 500$	$p \leq 1000$

This means that $p \geq 500$ and $p \leq 1000$, or $500 \leq p \leq 1000$.

Question 45 (page 109)

C Correct. The solution can be represented by graphing the inequality and identifying points in the shaded region.



Only the point (20, 50) is not in the shaded region of the graph.

If Brent sold 20 cans of popcorn, he would have raised $20 \cdot \$5 = \$100$. And 50 candy bars would have raised $50 \cdot \$2 = \$100$. He would have raised only \$200 in all, not more than \$300.

Question 46 (page 109)

D Correct. In general, the total cost of staying at the resort is equal to the cost of a night, n, times the number of nights, plus the cost of a meal, m, times the number of meals.

If a guest stays 2 nights and has 5 meals, it costs \$395. Represented by an equation in terms of n and m, this is $2n + 5m = 395$.

If a guest stays 5 nights and has 11 meals, it costs \$959. Represented by an equation in terms of n and m, this is $5n + 11m = 959$.

Question 47 (page 109)

D Correct. The total number of watermelons and cantaloupes bought is 13.

The number of watermelons, w, plus the number of cantaloupes, c, is 13.

$$w + c = 13$$

The total amount spent on watermelons is equal to \$2 times w, the number of watermelons, or $2w$.

The total amount spent on cantaloupes is equal to \$1 times c, the number of cantaloupes, or $1c$.

The total amount spent on watermelons and cantaloupes, \$20, is equal to the sum of these two amounts.

$$2w + c = 20$$

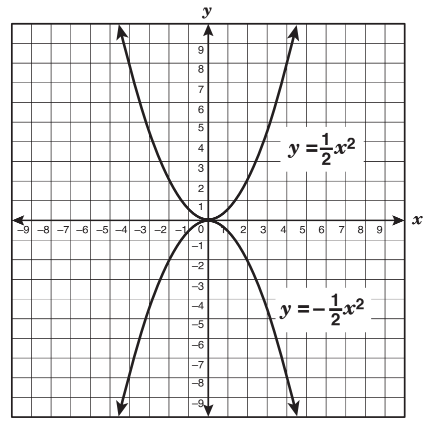
Objective 5

Question 48 (page 133)

A Correct. To answer this question, compare the coefficients of the two functions. Since $2 > \frac{1}{2}$, the graph of $y = 2x^2$ is narrower than the graph of $y = \frac{1}{2}x^2$.

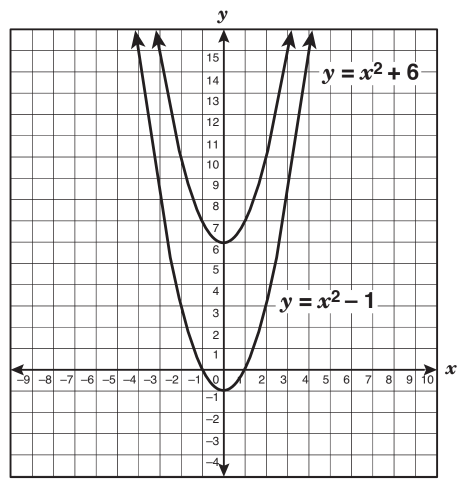
Question 49 (page 133)

B Correct.
The value of the parameter a in the function $y = \frac{1}{2}x^2$ is $\frac{1}{2}$. The value of a in the function $y = -\frac{1}{2}x^2$ is $-\frac{1}{2}$. Changing the sign of a results in a new graph that is a reflection of the original graph across the x-axis. The two graphs are congruent.



Question 50 (page 133)

D Correct.
The vertex of the graph of $y = x^2 + 6$ is 6 units above the origin, and the vertex of the graph of $y = x^2 - 1$ is 1 unit below the origin. The vertex of $y = x^2 + 6$ is 7 units above the vertex of $y = x^2 - 1$.



Question 51 (page 134)

B Correct. The maximum area is found at the highest point on the graph. The vertex is at (3, 9), which means the maximum area, 9 square yards, occurs when the width of the pigpen is 3 yards.

Question 52 (page 135)

A Incorrect. The horizontal axis represents time, not distance.

B Incorrect. The horizontal axis represents time, not distance. The interpretation of the graph as the path of the stone is not correct. In this instance, the path of the stone would not be a curving path, but a vertical line.

C Correct. The horizontal axis of the graph represents time in seconds from the time the stone is dropped. The stone's distance from the ground is 0 feet when the graph intersects the horizontal axis. The graph intersects this axis between 5 and 6, so the stone hits the ground between 5 and 6 seconds after it is dropped.

D Incorrect. The stone's distance from the ground does not decrease at a constant rate. If the distance decreased at a constant rate, the graph would be linear, not curved.

Question 53 (page 136)

B Correct. A zero of a function means a value of x where y equals zero. Looking at the table, notice that as x goes from -2 to -1, y goes from a positive value to a negative value. So somewhere in there, y has to be equal to zero. The other zero occurs when x is between 0 and 1 (when y goes from a negative value to a positive value), but this is not given as one of the answer choices.

Question 54 (page 136)

A Correct. One way to solve the equation $2x^2 - 15x - 16 = -11 - 24x$ is to first set the equation equal to 0.

$$2x^2 + 9x - 5 = 0$$

Factor the equation: $(2x - 1)(x + 5) = 0$

Set each factor equal to 0 and solve.

$$2x - 1 = 0 \quad \text{and} \quad x + 5 = 0$$

$$2x = 1 \qquad\qquad x = -5$$

$$x = \frac{1}{2}$$

Question 55 (page 136)

D Correct.
In the equation $x^2 - 4x - 5 = 0$, replace x first with -1 and then with 5 to see whether the equation is true.

Substitute $x = -1$	Substitute $x = 5$
$x^2 - 4x - 5 = 0$	$x^2 - 4x - 5 = 0$
$(-1)^2 - 4 \cdot -1 - 5 \stackrel{?}{=} 0$	$(5)^2 - 4 \cdot 5 - 5 \stackrel{?}{=} 0$
$1 + 4 - 5 \stackrel{?}{=} 0$	$25 - 20 - 5 \stackrel{?}{=} 0$
$0 = 0$	$0 = 0$

Both replacements make this equation true. Only the equation in choice D has -1 and 5 as solutions.

Question 56 (page 136)

A Correct.
Use the quadratic formula to determine the roots of the equation. Substitute $a = 1$, $b = -4$, and $c = 2$ into the quadratic formula.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The following result is obtained.

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$

$$\frac{4 \pm \sqrt{16 - 8}}{2}$$

$$\frac{4 \pm \sqrt{8}}{2}$$

Since $\sqrt{8} \approx 2.8$, substitute this value in the formula.

$$\frac{4 + 2.8}{2} \approx 3.4 \qquad \frac{4 - 2.8}{2} \approx 0.6$$

The roots of the equation are approximately 3.4 and 0.6, with 0.6 being the smaller root. The number 0.6 lies between the integers 0 and 1.

Question 57 (page 137)

A Correct. The vertex is the point $(-3, -1)$.

Question 58 (page 138)

D Correct. Substitute the given expression for the radius into the formula for volume of a sphere.

$$V = \frac{4}{3}\pi(3x^2y)^3$$

Following the order of operations, first simplify the expression for the radius cubed, $(3x^2y)^3$. When raising a term with an exponent to a power, multiply the exponents.

$$V = \frac{4}{3}(3x^2y)^3\pi$$

$$V = \frac{4}{3} \cdot 3^3x^{2 \cdot 3}y^3 \cdot \pi$$

$$V = \frac{4}{3} \cdot 27x^6y^3 \cdot \pi$$

$$V = 36x^6y^3\pi$$

Objective 6

Question 59 (page 153)

C Correct. Use the graph to find the coordinates of the triangle: A (0, −2), B (2, 2), and C (3, −2).

The length of side AC is $3 - 0 = 3$, and the length of side $A'C'$ is $6 - 0 = 6$. The ratio of these lengths is $\frac{6}{3} = 2$, which is the given dilation factor. Only the triangle in choice C has its side lengths in a 2:1 ratio to those of the original triangle.

Question 60 (page 154)

A Correct. The side lengths of the original rectangle and the dilated rectangle are as follows:
$RS = TV = 3$, $ST = RV = 6$, $R'S' = T'V' = 1$, and $S'T' = R'V' = 2$.

To find the scale factor, choose one pair of corresponding sides.

Find the ratio of the side length of the dilated figure to that of the original figure.

$$\frac{R'V'}{RV} = \frac{2}{6} = \frac{1}{3}$$

The side length of the dilated figure is $\frac{1}{3}$ the side length of the original figure. The scale factor is $\frac{1}{3}$.

Question 61 (page 154)

A Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{1} = 1 \qquad \frac{1.5}{3} = \frac{1}{2} \qquad \frac{2}{4} = \frac{1}{2}$$

The ratios are not all equal.

B Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{2} = \frac{1}{2} \qquad \frac{1.5}{4} = \frac{3}{8} \qquad \frac{2}{6} = \frac{1}{3}$$

The ratios are not all equal.

C Incorrect. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios.

$$\frac{1}{4} = \frac{1}{4} \qquad \frac{1.5}{4.5} = \frac{3}{9} = \frac{1}{3} \qquad \frac{2}{5} = \frac{2}{5}$$

The ratios are not all equal.

D Correct. If one triangle is a dilation of the other, then the two triangles are similar. The lengths of the corresponding sides of similar triangles must be in equal ratios. Only the side lengths in choice D form a proportion with the given sides.

$$\frac{1}{4} = \frac{1}{4} \qquad \frac{1.5}{6} = \frac{3}{12} = \frac{1}{4} \qquad \frac{2}{8} = \frac{1}{4}$$

Question 62 (page 154)

A Correct. The point (5, 6) in the given pentagon is 5 units to the right of the y-axis. The point (−5, 6) is 5 units to the left of the y-axis. The point (−5, 6) is the reflection of the point (5, 6) across the y-axis.

The point (4, 1) in the given pentagon is 4 units to the right of the y-axis. The point (−4, 1) is 4 units to the left of the y-axis. The point (−4, 1) is the reflection of the point (4, 1) across the y-axis.

The point (3, 5) in the given pentagon is 3 units to the right of the y-axis. The point (−3, 5) is 3 units to the left of the y-axis. The point (−3, 5) is the reflection of the point (3, 5) across the y-axis.

Question 63 (page 154)

D Correct. After a translation of 2 units to the right and 3 units down, 2 is added to the x-coordinate of each point, and 3 is subtracted from the y-coordinate of each point.

Point R (1, 1) of the original quadrilateral has coordinates of $x = 1$ and $y = 1$. Under a translation of 2 units to the right, its x-coordinate will be $(1 + 2) = 3$. Under a translation of 3 units down, its y-coordinate will be $(1 - 3) = -2$.

Point R' will have coordinates (3, −2).

Point T (5, 8) of the original quadrilateral has coordinates of $x = 5$ and $y = 8$. Under a translation of 2 units to the right, its x-coordinate will be $(5 + 2) = 7$. Under a translation of 3 units down, its y-coordinate will be $(8 - 3) = 5$.

Point T' will have coordinates (7, 5).

Question 64 (page 155)

C Correct. The scale factor of the dilation is greater than 1. The figure will be enlarged.

The ratios of the lengths of corresponding sides should be $\frac{2}{1}$.

Compare sides LM and $L'M'$ since they are vertical.

L (3, 4), M (3, 6), L' (6, 8), M' (6, 12)

Segment LM is vertical and has a length of $6 - 4 = 2$.

Segment $L'M'$ is vertical and has a length of $12 - 8 = 4$.

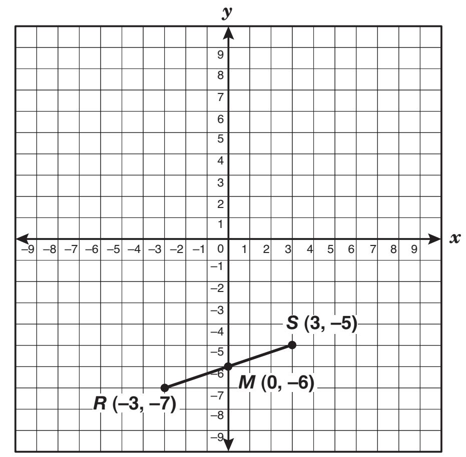
The side lengths are in the ratio of $\frac{2}{1}$.

Only choice C has all the corresponding side lengths in this ratio.

Question 65 (page 155)

D Correct. The midpoint of a line segment is found by averaging the x-coordinates and averaging the y-coordinates.

$$M = \left(\frac{-3 + 3}{2}, \frac{-7 + (-5)}{2}\right) = \left(\frac{0}{2}, \frac{-12}{2}\right) = (0, -6)$$



Question 66 (page 156)

A Incorrect. The coordinates of point R are $(3, -2)$. The x-coordinate of point R is 3. Since $3 \nless \frac{3}{2}$, point R does not meet the requirement that $-\frac{7}{2} < x < \frac{3}{2}$.

B Incorrect. The coordinates of point S are $(-4, -3)$. The x-coordinate of point S is -4. Since $-4 \ngtr -\frac{7}{2}$, point S does not meet the requirement that $-\frac{7}{2} < x < \frac{3}{2}$.

C Correct. The coordinates of point T are $(-2, 3)$. The x-coordinate of point T is -2.

$$-2 > -\frac{7}{2} \text{ and } -2 < \frac{3}{2}$$

Thus, $-\frac{7}{2} < -2 < \frac{3}{2}$. Only the x-coordinate of point T satisfies both of the given inequalities.

D Incorrect. The coordinates of point U are $(4, 2)$. The x-coordinate of point U is 4. Since $4 \nless \frac{3}{2}$, point U does not meet the requirement that $-\frac{7}{2} < x < \frac{3}{2}$.

Question 67 (page 156)

D Correct. To translate a point 2 units to the right, add 2 to its x-coordinate. The x-coordinate of the point $(m, 2n)$ is m. The x-coordinate of the translated point is 2 larger than m, or $m + 2$. The y-coordinate of the point is unaffected by a translation 2 units to the right. The y-coordinate of the point remains $2n$. The coordinates of the translated point are $(m + 2, 2n)$.

Question 68 (page 156)

C Correct. The x-coordinate of the given point is $\frac{8}{3} = 2\frac{2}{3}$. The point should be between 2 and 3 on the x-axis. The x-coordinates of points M and N are between 2 and 3. The y-coordinate of the given point is $-\frac{9}{5} = -1\frac{4}{5}$. The point should be between -1 and -2 on the y-axis. The y-coordinates of points L and N are between -1 and -2. Only point N has both the correct x- and y-coordinates.

Objective 7

Question 69 (page 164)

A Incorrect. This is the top view of the object.

B Incorrect. This is the front view of the object.

C Correct. This is not a top, front, or side view of the object.

D Incorrect. This is the right-side view of the object.

Question 70 (page 165)

A Correct. While a structure with this right-side view could exist with the given front view, it would require 11 cubes to build.

Question 71 (page 165)

D Correct.
Use the formula for the area of a rectangle to find the areas of the two panels she painted.

$$10 \text{ ft} \cdot 20 \text{ ft} = 200 \text{ ft}^2$$

$$4 \text{ ft} \cdot 15 \text{ ft} = 60 \text{ ft}^2$$

Find their sum.

$$200 + 60 = 260 \text{ ft}^2$$

Then subtract their sum from 400 to find the number of square feet that the paint left in the can will cover.

$$400 \text{ ft}^2 - 260 \text{ ft}^2 = 140 \text{ ft}^2$$

The remaining paint will cover at most 140 ft^2. Only the rectangle in answer choice D has an area that is less than or equal to 140 ft^2 because $10 \cdot 12 = 120$, and $120 < 140$.

Question 72 (page 166)

C Correct.

If the three lengths are to form the sides of a right triangle, then they must satisfy the Pythagorean Theorem, $a^2 + b^2 = c^2$.

The greatest number in the set is 16. The hypotenuse of a right triangle is the longest side. Let $c = 16$. The lesser numbers are 8 and 15. Let $a = 8$ and $b = 15$.

Does $8^2 + 15^2 = 16^2$?

Does $64 + 225 = 256$?

No, because $64 + 225 = 289$. Since $289 \neq 256$, the set of numbers 8, 15, and 16 could not be the sides of a right triangle.

Question 73 (page 166)

The correct answer is 10. Use a proportion to find the answer.

$$\frac{18 \text{ ft}}{4 \text{ in.}} = \frac{45 \text{ ft}}{x \text{ in.}}$$

$$\frac{18}{4} = \frac{45}{x}$$

$$18x = 4 \cdot 45$$

$$18x = 180$$

$$x = 10$$

The longer side of the scale drawing is 10 inches.

		1	0	.			
0	0	0	●		0	0	0
1	1	●	1		1	1	1
2	2	2	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Question 74 (page 167)

B Correct.

Since the tiles are in square inches, find all areas in square inches.

The area of the countertop is its length times its width.

$$A = lw$$

$$A = 5 \text{ ft } 4 \text{ in.} \cdot 2 \text{ ft } 2 \text{ in.}$$

$$A = 64 \text{ in.} \cdot 26 \text{ in.}$$

$$A = 1664 \text{ in.}^2$$

The area of the circular sink equals π times its radius squared.

$$A = \pi r^2$$

$$A = \pi \cdot 8^2$$

$$A = 64\pi$$

$$A \approx 201 \text{ in.}^2$$

The area of the countertop, not including the sink, is equal to the area of the rectangular top minus the area of the circular sink.

$$1664 - 201 = 1463 \text{ in.}^2$$

Each tile is 2 in. by 2 in., so it has an area of $2 \cdot 2 = 4 \text{ in.}^2$.

Divide 1463 by 4 to find the number of tiles needed.

$$1463 \div 4 = 365.75$$

Ed will need at least 366 tiles.

Question 75 (page 167)

B Correct.

If the three lengths are to form the sides of a right triangle, then they must satisfy the Pythagorean Theorem ($a^2 + b^2 = c^2$). The greatest number in the set is 22.5, so $c = 22.5$. Let $a = 18$ and $b = 13.5$.

$$18^2 + 13.5^2 = 22.5^2$$

$$324 + 182.25 = 506.25$$

$$506.25 = 506.25$$

Therefore 18, 13.5, and 22.5 do form a right triangle.

Objective 8

Question 76 (page 188)

B Correct.

To calculate the total surface area using a net, add the areas of all the surfaces.

The rectangle forms the curved surface of the cylinder. The length of the rectangle is equal to the circumference of the circular base.

$$C = \pi d$$
$$C = \pi \cdot 5$$
$$C \approx 15.7$$

The rectangle has a length of 15.7 m and a width of 2.8 m. The area of the rectangle is $15.7 \cdot 2.8 = 43.96$ m^2.

The cylinder has two circular surfaces. Use the formula for the area of a circle. Each circle has a diameter of 5 m. The radius is half the diameter. The radius is $5 \div 2 = 2.5$ m. Use 2.5 for r.

$$A = \pi r^2$$
$$A = \pi \cdot (2.5)^2 \approx 19.635 \text{ m}^2$$

The area of one circular surface is about 19.635 m^2.

To find the surface area, add the areas of all the surfaces.

$$S \approx 43.96 + 2 \cdot 19.635 \approx 83.23$$

Rounding to the nearest square meter, the surface area of the cylinder is 83 m^2.

Question 77 (page 188)

C Correct. The surface area of a prism is the sum of the areas of its surfaces.

To find the area of the triangular surfaces, measure the length of the base of the triangle and its height. The length of the base is 3 cm, and the height is 2.6 cm.

Use the formula for the area of a triangle.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \cdot 3 \cdot 2.6$$
$$A = 3.9 \text{ cm}^2$$

The area of each triangular surface is 3.9 cm^2.

Find the area of the rectangular surfaces. Measure the length and width of one of the rectangles. The length is 4.4 cm, and the width is 3 cm. The area of each rectangular surface is $3 \cdot 4.4$, or 13.2 cm^2.

Find the sum of the areas of all the surfaces to find the total surface area of the prism. The prism has 2 triangular surfaces and 3 rectangular surfaces.

$$S = 2 \cdot 3.9 + 3 \cdot 13.2 = 7.8 + 39.6 = 47.4 \text{ cm}^2$$

Choice C, 47 cm^2, is closest to this value.

Question 78 (page 189)

A Correct. The edge length of the cubical box is equal to the diameter of the base of the cone, represented by $2r$. As the dimensions of the cube are all $2r$, the volume would then be equal to the expression $(2r)^3$.

Question 79 (page 189)

B Correct.

The passageway is a rectangular prism. Calculate the volume of the prism.

$$V = Bh = (lw)h$$
$$V = (6 \cdot 4) \cdot 3$$
$$V = 72 \text{ ft}^3$$

The part of the pipe that is inside the passageway is a cylinder that is 6 feet long, $h = 6$ ft. The radius, r, of the cylinder is the diameter divided by 2.

$$r = 30 \div 2 = 15 \text{ in.}$$

Convert the radius to feet; divide 15 by 12.

$$r = 15 \div 12 = 1.25 \text{ feet}$$

Calculate the volume of the cylinder.

$$V = Bh = \pi r^2 h$$
$$V = \pi \cdot (1.25)^2 \cdot 6$$
$$V \approx 29.45 \text{ ft}^3$$

Subtract the volume of the pipe from the volume of the passageway to find the volume of insulating material needed to fill the space around the pipe.

$$72 - 29.45 = 42.55 \approx 43 \text{ ft}^3$$

To the nearest cubic foot, the volume of the space to be filled with insulating material is 43 cubic feet.

Question 80 (page 190)

B Correct.

On her walk to the scenic overlook, Jillian follows the sidewalk. She walks $300 + 475 = 775$ ft. Compare this distance to the shortcut.

Her journey along the sidewalk and her shortcut form a right triangle. The shortcut is the hypotenuse, c, of the right triangle.

Use the Pythagorean Theorem to calculate the length of the shortcut.

$$c = \sqrt{300^2 + 475^2} = \sqrt{90{,}000 + 225{,}625}$$
$$= \sqrt{315{,}625} \approx 561.81$$

Subtract the length of the shortcut from the distance walked on the sidewalk to determine how much shorter the trip back to the parking lot is.

$$775 - 561.81 = 213.19 \text{ ft}$$

To the nearest whole foot, the walk back to the parking lot is 213 feet shorter.

Question 81 (page 190)

D Correct. The triangles are similar, so the lengths of the corresponding sides are proportional. Write a proportion. Compare the ratio of a known pair of corresponding sides to the ratio of the unknown side and its corresponding side. Let x represent the length of side ST.

$$\frac{LN}{RT} = \frac{MN}{ST}$$

$$\frac{8}{10} = \frac{7}{x}$$

Use cross products to solve for x.

$$8x = 10 \cdot 7$$
$$8x = 70$$
$$x = 8.75$$

The length of side ST is 8.75 units.

Question 82 (page 190)

The correct answer is 21.

The rectangles are similar, so the lengths of the corresponding sides are proportional.

Let w represent the missing width.

$$\frac{\text{smaller width}}{\text{larger width}} = \frac{\text{smaller height}}{\text{larger height}}$$

$$\frac{18}{w} = \frac{24}{28}$$

$$24w = 18 \cdot 28$$
$$24w = 504$$
$$w = 21$$

The larger poster board is 21 inches wide.

		2	1	.			
0	0	0	0		0	0	0
1	1	1	**1**		1	1	1
2	2	**2**	2		2	2	2
3	3	3	3		3	3	3
4	4	4	4		4	4	4
5	5	5	5		5	5	5
6	6	6	6		6	6	6
7	7	7	7		7	7	7
8	8	8	8		8	8	8
9	9	9	9		9	9	9

Question 83 (page 190)

D Correct. If the dimensions of two similar figures are in the ratio $\frac{a}{b}$, then their areas will be in the ratio $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$. The diameters of the two circles are in the ratio of $\frac{3}{2}$. The area of the two circles will then be in the ratio $\left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$. The ratio of the area of the larger circle to that of the smaller circle is $\frac{9}{4}$.

Question 84 (page 191)

A Correct. When you multiply the dimensions of a figure by a scale factor, the perimeter of the figure changes by the scale factor. In this case, the scale factor is $\frac{2}{5}$.

To find the perimeter of triangle PQR, multiply the perimeter of triangle MNO by the scale factor $\frac{2}{5}$.

$$45 \cdot \frac{2}{5} = 18 \text{ cm}$$

The perimeter of triangle PQR is 18 centimeters.

Question 85 (page 191)

D Correct.

When you multiply the dimensions of a three-dimensional figure by a scale factor, the volume of the figure changes by the cube of that scale factor. The scale factor is 3. The cube of the scale factor is 3^3.

$$3^3 = 27$$

To find the volume of the larger tank, multiply the volume of the smaller tank by 27.

$$300 \cdot 27 = 8100 \text{ gal}$$

The volume of the larger tank is 8100 gallons.

Question 86 (page 191)

B Correct.

If the dimensions of two similar three-dimensional figures are in the ratio $\frac{a}{b}$, then their volumes will be in the ratio $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$. The dimensions of the two figures are in the ratio $\frac{2}{3}$. The ratio of their volumes will be the cube of that ratio.

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

To find the volume of the smaller box, multiply the volume of the larger box by $\frac{8}{27}$.

$$162 \cdot \frac{8}{27} = 48 \text{ in.}^3$$

The volume of the smaller box is 48 cubic inches.

Question 87 (page 191)

C Correct.

Substituting into $V = \frac{4}{3}\pi r^3$ gives $V = \frac{4}{3}\pi(6)^3$, since the radius is 6 inches. Therefore $V = \frac{4}{3}\pi \cdot 216$, which gives a volume of approximately 904 cubic inches.

Question 88 (page 191)

A Correct. Use the formula for the total surface area of a prism: $S = Ph + 2B$. The height of the prism is 4. The perimeter of the triangular base is $3 + 4 + 5 = 12$. The third side of the right triangle is 5 since this is a Pythagorean triple. The area of the triangular base is $\frac{1}{2} \cdot 3 \cdot 4 = 6$. Therefore $S = (12)(4) + 2(6)$, or 60 square feet.

Objective 9

Question 89 (page 211)

B Correct.

Use a proportion to determine how many hours Rhonda actually spent on the project.

Let n represent the number of hours Rhonda actually spent on the project. Write a proportion.

$$\frac{12}{n} = \frac{80}{100}$$
$$80n = 12 \cdot 100$$
$$80n = 1200$$
$$n = 15$$

Rhonda actually spent 15 hours on the project.

Question 90 (page 211)

B Correct.

Use a proportion to solve this rate problem. Write two ratios, each of which compares the number of switches to the number of hours. Let x equal the number of hours required to manufacture 210 switches.

$$\frac{35}{1.5} = \frac{210}{x}$$
$$35x = 1.5 \cdot 210$$
$$35x = 315$$
$$x = 9$$

It will take the worker 9 hours to manufacture 210 switches.

Question 91 (page 211)

A Correct. The events are dependent because the outcome of the first draw affects the outcome of the second draw. Find the probability of the first event. For the first draw, 4 of the 25 possible outcomes are numbers less than five: 1, 2, 3, and 4. The probability that the first number drawn will be less than 5 is $\frac{4}{25}$.

$$P(\text{1st} < 5) = \frac{4}{25}$$

Find the probability of the second event. One number was drawn, so now there are only 24 numbers in the box. If the first number drawn was a number less than 5, only 3 of the 24 possible outcomes can be numbers less than 5. The probability that the second number drawn will be less than 5 is $\frac{3}{24}$.

$$P(\text{2nd} < 5) = \frac{3}{24}$$

The probability that both numbers will be less than 5 is equal to the product of the two probabilities.

$$P(\text{1st} < 5 \text{ and 2nd} < 5) = P(\text{1st} < 5) \cdot P(\text{2nd} < 5)$$
$$= \frac{4}{25} \cdot \frac{3}{24} = \frac{12}{600} = \frac{1}{50}$$

Question 92 (page 211)

C Correct.

In 811 times at bat, Reggie got $210 + 20 + 1 + 6 = 237$ hits. The experimental probability that Reggie will get a hit during his next time at bat is the ratio of the number of times he got a hit (237) to the number of trials (811).

$$\frac{237}{811} \approx 0.292$$

The probability that Reggie will get a hit during his next time at bat is 0.292.

Question 93 (page 212)

C Correct.
Use the experimental probability to predict the number of seeds that will be 2 inches or more in height.

There were a total of $9 + 16 + 26 + 24 = 75$ plants in this experiment. Of these, $16 + 26 + 24 = 66$ plants reached a height of at least 2 inches after 3 months. The experimental probability that a plant will reach a height of at least 2 inches is $\frac{66}{75}$, or $\frac{22}{25}$.
Let n represent the number of plants out of 600 that will reach a height of at least 2 inches. Write a proportion.

$$\frac{n}{600} = \frac{22}{25}$$
$$25n = 600 \cdot 22$$
$$25n = 13{,}200$$
$$n = 528$$

Therefore, 528 plants out of 600 can be expected to reach a height of at least 2 inches in 3 months.

Question 94 (page 212)

C Correct. The median is the middle value when the data elements are listed in order. Half the scores are above the median, and half the scores are below it. Therefore, Jean should use the median of the data.

Question 95 (page 213)

D Correct.
Trish spent a total of

$600 + 200 + 360 + 40 + 50 + 250 = \$1500$.

The plane fare should be $\frac{600}{1500} = \frac{2}{5} = 0.4$, or 40% of the graph. This is larger than $\frac{1}{4}$, or 25%, of the graph and less than $\frac{1}{2}$, or 50%, of the graph. The cost of food should be $\frac{200}{1500} = \frac{2}{15} \approx 0.13$, or 13% of the graph. Together, plane fare and food should be 40% + 13% = 53%, or slightly more than half the graph. Hotel costs should be $\frac{360}{1500} = \frac{6}{25} = 0.24$, or 24% of the graph. Hotel costs will be slightly less than $\frac{1}{4}$, or 25%, of the graph. Only choice D fits these requirements.

Question 96 (page 214)

B Correct. The January bar is slightly above 1000, at approximately 1100. The February bar is less than halfway between 2000 and 3000, at approximately 2400. The March bar is slightly above 3000, at approximately 3100. The April bar is between 2000 and 3000, at approximately 2600; it is higher than the February bar.

Question 97 (page 215)

A Incorrect. The mean gives the average of a set of numbers, not the most frequently occurring data element.

B Incorrect. The median gives the middle value of a set of numbers, not the most frequently occurring data element.

C Correct. The mode tells which value occurs most frequently. In this case the mode is 7. The value 7 occurs 5 out of the 8 times listed. It shows that Ava most frequently spends 7 hours studying during a week.

D Incorrect. The range measures the spread between the highest and lowest values in the data, not the most frequently occurring data element.

Question 98 (page 215)

A Incorrect.
Sales increased at a constant rate of 5 CDs per week for the first three weeks, but they then decreased during the fourth week.

B Correct. Calculate the mean number of CDs sold per week.

$$\frac{280 + 285 + 290 + 285 + 295 + 300}{6} = 289.1\overline{6} \approx 289$$

C Incorrect. The length of the bar for Week 6 is three times the length of the bar for Week 1. However, read the numerical values of the bars from the graph. The value for the bar for Week 1 is 280. The value of the bar for Week 6 is 300. Since $300 \neq 3 \cdot 280$, one bar is not three times as large as the other.

D Incorrect. Calculate the total number of CDs sold.

$280 + 285 + 290 + 285 + 295 + 300 = 1735$

Since $1735 \ngtr 1800$, the store did not sell more than 1800 CDs during the six-week period.

Question 99 (page 216)

A Correct. The minimum for this data set is 3, and the maximum is 27. The median is 10, the lower quartile is 5, and the upper quartile is 20.

Objective 10

Question 100 (page 231)

C Correct.

Calculate the cost of the DVD player at Store A with the 10% discount.

Use a proportion to find 10% of 119.

$$\frac{10}{100} = \frac{x}{119}$$

$$100x = 1190$$

$$x = 11.9$$

10% of \$119 is \$11.90.

$$\$119.00 - \$11.90 = \$107.10$$

The cost of the DVD player at Store A with the 10% discount is \$107.10.

Calculate the cost of the DVD player at Store B with the 15% off coupon.

Use a proportion to find 15% of 138.

$$\frac{15}{100} = \frac{x}{138}$$

$$100x = 2070$$

$$x = 20.7$$

15% of \$138 is \$20.70.

$$\$138.00 - \$20.70 = \$117.30$$

The cost of the DVD player at Store B with the 15% discount is \$117.30.

This is more than the cost at Store A. Subtract to find the difference.

$$\$117.30 - \$107.10 = \$10.20$$

The DVD player costs \$10.20 more at Store B than at Store A.

Question 101 (page 231)

D Correct.

One way to solve this problem is to find the number of tiles needed to fill the large rectangle and then subtract the number of tiles that would have been needed for the small rectangle that will not be tiled.

If the tiles are 3 inches on a side, there are 4 tiles per foot ($3 \cdot 4 = 12$).

The width of the large rectangle is 2.5 feet. Multiply 4 tiles per foot by 2.5 feet to find the number of tiles needed for the width.

$$4 \cdot 2.5 = 10$$

The length of the large rectangle is 6 feet. Multiply 4 by 6 to find the number of tiles needed for the length.

$$4 \cdot 6 = 24$$

Multiply the number of tiles needed for the width by the number of tiles needed for the length to find the number of tiles needed to fill the large rectangle.

$$10 \cdot 24 = 240$$

In the same way, find the number of tiles needed to fill the small rectangle.

Number of tiles needed for the width: $4 \cdot 1.5 = 6$

Number of tiles needed for the length: $4 \cdot 2 = 8$

Number of tiles needed to fill the small rectangle: $6 \cdot 8 = 48$

Subtract the tiles needed to fill the small rectangle from the tiles needed to fill the large rectangle to find the total number of tiles needed.

$$240 - 48 = 192 \text{ tiles}$$

Jessica will need 192 tiles. The tiles are sold in boxes of 100 tiles per box; she will need 2 boxes of tiles.

Question 102 (page 231)

B Correct.

The cost of producing 250 staplers can be found by evaluating the original function for $n = 250$.

$$c = 1.12n + 300$$

$$c = 1.12 \cdot 250 + 300$$

$$c = 280 + 300$$

$$c = 580$$

If the fixed costs increase by 15%, use a proportion to find 15% of \$300.

$$\frac{15}{100} = \frac{x}{300}$$

$$100x = 4500$$

$$x = 45$$

If the fixed costs increase from \$300 to \$300 + \$45 = \$345, the increased cost per day of producing n staplers is given by the formula $c = 1.12n + 345$. The increased cost of producing 250 staplers can be found by evaluating the new function for $n = 250$.

$$c = 1.12n + 345$$

$$c = 1.12 \cdot 250 + 345$$

$$c = 280 + 345$$

$$c = 625$$

Find the percent by which the cost of producing 250 staplers increased.

$$625 - 580 = 45$$

What percent is 45 of 580? Write a proportion.

$$\frac{x}{100} = \frac{45}{580}$$
$$580x = 4500$$
$$x \approx 7.76$$

The cost of producing 250 staplers increased by about 8%.

Question 103 (page 231)

C Correct. If Jesse and Philippe start at the same time and meet along the road, they both travel the same amount of time. Let t represent the time each travels. The rate they each travel multiplied by the time they each travel is equal to the distance they each travel ($d = rt$). The distance Jessie travels plus the distance Philippe travels equals 5 miles. Write an equation that shows the sum of the distances they travel equals 5 miles.

$$4t + 6t = 5$$
$$10t = 5$$
$$t = 0.5$$

They will meet in 0.5 hour, or 30 minutes.

Question 104 (page 232)

C Correct. The function $h = 12 - 0.1m$ when rewritten as $h = -0.1m + 12$ is in the form $y = mx + b$, so it is a linear function.

To find the height of the candle when it started burning at $m = 0$, substitute 0 for m in the function.

$$h = 12 - (0.1)(0)$$
$$h = 12 - 0$$
$$h = 12$$

The candle was 12 inches tall when it started burning.

To find the height of the candle when $m = 120$ minutes, substitute 120 for m in the function.

$$h = 12 - (0.1)(120)$$
$$h = 12 - 12$$
$$h = 0$$

The candle can burn for at most 120 minutes because in that amount of time its height will have been reduced to 0 inches.

If the height of the candle were directly proportional to the number of minutes it burned, the function comparing them would be in the form $y = kx$. The function $h = 12 - 0.1m$ is not in this form.

Question 105 (page 232)

A Correct. Let x represent the cost of the shirt. Then $2x$ represents the cost of the pants. The sum of the costs of the three items, shirt + pants + shoes, equals the total cost, \$100. Use the equation $x + 2x + 40 = 100$. If like terms are simplified, this is the same equation as $3x + 40 = 100$.

B Incorrect. Let x represent the width of the rectangle. The length is 40 more than three times the width. The length is $3x + 40$.

To find the perimeter, use the formula $P = 2(l + w)$.

$$100 = 2(3x + 40 + x)$$
$$100 = 2(4x + 40)$$
$$100 = 8x + 80$$

Use the equation $100 = 8x + 80$ to solve this problem.

C Incorrect. Let x represent the number of years. To solve this problem, use the simple interest formula $I = prt$. The interest rate is 3%, or 0.03.

Use the equation $40 = 100(0.03)(x)$, or $40 = 3x$, to solve this problem.

D Incorrect. Let x represent the number of cups sold. If the student council earns \$3 for each cup sold, $3x$ represents the amount they earn for selling x cups. To find the profit, subtract the amount they spent on the cups. Use the equation $3x - 40 = 100$ to solve this problem.

Question 106 (page 232)

C Correct. One way to find the values of x for which $f(x) \geq g(x)$ is to write the following inequality and simplify it.

$$f(x) \geq g(x)$$
$$2(x - 3) + 7 \geq \frac{1}{2}x + 7$$
$$2x - 6 + 7 \geq \frac{1}{2}x + 7$$
$$2x + 1 \geq \frac{1}{2}x + 7$$

Question 107 (page 233)

B Correct. The graphs show a pattern. All the graphs are parabolas, and all are congruent. The

x-intercepts of the graphs equal the value by which x is decreased in the function.

The function $y = (x - 1)^2$ intersects the x-axis at the point $x = 1$.

The function $y = (x - 3)^2$ intersects the x-axis at the point $x = 3$.

The function $y = (x - 5)^2$ intersects the x-axis at the point $x = 5$.

Based on this pattern, the function $y = (x - 2)^2$ should intersect the x-axis at the point $x = 2$.

Question 108 (page 234)

A Correct.

Look for the pattern. The dimensions of the cylinders are increasing by a scale factor of 1.5.

$$\frac{3}{2} = \frac{4.5}{3} = \frac{6.75}{4.5} = 1.5$$

$$\frac{6}{4} = \frac{9}{6} = \frac{13.5}{9} = 1.5$$

To find the volume of the previous cylinder in this series, you could increase its dimensions by the same scale factor, 1.5.

Question 109 (page 234)

B Correct. Represent the number of visitors the site had each week in terms of x.

First Week: x

Second Week: $2x$
(twice as many as the first week)

Third Week: $2x + 22$
(22 more than the second week)

Fourth Week: $2.5x$
(2.5 times as many as the first week)

The mean of the number of visitors is equal to the sum of the number of visitors divided by the number of weeks for which records were kept, 4.

Represent their sum.

1st wk + 2nd wk + 3rd wk + 4th wk =

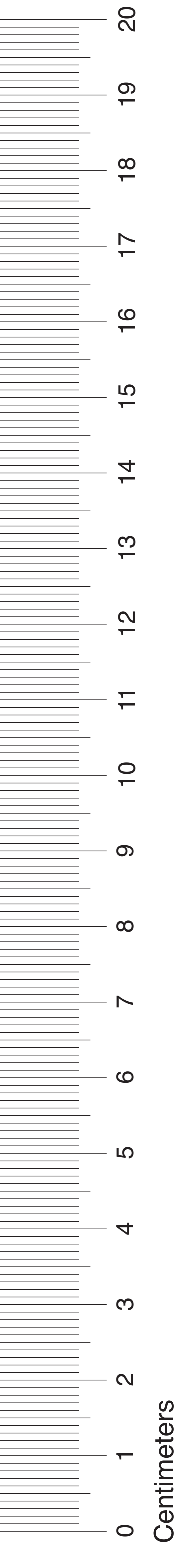
$x + 2x + (2x + 22) + 2.5x$

Represent their mean.

$$\frac{x + 2x + (2x + 22) + 2.5x}{4}$$

TAKS Texas Assessment of Knowledge and Skills

Grades 9, 10, and Exit Level Mathematics Chart



LENGTH

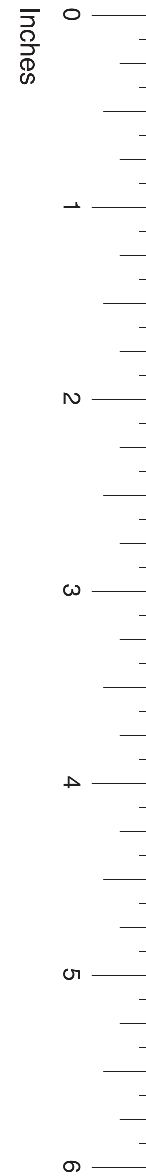
Metric	Customary
1 kilometer = 1000 meters	1 mile = 1760 yards
1 meter = 100 centimeters	1 mile = 5280 feet
1 centimeter = 10 millimeters	1 yard = 3 feet
	1 foot = 12 inches

CAPACITY AND VOLUME

Metric	Customary
1 liter = 1000 milliliters	1 gallon = 4 quarts
	1 gallon = 128 fluid ounces
	1 quart = 2 pints
	1 pint = 2 cups
	1 cup = 8 fluid ounces

MASS AND WEIGHT

Metric	Customary
1 kilogram = 1000 grams	1 ton = 2000 pounds
1 gram = 1000 milligrams	1 pound = 16 ounces



TIME

1 year = 365 days
1 year = 12 months
1 year = 52 weeks
1 week = 7 days
1 day = 24 hours
1 hour = 60 minutes
1 minute = 60 seconds

Continued on the next side

Grades 9, 10, and Exit Level Mathematics Chart

Perimeter	rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference	circle	$C = 2\pi r$ or $C = \pi d$
Area	rectangle	$A = lw$ or $A = bh$
	triangle	$A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$
	trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$ or $A = \frac{(b_1 + b_2)h}{2}$
	regular polygon	$A = \frac{1}{2}aP$
	circle	$A = \pi r^2$
P represents the Perimeter of the Base of a three-dimensional figure.		
B represents the Area of the Base of a three-dimensional figure.		
Surface Area	cube (total)	$S = 6s^2$
	prism (lateral)	$S = Ph$
	prism (total)	$S = Ph + 2B$
	pyramid (lateral)	$S = \frac{1}{2}Pl$
	pyramid (total)	$S = \frac{1}{2}Pl + B$
	cylinder (lateral)	$S = 2\pi rh$
	cylinder (total)	$S = 2\pi rh + 2\pi r^2$ or $S = 2\pi r(h + r)$
	cone (lateral)	$S = \pi rl$
	cone (total)	$S = \pi rl + \pi r^2$ or $S = \pi r(l + r)$
	sphere	$S = 4\pi r^2$
Volume	prism or cylinder	$V = Bh$
	pyramid or cone	$V = \frac{1}{3}Bh$
	sphere	$V = \frac{4}{3}\pi r^3$
Special Right Triangles	$30°, 60°, 90°$	$x,\ x\sqrt{3},\ 2x$
	$45°, 45°, 90°$	$x,\ x,\ x\sqrt{2}$
Pythagorean Theorem		$a^2 + b^2 = c^2$
Distance Formula		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope of a Line		$m = \frac{y_2 - y_1}{x_2 - x_1}$
Midpoint Formula		$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Quadratic Formula		$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Slope-Intercept Form of an Equation		$y = mx + b$
Point-Slope Form of an Equation		$y - y_1 = m(x - x_1)$
Standard Form of an Equation		$Ax + By = C$
Simple Interest Formula		$I = prt$